238

BOSTON STUDIES IN THE PHILOSOPHY OF SCIENCE

History of Science, History of Text

Edited by
Karine Chemla

Springer

History of Science, History of Text

BOSTON STUDIES IN THE PHILOSOPHY OF SCIENCE

VOLUME 238

HISTORY OF SCIENCE, HISTORY OF TEXT

Edited by

KARINE CHEMLA
REHSEIS — CNRS & University Paris 7



A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 1-4020-2320-0 (HB)
ISBN 1-4020-2321-9 (e-book)

Published by Springer,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Printed on acid-free paper

Printed in the Netherlands.

TABLE OF CONTENTS

KARINE CHEMLA

HISTORY OF SCIENCE, HISTORY OF TEXT: AN INTRODUCTION

Practitioners of the sciences have always been "scriptors" in the largest sense of the term. In the course of their research, they wrote inscriptions on temporary surfaces such as sand, wax or scratch paper. Their thoughts and results were also set out and recorded in more permanent forms and materials, whether for wider circulation or for archival purposes.

Let us call all such items texts. They fulfilled all kinds of functions, e.g., exposé, record, exploration. Each such function required some specific form of textual processing that conformed to some norms and related to communities of scholars. According to the period of time and social group in which they took shape, texts produced in relation to these various functions were linked in different ways, ranging from radical discontinuity to strong continuity, depending, for instance, on whether they were inscribed on the same support or not.

These products, in all their variety, constitute the topic of this book.

In recent years, the texts produced by the various practitioners of the sciences have been at the center of an increasing number of publications in science studies.[1] This volume aims at bringing a specific contribution to this emerging field in several ways.

First, the volume is predicated on the conviction that, in order to approach any topic in the history of science, and above all the analysis of the texts produced in the context of scientific activities, it may be fruitful to consider it from a global perspective. This is the reason why the various papers gathered hereafter deal with not only Western sources, but also texts produced in a great variety of places on the planet, and during a large time span.

Besides Greek geometrical texts of antiquity, sixteenth century French books in algebra, seventeenth century scientific manuscripts and paintings, eighteenth and nineteenth century memoirs published by European academies or scientific journals, and Western *Opera Omnia*, the reader will discover the problems of interpretation relating to reading Babylonian clay tablets and Chinese books and illustrations. Through analysis bearing on ancient Chinese and Sanskrit sources, he or she will explore the diversity of forms which texts have taken in history and the wide range of uses they have inspired. Spreading a large spectrum of extant texts under our eyes may help us to realize how particular each kind of text is. Moreover, as we shall see, it should contribute to highlighting the difficulties attached to interpreting our sources.

Secondly, this volume opts for a specific approach to scientific texts, in that it intentionally breaks with the assumption that there be two aspects of text that may be treated separately. It rejects the divide between, on the one hand, a "content", which would consist

of ideas, results, concepts, theories and the like and could be considered independently from the physical constitution of the text itself, and, on the other hand, the appearance of the text, be it its material realization or its rhetoric, that would approach the text as a social or an economic product, independently of the scientific issues it deals with. There are two main reasons to reject this divide.

The first one is fairly obvious: there can be no satisfactory definition of the "content" of a given text, its interpretation being by definition open. Since a new interpretation generally requires going back to material elements of the text itself, this divide is meaningless for the historian. Were it necessary, some examples analyzed in the volume leave no room for doubt on this account.[2]

The second reason for rejecting this divide takes us closer to the topic on which this book concentrates and to a fundamental assumption that lies at its basis. One idea that guided its composition is that practitioners of the sciences are shaping their texts, as texts, in the same process as they are carrying out their intellectual activities. The design of written inscriptions is a constitutive part of the scientific work and an essential condition for research to be carried out.

This volume is thus based on the hypothesis that texts, as such, are to be presumed to be historical objects in every respect. In a first, weaker sense, they are historical objects simply because they were produced at different times and within given working communities. However, in a second, stronger sense —and more importantly for the research project envisioned by this book— they are historical objects because they were produced, as texts, at the same time as the concepts, results and theories which they contain were, and essentially contribute to the shaping and sharing of these ideas.

In other words, we reject the presupposition that, once concepts, results or theories have been obtained by other ways, in an immaterial space, they are merely transcribed in a textual form that remains indifferent to them. In contrast to such a stand, this book assumes that the texts elaborated in the course of the practice of science belong, as such, to the outcome of the scientific work, along with the concepts, results, or theories, in intimate interaction with which they were shaped. In this sense, we believe that the history of the kinds of texts produced by the practitioners of the sciences constitutes a full-fledged subdomain of the history of science itself.

In exactly the same way as for the other outcomes of scientific activities, all kinds of factors, cognitive as well as cultural, technical, social or institutional, enter the shaping of texts. The papers gathered in this volume shed light on several such factors. More generally, the aim we can assign to the subdomain just outlined is thus to identify these factors and to understand how they conjoined in shaping texts. Furthermore, our research program should include exploring how these texts underwent transformations when circulating in space and time, and how they were received and read in different social and professional settings. Fulfilling this program requires putting into play disciplines as different as the linguistics of text, cognitive psychology and sociology of knowledge. To these disciplines, I would suggest adding the history of text, as one field from which the history of science can benefit in developing this research program. As is clear from its title, this relates to a specific focus of this book, which I will now seek to explain.

For the most part, scientific activity goes along with the production, the manipulation, and the use of texts. As already stressed, the texts produced are working tools in at least

two main (interacting) ways. They constitute that with which exploration into problems or questions is carried out. But they also constitute that which is exchanged between scholars or, in other terms, that which is shaped by one (or by some) for use by others.

In these various dimensions, texts obviously depend on the means and technologies available for producing, reproducing, using and organizing writings. In this regard, the contribution of a history of text is essential in helping us approach the various historical contexts from which our sources originate. However, there is more to it.

While shaping texts as texts, the practitioners of the sciences may create new textual resources that intimately relate to the research carried on. One may think, for instance, of the process of introduction of formulas in mathematical texts. This aspect opens up a whole range of extremely interesting questions to which we will return at a later point. But practitioners of the sciences also rely on texts produced by themselves or others, which they bring into play in various ways. More generally, they make use of textual resources of every kind that is available to them, reshaping them, restricting, or enlarging them. Among these, one can think of ways of naming, syntax of statements or grammatical analysis, literary techniques, modes of shaping texts or parts of text, genres of text and so on. In this sense, the practitioners depend on, and draw on, the "textual cultures" available to the social and professional groups to which they belong.

This interrelation constitutes a dimension that this book specifically aims at elaborating. In addition to the fact that texts drastically varied along history from a material point of view, they were also not composed, organized and used in the same way within the different scholarly milieus, say, of Vedic or classical India, antique or medieval China, Hellenistic Greece, or archaic Mesopotamia. This may seem obvious for a remote past. However, as is shown in the book, it also holds true for modern Europe, where local traditions of text developed that have an echo in the production of scientific writings.

The practitioners of the sciences hence shaped their inscriptions within the context of utterly different textual traditions. This partially explains why, when considering texts from a large time span, historians are in fact confronted with wholly different types of object. Such an observation immediately raises two kinds of issue.

On the one hand, we must describe these texts in their specificity. This issue is perhaps more intricate than it first appears, especially regarding ancient texts, as some of the contributions of this volume bring to light. Due to the successive regimes of the text through which some of the writings of antiquity came down to us, they underwent a series of transformation: transliteration, reshaping, editing and so on. This process made them look familiar to the successive generations of users, at the cost of altering them substantially from a textual point of view. It hence requires a critical eye to capture the hints that allow restoring some of their original features. We shall indicate below some of the consequences of this process.

On the other hand, we must elaborate methods for interpreting the various writings. Texts took shape in working communities that developed conventions for reading them and working with them. A naive approach, based on the careless assumption that texts are basically ahistorical objects which were read yesterday as they are today, is likely to miss the actual import of a given text. On this front, much is to be expected from the development of an anthropology of reading which could help us take some critical distance from our usual modes of reading and situate more adequately our sources

within the contexts for which they were composed or in which they were subsequently used.

These are the points concerning which history of science and history of text can develop conjointly. The history of scientific texts certainly falls under a general history of text. Moreover, its development would certainly benefit from a history of text exploring the variety of texts produced in various social groups and the modes of reading that were applied to them through time. Conversely, however, the way in which specific textual resources were put into play in scientific texts can help better understand how these resources have been more generally used in a given group. Their study within this context, because of the specificity of the subject matter, may hence contribute to a general description of these textual resources.

These are the main issues at stake in this collection of papers. The book deals with them in four main parts. After a first part devoted to highlighting how texts and readings may differ from our present-day expectations, the second part considers types of text produced at different moments of scientific activity and exhibits a variety of general factors at play in shaping them. The third part explores several ways in which texts and their circulation can depend on the textual cultures of the milieus producing or using them. In a fourth part, different methods of analysis are used that bring to light specific and unexpected textual features in the most varied scientific sources, which pose distinct problems of interpretation. A final contribution will lead us to formulate a challenging issue that discloses some of the stakes at play in the further development of the research program outlined. Before getting to this final issue, however, let me stress how the various papers gathered below contribute to the questions which form the focus of the book.

The collection of papers thus opens with a first part devoted to a case illustrating clearly how cautious we should be before assuming that our present-day experience equips us with enough insight about the production and uses of texts in the past.

The paper dealing with this case issues this warning regarding the nature of ancient texts. Vera Dorofeeva-Lichtmann's contribution carefully and convincingly argues that some Chinese texts of antiquity that came down to us under the ordinary form of linear discourse, neatly arranged in the sequence of pages of a book, had in fact been originally composed with reference to non-linear spatial schemes. She thereby presents us with a corpus of writings whose composition and reading certainly challenge naive ideas about the texts.

Internal analysis reveals that these texts are constituted of textual components linked according to a non-linear pattern. In fact, the schemes in which the relationships among the textual components can be best displayed turn out to be ancient symbolic representations of geographical space. The texts thus seem to have been composed as a set of textual components, perhaps materially distinct pieces, to be arranged according to a cardinally oriented scheme.

Bringing to light these patterns discloses a feature essential for the interpretation of the texts. On the one hand, the various textual components cannot be interpreted without paying due attention to their interrelationship. Since the text was composed on the basis of a diagram, the requirement of having to fill up the scheme influenced its composition. On the other hand, the scheme according to which the text appears to have

been composed itself constitutes a distinctive aspect of its meaning and must be read as such. In particular, since the patterns emulated usually formed a representation of space, the texts which materially embodied this representation can hence be considered as an early genre of geographical texts.

In order to support her thesis, V. Dorofeeva-Lichtmann relies on two types of argument. First, she surveys Chinese texts of antiquity that were unearthed in archeological discoveries of the last century and present non-linear layouts. In each case, she shows that the scheme according to which the textual components are arranged conveys a spatial dimension and is oriented according to points of the compass. Furthermore, as she stresses, the same features are also shared by all of those texts which later scholarship showed must be reconstructed in a diagrammatic arrangement. Such continuity places texts of this type within a tradition that goes back to high antiquity.

V. Dorofeeva-Lichtmann's second type of argument is based on considering the evolution of the media for writing in ancient China. It leads her to formulate assumptions regarding the way in which such texts were used in antiquity. In brief, V. Dorofeeva-Lichtmann identifies in the shift from rolls of bamboo slips to block-printed books a key factor for transforming the writings once presented in a diagrammatic layout into a linear sequence of sentences. As she convincingly argues, texts written on bamboo slips were susceptible of having their parts rearranged by a simple change in the disposition of the physically separate units. Moreover, a writing composed of several rolls could be read on the basis of a preliminary display of these components according to a spatial scheme. Users may have learnt the procedure for arranging the parts of a text adequately, before reading it. Paying due attention to the materiality of the writings, both at the time of their production and throughout the process of their transmission, appears here to be helpful for approaching our sources. In addition to this, V. Dorofeeva-Lichtmann sketches several consequences of these assumptions. First, it follows that the reading of the text presupposed certain gestures and a certain way of traveling throughout its pieces: most probably, the reader would move around the diagram to read the text in a prescribed way. Secondly, the text, as object, would be inscribed in the cosmos as well as in the social activities, such as rituals, to which it adheres.

In conclusion, this example clearly shows how —as regards both the type of text considered and its uses— a critical awareness is badly needed if we do not want to project onto these writings anachronistic contemporary approaches to texts. Moreover, in this case, only such awareness discloses the relevance of these sources for a history of geography. We see here a first way in which history of science can benefit from such a critical approach to texts.

The same critical awareness is required to analyze how readers of the past carried out their reading. This can be best illustrated by a case where traditions developed of readers who had, towards a given text, expectations that may seem odd to us —a case which should incite us to suspend any a priori idea regarding readers, as much as we just suggested doing for texts. The case I have in mind is the Bible. It may appear as an exception, and its relevance to discussions concerning scientific texts may seem far-fetched. Let us first highlight some of the points made on it by Moshe Idel in his *Absorbing Perfections: Kabbalah and Interpretation*,[3] before coming back to these questions. Dealing with the post-biblical period of the history of Judaism, in which a "textualization of religion"

took place, he discusses how a new conception of the Hebrew bible emerged that saw the canon, rather than the sanctuary, as the place where God was to be encountered. This constituted a major shift in the attitudes towards, and approaches to, the text, which discloses a rupture in the readers' apprehension of the Bible. In fact, M. Idel develops an anthropological approach of the reading of the sacred text that successfully brings to light how, over centuries, various traditions of exegesis elaborated drastically different conceptions and interpretations of the canon as text. Moreover, he sketches some major characteristics of the apparently new representation of the Torah which developed in these rabbinical literatures. Two of them deserve some comments here, since they bring us back to the readers' expectations and to history of science.

One new —and apparently odd— idea concerning the Torah that emerged in the post-biblical Rabbinical writings held it that it encompassed the "whole range of supernal and mundane knowledge". The influence of such a belief on how these scholars approached the natural world certainly constitutes a topic of importance for the historian of science. In fact, M. Idel concentrates on a second assumption of this rabbinical literature, and the investigation of this second assumption helps us to interpret the first idea. These scholars saw, in the Torah, an entity that preceded the creation of the world and that God used as a tool for the fulfillment of this task. M. Idel shows how various interpretations developed to account for how God actually used the canon as text for carrying out the creation.

Still, the premise that a single text encompass the whole of created reality may seem to take us far from scientific texts. In fact, such is not the case. Even in this respect, the example of the Bible brings us back to our topic. Indeed, readers of ancient China developed similar attitudes towards the books to which they granted the status of Canons (*jing*). This holds true not only for the Confucian canons, but also for the canons of scholarly disciplines such as mathematics.[4] In the latter case, for example, the commentators read the oldest of all mathematical Canons, *The nine chapters on mathematical procedures* (first century B.C.E. or C.E.), under the assumption that it should contain the whole of mathematics. We hence find ourselves confronted with exactly the same type of expectation as for the Bible. Even as regards scientific texts, readers of the past thus developed attitudes towards, and approaches of, their texts that we must seek to recover, if we want to avoid practicing an anachronistic reading of our sources. Instead of discarding such statements as strange and concentrating solely on the mathematical content of the Canon, I hence suggest attempting to recover our actors' conception of both mathematics and canonicity that can account for this premise.[5] To this end, comparative approaches of various kinds are most helpful, as J. Henderson successfully showed. Not only should we compare, as he did, the attitudes of readers that adopted similar assumptions towards canonical texts within different cultural contexts. But, when it is possible, as in the case of ancient China, we should also compare the attitudes towards scientific canons and other kinds of scriptures within the same context. Such comparisons can benefit both history of science and history of text in general.

This example hence indicates that our actors' modes of reading can by no means be assumed to be, without further inspection, for any time and any place, identical to those we spontaneously adopt. They might differ from ours, and the systematic observation of their uses of texts, within the context of a historical anthropology of reading, can give us

precious information concerning how readers of the past made sense of the writings that are now our sources.

In conclusion, as this first part shows, neither the definition of a text nor the uses to which it may be submitted are simple matters to handle. As already stressed, this book is based upon the assumption that considering texts selected in a large worldwide corpus allows us to formulate such warnings in the most efficient way. The more the reader will enter into the collection of papers, the clearer the warnings should become.

The second part of the book turns to a consideration of the shaping of scientific texts at two drastically different moments in the process of textual production, namely: the stage of the draft and the completion of *Opera Omnia*. Considering a variety of stages helps us grasp how a wealth of factors is at play in text production, from cognitive factors to social, political or economic ones. In fact, the various kinds of texts considered do not shed a uniform light on each factor.

There are corpuses of scientific texts where the cognitive processes at play in the design of text are more conspicuous. Such is the case in the corpus selected by Eberhard Knobloch, whose article surveys Leibniz's mathematical manuscripts to capture indications concerning how the surface of the page is put into play for the production of knowledge in written form. In doing so, E. Knobloch can rely on one of the most formidable "intellectual workshops" for the production of texts available to historians of science, thereby opening an immense field of research that can be exploited in several directions of research.

A first option would be to consider the process of genesis of a given text. E. Knobloch indicates several instances in which, owing to Leibniz's working style, successive drafts for the same piece are extant and can be ordered chronologically. This constitutes a precious source for studying the driving forces at play in the shaping of a given text, thanks to the rewriting that the successive previous versions made possible. As E. Knobloch explains, Leibniz's continuous written internal dialogue in the process of writing regularly makes explicit the values and motivations at play in his rewriting of the work.

However, Leibniz's manuscripts also allow studying the production of text that goes along with scientific research at another, more abstract, but most promising level. In fact, E. Knobloch gathers several instances where we can observe Leibniz *in the process of* designing, or even tuning, forms of text such as specific tables, figures or graphical representations precisely to inquire into given mathematical questions or domains. The inscriptions that Leibniz works out on the surfaces of his pages, E. Knobloch shows, are the basis for various scientific activities: finding out regularities, checking, easing work, computing, visualizing, proving, recording results in a form appropriate for subsequent use. They serve the "art of invention" as well as the "fixation of insights". We hence have here invaluable source material for dealing with one of the main questions we have in mind, i.e., the dimension of textual invention that adheres to the practice of scientific activities, or, seen from another angle, the intimate relationship between the production of scientific knowledge, in all its dimensions, and the design of inscriptions appropriate for a given range of questions that goes along with it. Not only does Leibniz's archive illustrate in a magnificent way the process of the text production necessary for carrying out scholarly work, but the philosopher Leibniz also developed a theory for it with his

elaboration on the universal characteristic, which turns this case into probably one of the richest ones we may ever expect to consider.

At this end of the spectrum of extant inscriptions, research on the design of text could clearly benefit from the insights provided by cognitive psychology. No matter how private this aspect of text production may seem, however, it can end with the production of forms of text to be circulated in a community. In fact, Leibniz does not only offer source material for observing how the lonely working scholar designs inscriptions in the process of producing knowledge. He also devised notations for public use in the scholarly community. This is best illustrated by the notice he anonymously published in the *Acta eruditorum*, to describe the mathematical notations that could be used for writing articles to be included in this journal.[6] These notations had, in Leibniz's view, the property of fitting the constraints met by typographers, as well as the requirements of scientific work, a point which he established mathematically. This example hence demonstrates how the private workshop where inscriptions are shaped for the sake of carrying out scientific activity is in continuity with the public space where types of text circulate as social objects. The notations described by Leibniz, as material objects, resulted from a negotiation where mathematical and typographical constraints meshed with each other.

At the other end of the spectrum of archives that historians of science handle, other types of factors and actors participating in the shaping of texts appear more conspicuously. Such is the case with the corpus of *Opera Omnia,* on the history of which Michael Cahn presents here some pioneering research. His analysis of this special genre of text, which, in the West, flourished after the spread of printing, discloses a rich sample of such factors and actors. As he emphasizes, this should not surprise us. Generally, *Opera Omnia* do not publish texts, but re-publish them. The production of collected works thus offers a vivid example of how a given text has meaning as object, beyond the letters and figures it contains. Its reprinting in the context of *Opera Omnia* produces a batch of new meanings, some of which are attached to the event of a publication as such.

The constitution of collected works, M. Cahn shows, relates to the fashioning of an author as authority. Even though some scholars may act to promote the publication of their own *Opera Omnia* and manipulate their shaping, it usually requires a social group, a scholarly community or an institution, to support the claim of authority bestowed upon an author and to gather the vast energy essential for the preparation of collected works. The magnitude of the energy needed makes the main actors involved in the production of a text more visible than in other cases. As M. Cahn illustrates, scholarly communities engage in, or support, such endeavors as the production of *Opera Omnia*, since such publications contribute to constituting them, structuring them and making them visible. Communities or institutions usually receive support for *Opera Omnia* from the political power, not least because sculpting scientific authorities participates in the process of nation building. M. Cahn also stresses the specific part played by publishers in the process of producing these particular items. The prestige attached to the enterprises of publishing *Opera Omnia* and the economic consequences that can be expected account for publishers' engaging enormous resources in these achievements. In this sense, the production of such a text is generally a social, economic and political event.

However conspicuous these factors may be, due to the magnitude of the energies engaged in the process, they should not conceal the transformations of the texts as

scholarly items that their reprint under this new form entails. *Opera Omnia* also remain a genre of book, which requires a designer and allows new and specific modes of reading. The shaping of such a work involves specific acts of writing carried out by a kind of actor who has hardly been studied as such in the history of science: the editor. How can the editing of texts, the composition of pieces into a new whole, according to a chronology or by topics, and the adjunction of editorial aids such as indexes, footnotes, tables and the like create new meanings?[7] This fundamental question relates to the vexed issue of the transformation in the meaning of a text caused by changes in its form or its textual environment. If the question remains open, M. Cahn offers hints for carrying out research further along these lines.

In conclusion, as in the case of Leibniz' workshop, analyzing the shaping of *Opera Omnia* reveals a great variety of factors at play in the making of the text. This illustrates why we suggest systematically addressing the production of scientific texts in all their dimensions, since these cannot be taken apart. Texts are not a set of layers nicely separated and corresponding each to one factor. They result from a complex process in which all factors conjoin. Concentrating on only some of them, whichever one may choose, would create a truncated image of the process involved in their design.

As a commentary to Michael Cahn's paper, Hans-Joerg Rheinberger offers several cases in which the authors themselves set out to act as editors, or even authors, of their own *Opera Omnia*. The way in which authors fashioned or re-fashioned their writing self by selecting, arranging and rearranging their previous publications is correlated with motivations comparable to those displayed when institutions or editors carry out this editorial act. H. J. Rheinberger outlines how these motivations can account for the actual shaping of these *Opera Omnia*. The last example he considers remarkably highlights the complexity of the phenomenon of the production of *Opera Omnia*, in history. With the figure of Georges Louis Leclerc de Buffon, Rheinberger discusses the case of an author setting out to writing straightforwardly the "Collected works of natural history", soon to be printed, in one of its numerous versions, under the title *Oeuvres complètes de M. le C^{te} de Buffon*. Opting for this mode of writing clearly relates to the scientific project of the author, and has consequences on how it is fulfilled. It also expresses the social and institutional setting within which Buffon evolves. However, the import of this case goes far beyond the mere phenomenon of showing an author conceiving his written production directly within the form of *Opera Omnia*. H. J. Rheinberger exhibits how, in nineteenth century natural history, a whole tradition of research, to which the greatest names are associated, expressed itself under the form of re-editing the *Oeuvres complètes de Buffon*. The dozens of editions produced under this title correspond, in fact, to deep transformations of the text, including additions, reorganizations and so forth. Producing an edition of the *Oeuvres complètes de Buffon* hence became for several decades a genre of scientific writing in a given community, a genre requiring for the historian to develop specific modes of reading. The phenomenon is not unique: suffice it to evoke the multicentenary tradition of composing *Elements of geometry* under the name of Euclid to show how much these genres, prominent as they were, still await to be studied as such in history of science. Ironically, H. J. Rheinberger suggests, the philological enterprise of restoring the *Oeuvres complètes de Buffon* to their original shape marked the end of the scientific life of the collection as a support for the expression of active research in natural history. Moreover,

the case of the *Oeuvres complètes de Buffon* leads H. J. Rheinberger to stress a shift in the forms of scientific writings, and consequently in the genre of the *Opera Omnia*, in life sciences between the eighteenth and the twentieth century. With the emergence of the "article" to be published in a journal as the main genre of scientific literature, the *Opera Omnia* were to take new shapes and, correlatively, different meanings. As these remarks clearly demonstrate, again in this case, the same genre of text may be the result of different writing acts and relate to different realities. Here again, we meet with the conclusion that, even as regards texts composed in modern and contemporary Europe, developing research on the history of scientific texts should provide us with tools for interpreting our sources in a more rational way.

The previous section was devoted to highlighting general factors at play in the shaping of texts. The third part of the book turns to emphasizing how scientific writings, as objects as well as texts, adhere to the local culture of the social groups in the context of which they were produced or circulated. Again, the various case studies assembled in this part disclose this link in distinct ways. They all show why this dimension is of essential importance for interpreting our sources.

Craig Clunas' s contribution focuses on the literate elite of fifteenth–sixteenth century (Ming) China and to its attitudes towards books devoted to scientific or technical matters. His main emphasis consists in showing how books, both by their topics and their design, can shape, and conversely be shaped by, the social identity of members of a given group.

Seeking to understand what possessing mathematical books meant for a member of this social elite, C. Clunas concentrates on an incident in which a father ordered that his books on mathematical subjects be burnt because his son had refused to learn what his father had learned about numbers. Understanding this episode yields startling insight into the attitude of members of the Ming elite towards knowledge concerning numbers. The episode also helps us investigate the social meaning of the possession of mathematical books in Ming China.

To account for the episode, C. Clunas examines some of the activities that, at the time, were related to numbers. Among them, he lists mercantile activities and surveying. The former, which, in fact, furthered the development of mathematical knowledge in Ming China, certainly constituted a sphere from which members of the educated elite wanted to distance themselves. As to surveying, C. Clunas designates it as a key locus, around which tensions between the bureaucracy and the landholding families developed. The social uses to which numbers were put tainted the perception that the elite could form of them and of the books about them. C. Clunas hence suggests that the development of negative attitudes towards specific domains of knowledge and the cultural artifacts conveying them, such as books, can be interpreted as participating in the shaping of the social identity of members of the elite. C. Clunas also notes how similar meanings can be read in other aspects of the management of books as objects, such as the comparatively rare printing of mathematical books, their mode of inclusion in private libraries, or the way in which they were catalogued. These examples clearly show how the circulation of scientific books in a given society discloses the values that various social groups attached to their possession and to the knowledge that they present. In this vein, the same texts appear to have changing values, according to the time and the milieu.

Considering other books dealing with technical matters and written for the same elite enables C. Clunas to reach a nuanced conclusion. The most interesting case of the *Records of lacquering* (1625) illustrates the appearance, at the time in China, of a new type of technical text, especially designed in relation to the consumption of this social group. Even though it dealt with a topic that might have provoked the contempt of the Ming literate elite, this book approached technology from a new perspective that its members could accommodate, that is, the insight yielded could be used for the appreciation of the artifacts produced rather than for the explanation of how to carry out technical procedures. Such an example again brings us back to our hermeneutic concern: as C. Clunas stresses, when the book was rediscovered in the twentieth century, historians read it as a text composed for workmen, which led to completely misconstruing its description of techniques. This example illustrates the problems arising when considering books independently from the reading communities for which they were written. Interpreting their text today requires retrieving the readership for which they were composed as well as the groups that came to be interested in them.

Furthermore, C. Clunas emphasizes how such technical books, in both the approach of their topic and their design, conformed to the textual culture characteristic of the Ming literate elite. More generally, the way in which texts are shaped also discloses modes of interaction with the culture of the social milieus by which they are produced. The following two contributions, based on two radically different cases, each bring to light distinct aspects of this interaction.

Giovanna Cifoletti's article focuses on the correlations to be established in sixteenth century France between the transformations undergone by algebra and the emphasis placed, in the humanistic culture of the time, on the disciplines of the text. Her article outlines several modes of interaction between these two ranges of phenomena, through the analysis of two books by a same author, Jacques Peletier du Mans: *L'algèbre* (1554) and *L'art poétique* (1555).

First, she suggests capturing the innovations brought about by Peletier in the domain of algebra through the series of transformation he introduced in the *text* of the discipline itself. Among these transformations, one can find symbolic notations and new modes of structuring the presentation of a process of reasoning. It is only by paying attention to the rewriting which Peletier carried out that one can approach his contribution to algebra. The historian thus needs to read these new meanings into what may appear to be a mere reformulation. Interestingly enough, this line of interpretation is in agreement with Peletier's own theory of intellectual innovation, developed in his *L'art poétique,* which, as he suggests, can consist in a rewriting (an 'imitation') that produces new meanings. As G. Cifoletti notes, this orientation, rooted in humanistic culture, accounts precisely for one of the main lines of innovation in algebra after Peletier: changes in the text used to write down algebraic reasonings. This historical case bears witness to the conscious creation of a new type of text for a scientific discipline, in relation to the formation of more general ideas concerning textuality in a given milieu. It clearly shows how the shaping of a kind of text can constitute an essential aspect of the scientific activity itself, thereby designating its description as a task that the historian of science cannot ignore. Placing Peletier's contribution to algebra within the context of the contemporary disciplines of the text not only makes it possible to interpret his writings in a more informed way. It

also brings to light the intimate connection between his influential contribution to algebra and his theories of the text.

On another front, G. Cifoletti shows how Peletier refers to the changes that he made in the text of algebra through the use of terms employed to describe the *oratio*, the production of which constituted the ultimate goal of the humanistic education. This indicates that he was conceiving of his reshaping of the formulation of algebra in humanist terms and with humanist requirements in mind. The text of algebra was hence approached by Peletier himself as belonging to the general category of text on which the humanists focused. Conversely, algebra will emerge as the discipline offering a new model of text which could be imitated in shaping a new kind of *oratio*. This case thus also brings to light how a kind of text, consciously shaped within the context of a scientific discipline, can be exported as a textual model for wider uses.

In his analysis of the mathematical texts produced in ancient India, Pierre-Sylvain Filliozat describes a situation in many respects comparable to that analyzed by G. Cifoletti, even though it developed in a completely different cultural context. His article emphasizes that it is only by reference to the disciplines of the text, i.e., grammar, exegesis and logic, which formed the core of the training in the scholarly communities of Sanskrit pandits, that the specific form taken by mathematical texts and, consequently, the practice of mathematics can be understood. P. S. Filliozat shows how this holds true in two distinct periods in which the practice of texts utterly differed.

In a first, Vedic, period of more than one thousand years, which ended roughly at the beginning of the common era, the term "text" needs to be understood exclusively as "oral text". There developed a complex scholarly culture based on orality and memorizing techniques, for which the pandits received a specific training. This culture elaborated the means necessary for handing down the Vedic scriptures—that is, on one hand, it molded traditions of interpretation and, on the other hand, it yielded means for practicing scholarly activities in the disciplines that took shape for preserving the sacred texts. P. S. Filliozat emphasizes that this constitutes the context in which, roughly between the seventh century and the fourth century B.C.E., scientific literature emerged, for disciplines of the text as well as the ritual, which included mathematics. In other terms, texts dealing with scientific matters first appeared in connection with practices relating to the conservation and enactment of the most fundamental oral archive, i.e., the Veda.

For the case of the most famous and central example of such scholarly texts, Panini's grammar, P. S. Filliozat strikingly describes how its composition reflects the oral textual culture within which it was produced. Reading and interpreting it today demand approaching it as an oral text and restoring the specific technical procedures devised in ancient India for using it as such. The same characteristics hold true for texts dealing with geometrical topics. This case emphasizes that, if speech was not written, oral texts referred to other practices of inscription such as drawing figures with cords and pegs. Mathematical activity hence required the articulation of several forms of text. Ancient India thus again provides a most efficient warning for avoiding oversimplification regarding both orality and the nature of scientific texts.

If the second, "classical", period that P. S. Filliozat considers inherited and preserved the rich culture of oral techniques elaborated in the preceding centuries, it bears witness

to a considerable shift since it experienced the development and generalization of the use of writing. P. S. Filliozat describes how a new type of mathematical literature hence emerged that combined both aspects. In fact, as for any other discipline, these new texts were composed of two components: they recorded a part that was meant to be memorized and expressed in verses, and added to it a part that was more consistent with a written style of a commentary type. Again, this specific kind of text should be considered against the background and rules of the textual culture to which it pertains. This precaution is crucial in order for the historian to interpret the text adequately. But there is more to it: as P. S. Filliozat shows, the exercise of exegesis led commentators to strive to interpret certain stylistic properties of the verses as conveying mathematical meanings. The type of stylistic events that could be deemed meaningful was dictated by the disciplines of the text. Their interpretation required practicing and developing mathematics in a specific way. Here again, therefore, we can observe a close and quite specific interaction between the practice of mathematics and the disciplines of the text.[8] Furthermore, P. S. Filliozat suggests that the new practice of writing and the various forms of expression recorded (such as commentaries) were not restricted to the transcription of speech. Writing allowed the generation or recording of inscriptions specifically belonging to the domain of the written, such as the place-value numeration system. It hence allowed gradually articulating, within the text, the transcription of what previously had been oral speech as well as representations of what once had been attached to other material supports, such as drawings or the successive stages of computations formerly practiced on sand. This attests to a dramatic change in the composition of mathematical texts, to which we will later return.

In the previous two examples, the specificities of mathematical texts could be correlated with the disciplines of the text that developed in the same scholarly communities. Reading the sources therefore requires acquiring some familiarity with this local textual culture, which could provide tools for interpreting the texts and describing the scientific practices to which they bear witness. More generally, our third part illustrated various ways in which history of science could benefit from a history of text, along the lines outlined at the beginning of this introduction.

To recapitulate, the various papers gathered in the book up to this point shed light on the factors at play in the shaping of text. In addition to general factors, from cognitive to social ones, we saw various ways in which the local cultures and the agenda of specific social groups could contribute to the design of the texts. In each case, interpreting the writing in all its dimensions proved to be a tricky issue.

Our fourth and last part will bring us back to the warnings issued on the basis of our first part, in that it presents us with sources that clearly adhere to textual cultures, and the reading of which poses a challenge to the modern exegete.

Each of the following contributions describes a distinctive method of analysis especially designed for bringing to light unexpected specificities of a text or a corpus of texts. Once identified, these specific features, which an ordinary reading would have missed, require interpretation. How are we to read a meaning in these features of the texts? How are we to account for the production of these specificities? However tricky they may appear, they notably help to demonstrate to what extent the texts designed in relation to

the practice of scientific activities may present striking features and can fail to conform to simplistic ideas about the nature of texts.

In his paper devoted to analyzing some Greek geometrical writings of antiquity, Reviel Netz introduces a sophisticated method that brings to light unexpected structural properties of the texts as such. He originally designed his method to address a question regarding the nature of the relationship between the verbal component of these texts and the figures. The question can be formulated as follows: in these Greek geometrical writings, is the verbal component autonomous, in the sense that the diagram would simply be helpful but not necessary for following it, or a mere illustration that could be drawn progressively while reading the text? Or, on the contrary, is the diagram not fully specified by the text and necessary for reading it? Before close inspection, we may be tempted to surmise that, since the logical structure of Greek geometrical writings constitutes a prominent characteristic feature, the verbal components of the text should be self-contained, the drawings being dispensable. Contrary to such an expectation, R. Netz establishes that, in these texts, the discourse relates in such a way to the diagram that its progressive reading *requires* relying on the completed drawing. In other words, from the perspective of the production of the text, R. Netz shows that the way in which a Greek proof is written indicates that the proof was previously already carried out, and the drawing underlying it completed, before the discursive part of the text was written down. This result indicates that the text functions in a way that is radically different from the representation that we would spontaneously develop. Bringing to light the constitutive solidarity between the discourse and the diagram in these cases discloses an unexpected specificity of the text, which needs to be accounted for in terms of the mathematical practice in relation to which the text was produced. However, this by no means exhausts the significance of R. Netz's method. More generally, the method also provides a tool for *analyzing* the intricacy of the relationship of the verbal component of the text to the figure and outlines a typology of the modalities of reference of a discourse to a figure, which can be extended to other cases.

It is not surprising that such methods emerge in history of science in relation to the study of ancient texts. As already stressed above, the process of transmission was such that we often only find evidence that has experienced drastic material transformation with respect to its original form. The historian must hence develop refined "methods of interrogation", in R. Netz' own terms, to draw from the extant sources any possible information regarding the shaping of the original documents. Analyzing the texts as such —our research program— hence appears as yet another means by which the historian can elicit new insights from his or her sources.

In the present case, the question addressed by R. Netz can be of assistance in discussing the exact nature of the original texts. The analysis carried out brings to light the solidarity of the received text with a drawn figure. Without yet solving the question of the original, since we lack information about the material environment of Greek geometrical practices and the uses of text, this result gives us a better command of the possible options. In *The shaping of deduction in Greek mathematics*,[9] R. Netz puts into play other features of the relationship between the narration and the figure, i.e., the assignment of letters to geometrical entities on the figure, to bring out a further indication of the intimate relationship between the two. He shows that, if the drawing was completed *before* the verbal component of the text was composed, its points, however, were named

progressively while writing the exposé of it. These results indicate all that might be expected, in terms of the recovery of ancient scholarly practices, including the shaping of the text, from a careful and detailed analysis of the extant documents.

The form of texts generally reflects to some extent the material environment of the working scientist and the actual practice of scientific activity in relation to which the texts were produced. Regarding ancient periods, these latter aspects are usually those on which we are the most poorly informed, but the previous analysis gives some hope that scrutinizing the extant sources as such may provide insight into these matters. This is also what is at stake in the second approach sketched by R. Netz. In terms of a particular text, the article outlines a description of the modalities of reference to other parts of the same text and to other texts. In the special case of Euclid's *Elements* discussed by R. Netz, the issue is crucial. References of theorems to former ones on which they depend may seem to us intimately related to Euclid's project, at least according to our common view of Euclid. However, in this respect, R. Netz shows that the ancient sources dramatically differ from the critical editions that were produced in the nineteenth century. In fact, not only do they hardly include any internal reference, but the basic tools for referencing, such as numbering the propositions, seem to be a very late development. This radical transformation of the texts through the process of transmission, which R. Netz analyzes in a particular case, still awaits a much more systematic study in the history of science. As regards Euclid's *Elements*, R. Netz suggests interpreting this mutation as the reflection of a transformation in the uses of the text, assuming that the absence of explicit references in ancient sources and the nature of the quotations may relate to an oral practice with respect to some fundamental texts. More generally, R. Netz tries to delineate the border between the written and oral aspects of ancient geometrical practices, by capturing its reflection in the shape of the original text. This question echoes the issues raised by the Indian case as treated by P. S. Filliozat and leads us into much wider issues to which we return below.

In his paper centered around a Babylonian mathematical clay tablet of the second millennium B. C. E., Jim Ritter puts into play another type of method, which he elaborates on the basis of a general remark regarding that which is involved in any reading. This method also leads him to bring to light several distinct kinds of specificity that the text under scrutiny shares with various others of the same type.

For his presentation, J. Ritter chooses a unique tablet that contains a problem and its solution in the form of an algorithm, and he examines several distinct possible readings for it. Since there existed no discipline of the text or second-order commentary that would be roughly contemporary with the text and would orient our way of interpreting it, the author suggests that this case allows for an ideal observation of how historians carry out their reading. Principally, J. Ritter emphasizes the extent to which each reading involves placing the text in a given context of other texts. The paper demonstrates how different textual contexts shaped by the historian each lead to capturing different features of the text. Moreover, J. Ritter's work analyzes the relation between both aspects through a systematic variation of contexts.

The first context is constituted by the corpus of mathematical problem texts of the same type, coming from the same cultural area and dating from roughly the same period. Interestingly enough, placing the text in this context discloses several structural features of the algorithm as text which hold true for the whole corpus. These stable characteristics

in carrying out the description of the lists of operations seem to indicate that a form of text was designed by working communities for handling algorithms. The simple management of the description of algorithms requires, as such, bringing mathematical knowledge into play. Furthermore, the text is not mere description, but constitutes a basis for carrying out mathematical activity. A recurring feature of these texts —that is, the embedding of certain texts of algorithm into the others— most probably implied that these descriptions of algorithms were, as such, the support of mathematical operations. The specificities of the text hence resonate with those of the practice of mathematics itself.

These conclusions are reinforced by the reading of the tablet that J. Ritter carries out in the second textual context, that of roughly contemporary Egyptian mathematical texts. Making explicit the contrast between the two contexts reveals new types of specificity into the text studied. Let us concentrate here on one of the differences between the descriptions of algorithms in the two worlds compared, to which J. Ritter draws our attention. The Egyptian text regularly articulates the prescription of the operations with the display of their performance on the surface of the papyrus itself. The computing techniques are, in a strong sense, a specific component of the mathematical text of the algorithm as such. In contrast to this, the Babylonian text chosen includes no comparable display of the carrying out of operations. The way in which this text refers to the operations themselves indicates, in my view, that they are performed either by making use of a counting instrument, still to be identified, or by reading another type of text: the tables —either or both. The latter case hence implies that in the Babylonian corpus, different types of mathematical texts were shaped, which were specialized for different uses and the production of which required different activities. More generally, the comparison between the two texts shows that the description of an algorithm can develop at different levels simultaneously, and that the two sets of texts compared reveal different, but stable, options as to the levels at which the formulation of the algorithms is shaped. For the same type of mathematical text, i.e., algorithm, different cultures of description took shape, which reveals different ways of organizing the practice of mathematics in relation to inscriptions.

The third reading discussed by J. Ritter highlights yet another feature in the design of the text studied: a specific structure in the distribution of verbal forms. Without entering into the details, for which I refer the reader to the paper itself, I would like to stress the striking fact that the discovery of this structure allows J. Ritter to assemble a corpus of roughly contemporary Babylonian texts that share a similar textual feature. On the one hand, the design of the text reveals a link between a family of disciplines in which topics are explored in a similar way. On the other hand, conversely, the same mode of rational exploration in these various fields receives a stable, "formulaic" expression, which indicates that a text was designed in relation to a specific type of scholarly practice. This remark reveals features of the text that a naive reading may not have shown to be relevant. Disclosing this structure, common to several types of text, requires that an interpretation of each text be given that accounts for it.

In conclusion, if each of these readings sheds light on specific features of the text as such, they all manifest an intimate link between the text and an aspect of scholarly activity, of which the text constitutes an outcome.

My own paper turns to a discussion of yet another method of analysis that brings to light features of a text that would not appear upon a progressive reading. In fact, it

deals with two different types of archive: on the one hand, a Chinese text of the thirteenth century and, on the other hand, a set of *Mémoires* and papers published in the mid-eighteenth century and the 1820s. In both cases, the texts can be shown to have structures that convey, as such, a part of their meaning. In other words, it is by making use of non-discursive means of expression that a whole component of the meaning is given to read. Furthermore, in both cases, it is by injecting some mathematical knowledge that is posterior to that displayed by the text that we can disclose, and make sense of, these structures. How, then, are we to interpret the meaning that they convey? It would be naive to attribute to the ancient authors knowledge of the mathematical tools with which the structure was revealed or read. But it would be equally inadequate to reject this non-discursive aspect of the text as meaningless. Such is the challenge confronting us here, in terms of interpretation.

The European archive considered is rich enough for us to be able to establish that the various authors deliberately opted for a specific non-discursive mode of writing, in order to inquire into a phenomenon that systematically caused structural patterns to appear in various domains of geometry. In this case we can show, therefore, that a specific form of text was shaped to inquire into a given mathematical phenomenon and convey the results of this research to a particular group of readers. We can even find hints regarding how the texts, as such, were composed. Moreover, geometers carried on research on the topic with the help of similar means for decades, before it be addressed discursively in the 1820s. This indicates that the non-discursive parts of the text were read as such by the actors and, accordingly, gave rise to further inquiry. Thus the historian must necessarily take them into account.

As for the Chinese text of the thirteenth century, the case is made more complex by the fact that it stands in relative isolation. Almost no contemporary mathematical text survived that would enable us to put such means of expression in a context or to capture how contemporary scholars read it. However, there is no doubt that the composition of the book puts into play several non-discursive modes of expression and this finding reinforces the idea that the historian must necessarily consider them.

One of these structures is shaped by making use of a widespread literary technique, namely, the use of parallel statements. It is interesting to note that nineteenth century Chinese mathematicians also found this parallelism meaningful. Even though these readers carried out their analysis several centuries after the composition of the book, it remains of interest to realize that they also interpreted mathematical meanings in the structures of the book.

As in the previous case, another structural feature of the text is brought to light through the use of modern mathematics. The question is still open to decide how to interpret this structural feature. Another issue may never be settled: why did the author make use of such means of expression? This question will only be answered if other non-mathematical contemporary texts be discovered that share similar modes of expression. Perhaps, the description of these modes in the context of mathematics can be precise enough to make us aware of their existence and once we have identified them in mathematics perhaps we can identify them in other sources as well. This is one of the ways in which we may hope that history of science can positively contribute to a history of text.

In conclusion, the evidence analyzed in this part of the book indicates how complex the texture of scientific writings may be. On the one hand, these writings reflect the research carried out and the practices of scholarship from which they derive. On the other hand, they depend on the technologies of the writing available, and the textual cultures of the groups within which they take shape. As a consequence, their interpretation becomes quite challenging. It is clear, however, that much is to be expected from developing a history of scientific text, along the lines outlined by the various contributions gathered in this volume.

It is to sketching the outlines of such a history that David Olson's concluding paper is devoted, and it allows deriving from the volume a range of open questions. Drawing on previous research in the domain of the history of text in Europe, David Olson invites historians of science to pay greater attention to the shift in the uses of writings that, according to mediaevalists, occurred in Europe between the eleventh and fourteenth centuries. D. Olson argues that the function of texts shifted from being mainly an aid for memorizing oral scriptures, the exegesis of which did not rely on textual analysis, to representing the world in ways that presupposed precise rules of interpretation and allowed actors to operate on the world via its representation. Accordingly, Olson correlates this transformation from a culture mainly based on orality to a world in which writing gained prominence with the emergence of a new way of reading. The main question D. Olson discusses, in relation to this transformation, concerns its impact on the emergence of modern science.

Bringing together questions and results from the research program that developed in mediaeval history in the last decades, on one hand, and the issues discussed in this volume, on the other hand, could be an extremely fruitful move that would be of great interest for the history of science. In my view, the main reason for this is that research work relating to this shift in the use of writing recently focused on the development and prompt dissemination, in mediaeval Europe, of "practical writings", that is, specific kinds of text designed for carrying out a specialized activity such as accounting or establishing contracts. In doing so, mediaevalists interestingly emphasize the historicity of the kinds of text used in any given society. If this issue has been studied regarding several domains of practical life (law, commerce, administration),[10] it has not yet, as far as I know, been considered with respect to scientific texts. This dimension of the problem has a direct bearing on the questions debated in several contributions of the volume.

As stressed above, one of the issues at stake in R. Netz's examination of his sources was to understand where, in the ancient Greek practice of geometry, the divide between oral and written once was, and in relation to which form of inscriptions. In addition, the transformations undergone by Greek geometrical texts through the process of transmission bear witness to a progressive preeminence of the written text, or, to be more precise, of a new type of written text, in scientific activities. This process could also be studied from the perspective of the emergence of a kind of "practical text" for carrying out mathematical activity.

The same issue presented itself as crucial in P. S. Filliozat's description of the transformation of mathematical texts in ancient India from oral practice to written documents. Texts underwent radical mutations in this process, not only in their discursive style,

but also in their constitution and in their mode of composition. Elements that had once been inscriptions on other surfaces, like figures or configurations of numbers, were to be included within the document, in a manner that was connected to the discourse. In fact, a similar process can be documented for Chinese mathematical texts.[11] These phenomena could all be examined from the perspective of the emergence of "practical texts" in the form of a written document, likely to be both support for practical work and object to be passed along to others. In contrast, in her paper included in this volume, Vera Dorofeeva-Lichtmann describes another way in which such a process took place, when she suggests that texts, originally non-linear in their structure and materiality, lost their graphic dimension through a process of transmission that molded them in the form of the linear discourse of a book.

To sum up, the worldwide study of scientific texts seems to bring to light processes similar to those found in mediaeval Europe in other times and places. In most of the traditions, one can note a shift from texts merely composed of verbal elements, perhaps recording oral practices, to texts incorporating on the surface of the same page distinct kinds of inscriptions with which operations were carried out. This naturally opens up several issues.

On one hand, the study, from this angle, of documents provided by archaeology would prove essential for capturing these various shifts more accurately. This is what Vera Dorofeeva-Lichmann demonstrates clearly in this volume. And this approach must be generalized if we are to examine these various turns in the use of writing. In this respect, texts coming from Mesopotamia or ancient Egypt, in their variety, appear to provide a different type of evidence, the study of which is essential for the project.

On the other hand, bringing to light such shifts in the various regimes of the written should help to describe the regimes through which ancient texts were handed down, a question essential for appreciating both the nature of these regimes and the nature of the ancient documents.

In conclusion, the transfer, in the sphere of the written document, of inscriptions designed for practicing the sciences and carried out on other material surfaces remains to be studied as such. It clearly shows, I believe, how much a history of the uses of writing can benefit from a history of scientific text, carried out at the surface of the planet.

REHSEIS, CNRS & Université Paris 7

ACKNOWLEDGMENTS

This volume consists in a selection of papers presented at the workshop held in Berlin, from March 30 to April 2, 1995. This workshop would not have been possible without the support of the Wissenschaftskolleg zu Berlin, the Einstein Forum, and the Otto and Martha Fischbeck foundation. It is my pleasure to take this opportunity to express my deepest gratitude to each of these institutions. A presentation of the whole workshop can be found in "Histoire des sciences, histoire du texte", *Wissenschaftskolleg. Jahrbuch 1994–1995*. Berlin: Nicolaische Verlagsbuchhandlung, 1996:194–199. Robert Cohen expressed interest in the project, as soon as he heard of it, and I am glad to be able to thank him for his ongoing support over so many years, by publishing the volume in the series to which his name is so intimately linked. My deepest thanks also to all those who accepted

to referee the various papers contained in this volume and, last but not least, Peter Quint and Don Zagier, who helped in making my English bearable.

NOTES

[1] (Chemla 1995) discusses some research programs relating to this field that developed in history of science. We shall not attempt to give an exhaustive bibliography here, but refer the reader to some books published more recently and providing a perspective on some of the main directions taken by research on the topic in the last years. Let us mention two books that concentrate on "books", printing, and their import for the practice of science: see (Johns 1998; Frasca-Spada & Jardine (eds.) 2000). Further, the reader can find in (Lenoir 1998) an approach to the phenomenon of inscription from as wide a perspective as the one we advocate.

[2] Cf. also the second part of (Chemla, Morelon, Allard 1986), which discusses the vexed issue of the "content" of Diophantos' *Arithmetics*.

[3] See (Idel 2002).

[4] Compare (J. Henderson 1991), who studies in a comparative way approaches to various scriptural texts, including the Bible and Confucian canons.

[5] See my treatment of this case in (Chemla forthcoming-a).

[6] (Chemla 1995: 176–177).

[7] (Netz 1998) outlines such a research program and shows its relevance for the case of Greek antiquity. The same issue is addressed in several articles included in (Frasca-Spada and Jardine 2000). See the introduction and the afterword by N. Jardine (Frasca-Spada and Jardine 2000: 4–5, 399 *sq.*).

[8] In her Ph. D. thesis, Agathe Keller explored this interplay between the composition of a commentary and the practice of mathematics in ancient India (Keller 2000).

[9] (Netz 1999).

[10] (Cocquery, Menant & Weber forthcoming) develops research along these lines and contains several papers on these domains. The reader can find there a bibliography of these new trends in historiography.

[11] See Chemla 1996, 2001, forthcoming-b.

REFERENCES

Chemla, K. 1995. "Histoire des sciences et matérialité des textes. Proposition d'enquête." *Enquête* 1: 167–80.

Chemla, K. 1996. "Positions et changements en mathématiques à partir de textes chinois des dynasties Han à Song-Yuan. Quelques remarques." *Extrême-Orient, Extrême-Occident* 18: 115–47.

Chemla, K. 2001. "Variété des modes d'utilisation des *tu* dans les textes mathématiques des Song et des Yuan." Preprint for the conference "From Image to Action: The Function of *Tu*-Representations in East Asian Intellectual Culture", Paris, September 2001, to be found at http://hal.ccsd.cnrs.fr/, Philosophie/Histoire de la logique et des mathématiques. A revised version is in preparation.

Chemla, K. Forthcoming-a. "Classic and commentary: An outlook based on mathematical sources." In *Critical Problems in the History of East Asian Science*, edited by KIM Yung Sik (Proceedings of a conference at the Dibner Institute, November 2001, 16–18). Cambridge (Mass.): MIT Press.

Chemla, K. Forthcoming-b. "Postface. Ecritures pratiques et histoire des sciences." In *Ecrire, compter, mesurer. Le calcul économique à l'épreuve de l'histoire et de l'ethnographie,* edited by N. Coquery, F. Menant & F. Weber. Paris: Editions Rue d'Ulm & PUF.

Chemla, K., Morelon, R. & Allard, A. 1986. "La tradition arabe de Diophante d'Alexandrie." *L'Antiquité Classique* LV: 351–375.

Coquery, N., Menant, F. & Weber, F. (eds.). Forthcoming. *Ecrire, compter, mesurer. Le calcul économique à l'épreuve de l'histoire et de l'ethnographie.* Paris: Editions Rue d'Ulm & PUF, 2004 Forthcoming.

Frasca-Spada, Marina & Jardine, Nick (eds.). 2000. *Books and the sciences in history*. Cambridge (UK): Cambridge University Press.

Henderson, John B. 1991. *Scripture, Canon and Commentary. A Comparison of Confucian and Western Exegesis*. Princeton, NJ: Princeton University Press.

Idel, Moshe. 2002. *Absorbing Perfections: Kabbalah and Interpretation*. New Haven—London: Yale University Press.

Johns, Adrian. 1998. *The nature of the book. Print and knowledge in the making*. Chicago & London: Chicago University Press.

Keller, Agathe. 2000. *Un commentaire indien du VIIième siècle. Bhaskara et le Ganitapada de l'Aryabhatiya*, Ph. D. Thesis, University Paris 7 (to appear in book form. Basel: Birkhäuser).

Lenoir, Timothy (ed.). 1998. *Inscribing science. Scientific texts and the materiality of communication*. Stanford: Stanford University Press.

Netz, R. 1998. "Deuteronomic Texts: Late Antiquity and the History of Mathematics." *Revue d'histoire des mathématiques* 4: 261–288.

Netz, R. 1999. *The shaping of deduction in Greek mathematics*. Cambridge (UK): Cambridge University Press.

Part I

WHAT IS A TEXT?

VERA DOROFEEVA-LICHTMANN

SPATIAL ORGANIZATION OF ANCIENT CHINESE TEXTS (PRELIMINARY REMARKS)[1]

ABSTRACT

Ancient Chinese texts provide some striking examples of what may be referred to as non-linear textual structures. These peculiar textual structures differ markedly from texts organized in a linear way, that is, those whose constituent elements (chapters, paragraphs, phrases etc.) are connected like links in a chain. The constituent elements of these non-linear textual structures, in contrast, are related in complex multi-dimensional ways. These relationships are like those found between the units of a scheme (or diagram, map, table, chart, design, sketch, picture, etc.), that is, a class of graphic representations designated in the Chinese language by the character *tu*. This implies that the interconnections between the constituent elements of the textual structures in question are manifested through appropriate non-linear layouts corresponding to a specific *tu*. Under these circumstances a text serves a dual function, that of description and graphic representation. In order to highlight the complementary facets of this textual type, I propose to define it as a *text-scheme*. This paper is primarily concerned with texts whose structures emulate ancient Chinese models of space, the latter characterized by a remarkable regularity and orientation to the cardinal directions.

0.

Ancient Chinese texts provide some striking examples of what may be referred to as *non-linear textual structures*. These peculiar textual structures differ markedly from texts organized in a linear way, that is, those whose constituent elements (chapters, paragraphs, phrases etc.) are connected like links in a chain.

The constituent elements of the non-linear textual structures, in contrast, are related in sophisticated multi-dimensional ways. These relationships are like those found between the units of a chart (or scheme, diagram, map, table, design, sketch, picture etc.), that is, a class of graphic representations designated in the classical Chinese language by the character *tu*.[2] This implies that the interconnections between the constituent elements of the textual structures in question are manifested through appropriate non-linear layouts corresponding to a specific *tu*.

Such a layout constitutes a “graphic representation” (*tu*) built of textual passages. In other words, this textual layout combines the properties of a “graphic representation” and relevant “elucidations” (*shuo*)[3]. “Elucidations” associated with definite units of the *tu* are arranged according to the placement of these units, and are used as a sort of “filler”

K. Chemla (ed.), History of Science, History of Text, 3–47.

for delineating their contours. It is of central importance to note here that a "graphic representation", while determining interconnections within textual structures, will also have a specific *meaning* conveyed through its depictive properties.[4] Consequently, a non-linear textual structure serves a dual function, namely, depictive and elucidative. It can only be fully comprehended through considering both of these facets simultaneously. Thus, the textual passages of a non-linear textual structure "make sense" when regarded as units of a graphic image which links them together.

Unfortunately, most non-linear textual structures, with very few exceptions, have reached us in a "deconstructed" form. It seems likely that, to a considerable extent, this "deconstruction" resulted from changes in writing media in the course of the development of Chinese written culture.[5] Chinese society experienced considerable transformations throughout its history, many of them deeply affecting its written culture, and the original form of texts in particular.[6] Distinguishing non-linear structures from linear texts (or linear parts of texts) requires careful investigation of the body of surviving ancient Chinese texts from the point of view of their organization, or, in other words, considering their formal aspect.

Despite the considerable concern of some researchers with the formal aspect of Chinese texts,[7] non-linear textual structures have more often than not been overlooked in sinological literature. Yet, there is no question that the failure to distinguish between linear and non-linear textual organization may result in considerable confusion in the interpretation of certain texts. Thus, if a text organized in a non-linear way is approached as if it were linear, the distortion in its interpretation may be compared to inadequate linguistic translation.

What relevant research exists is, however, far from providing a consistent view of the subject, as it is done with differing approaches and motivations in mind. Thus, there is no more or less generally-accepted taxonomy for categorizing variables of non-linear textual structures.

Under these circumstances, it seems reasonable to survey the extant structures in question, both original non-linear layouts of ancient texts and reconstructions of such layouts, in an attempt to determine at least one specific type of these structures. This paper focuses on cardinally-oriented non-linear structures related to spatial models.

1.

The continuity of Chinese written culture suggests a strong and lasting impact of its origins on its further development. From this point of view, it seems to be especially noteworthy that non-linear textual structures are clearly present in the initial period of Chinese writing (ca. the second half of the 2nd millennium B.C.). This period is represented by divinatory inscriptions on turtle plastrons and scapulas *(jia gu wen)*. These inscriptions are characterized by a formulaic, uniform and lapidary style, and their content does not vary a great deal.[8] A considerable number of the inscriptions on turtle plastrons are distinguished by a regular and explicitly displayed non-linear layout.[9] This is not coincidental, as the symmetry of the plastron's shape and its natural segmentary structure served as an appropriate pattern for such arrangements.

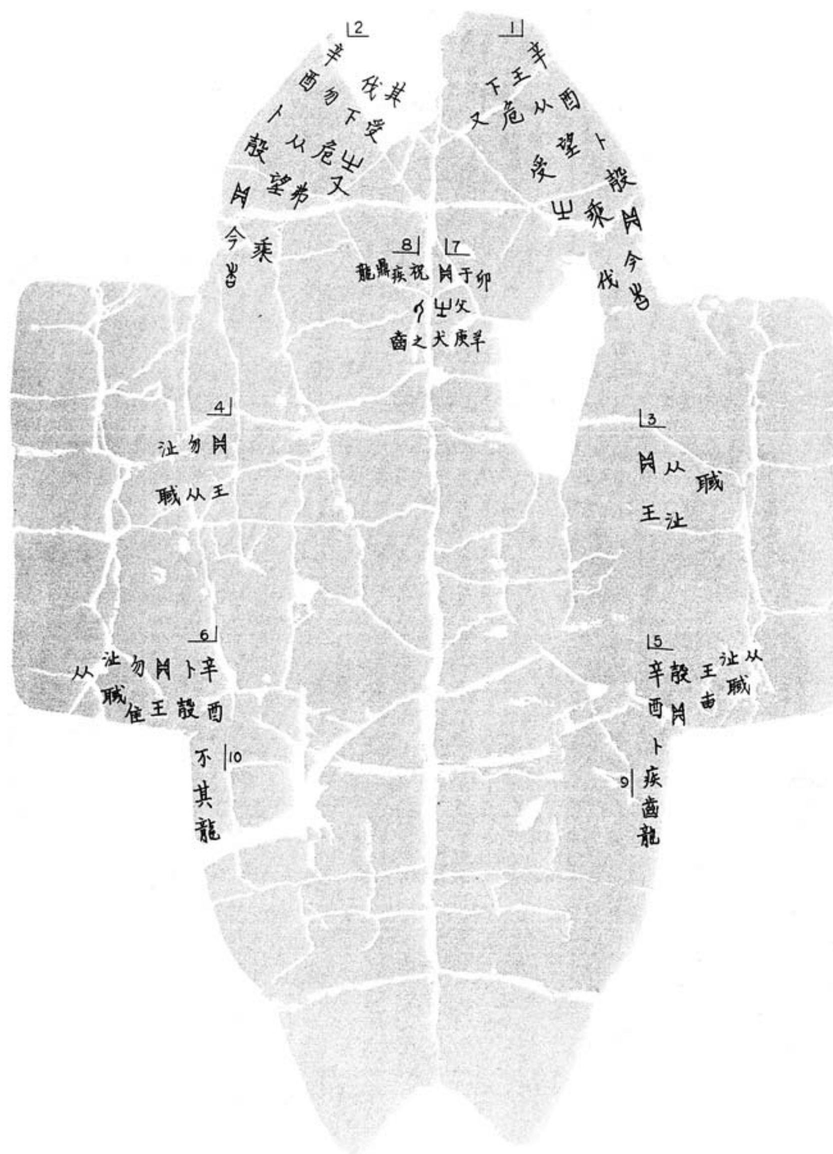


Figure 1a. A turtle plastron with oracle inscriptions (Reproduced from (Keightley 1985, figure 15)).

Let us take a closer look at a typical example of such an inscription (cf. **figure 1a–b** showing the inscription and its translation). It consists of two correlating series of charges, "positive" and "negative" ones. The "positive" charges are placed on the left side of the plastron, the "negative" ones on the right. A pair of correlating charges is engraved

	PAIRS OF COMPLEMENTARY CHARGES	
	Negative charge (left side)	Positive charge (right side)
(Preface:)	(2) Crack-making on hsin-yu (day 58), Ch'üeh divined:	(1) Crack-making on hsin-yu (day 58), Ch'üeh divined:
(Charge:)	"This season, the king should not follow Wang Ch'eng to attack the Hsia Wei, (for if he does, we) will not perhaps receive assistance in this case."[85]	"This season, the king should follow Wang Ch'eng to attack the Hsia Wei, (for if he does, we) will receive assistance in this case."
(Preface:)	(4) Crack-making on hsin-yu (day 58), Ch'üeh divined:	(3) Crack-making on hsin-yu (day 58), Ch'üeh divined:
(Charge:)	"(This season) the king should not follow Chih Kuo (to attack the Pa-fang, for if he does, we will not perhaps receive assistance in this case)."[86]	"(This season) the king should follow Chih Kuo (to attack the Pa-fang, for if he does, we will receive assistance in this case)."
(Preface:)	(6) Crack-making on hsin-yu (day 58), Ch'üeh divined:	(5) Crack-making on hsin-yu (day 58), Ch'üeh divined:
(Charge:)	"(This season) it should not be Chih Kuo that the king follows (to attack the Pa-fang, for if he does we will not perhaps receive assistance in this case)."[87]	"(This season) it should be Chih Kuo that the king follows (to attack the Pa-fang, for if he does we will receive assistance in this case)."
(Preface:)		(7) Divined:
(Charge:)	(8) "Praying to lead away this sick tooth (?), the *ting* sacrifice will be favorable."[89]	"*Yu* sacrifice a dog to Fu Keng (and) *mao* sacrifice a sheep."[88]
(Charge:)	(10) "(Sick tooth) will perhaps not be favorable."	(9) "Sick tooth will be favorable."

Figure 1b. Pairs of complementary charges on a plastron (Reproduced from (Keightley 1985, 78–79)).

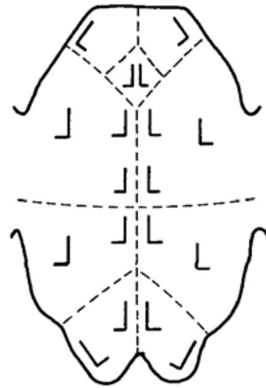


Figure 1c. Ideal placement of inscriptions on a plastron (Reproduced from (Keightley 1985, figure 17)).

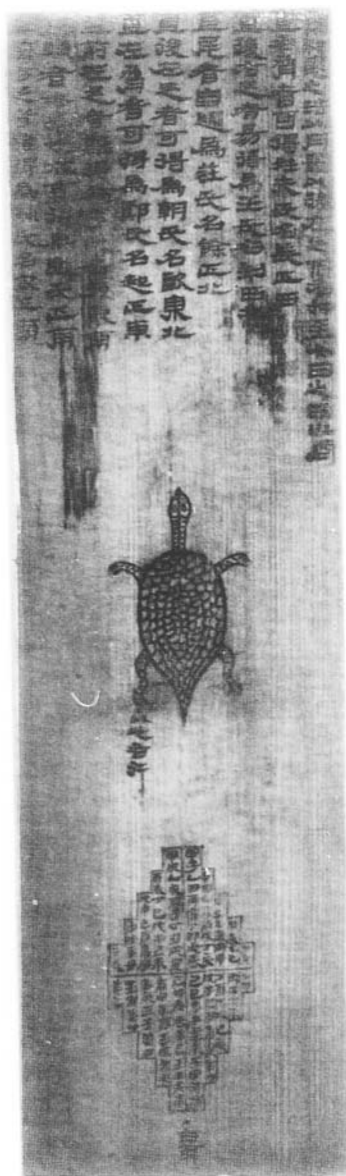
symmetrically, as mirror images,[10] with respect to the central lengthwise axis of the plastron. Furthermore, the charges are placed at specific "points" on the plastron (cf. **figure 1c** showing the ideal placement of charges). The placement of a charge determined the direction of writing, as shown by the L-shaped symbol on **figure 1c**. Thus, the charges placed at the upper and the lower sections of a plastron are written from its borders towards the central axis (e.g., charges 1 and 2 on **figure 1a**), all the others going in the opposite direction—from the central axis towards the borders (e.g., charges 3 and 4; 5 and 6; 7 and 8 on **figure 1a**). The numbering of charges according to their succession in the oracle procedure also gives an impression of following certain rules. The charges were "processed" by pairs, first a "positive" charge, then its "negative" counterpart, beginning from the upper pair (charges 1 and 2), then the two flanking pairs (charges 3 and 4, 5 and 6), the central pair (7 and 8), and, finally, the lower pair (9 and 10).

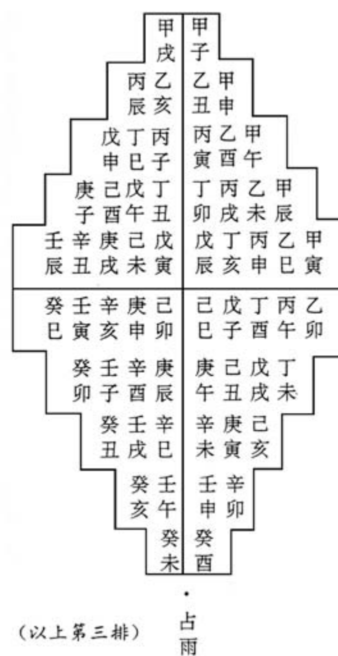
In brief, the examined inscription on a turtle plastron is a sophisticated textual construct of remarkably regular configuration whose constituent elements (charges) are interrelated in a non-linear way.

There is good evidence to suppose that the Shang conceived of the Earth in the shape of a cardinally-oriented cross, and that the turtle plastron functioned as a model of the cruciform Earth's surface.[11] Additional support for this supposition may be found in one of the divinatory texts excavated at Yinwan.[12] This text inscribed on two faces of a wooden tablet (cf. **figure 2a–b**) is concerned with divination according to "the rules of the sacred turtle" *(shen gui zhi fa)*. These rules are based on correlation between parts of the turtle's body and the cardinal and semi-cardinal directions, as specified at the beginning of the text (the upper part of face A of the tablet, cf. **figure 2a**):

Parts of the turtle's body	Cardinal orientation
right side of the body	west
right back leg	north-west
tail	north
left back leg	north-east
left side of the body	east
left front let	south-east
head	south
right front leg	south-west[13]

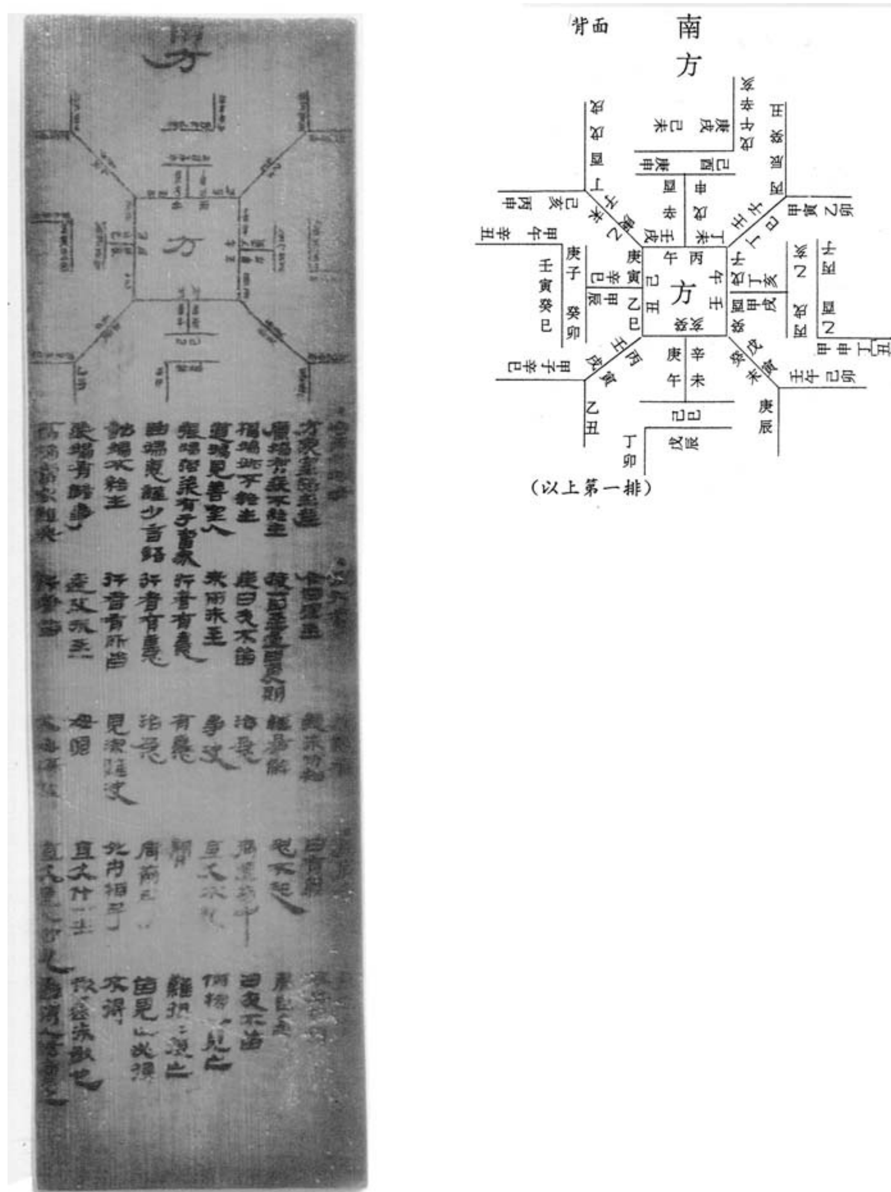
The specification of correspondences between the turtle's body and the cardinal directions is followed by a rather realistic picture of a turtle and then by a diamond-shaped scheme filled up by cyclical signs (the middle and the lower part of face A of the tablet, respectively, cf. **figure 2a**).[14] This scheme obviously provides a conventional representation of the cardinally-oriented turtle above. Thus, the squares of the scheme, each square containing a cyclical sign, resemble prominent segments on the turtle's shell, and a cross delineated in the middle of the scheme accentuates the south-north and the east-west directions, according to the turtle's body. One more scheme also containing cyclical signs is found on face B of the tablet (cf. **figure 2b**). This scheme reproduces the so-called TLV design of "magic" mirrors.[15] The TLV design serves as a demarcation of the four





1. 神龟占卜法（正面）

Figure 2a. Wooden tablet from Yinwan ("The rules of the sacred turtle"), face A (Reproduced from *Wenwu/*Cultural Relics 1996.8, coloured plate 1 and p. 30).



2. 神龟占卜法（背面）

Figure 2b. Wooden tablet from Yinwan ("The rules of the sacred turtle"), face B (Reproduced from *Wenwu*/Cultural Relics 1996.8, coloured plate 1 and p. 30).

cardinal and the four semi-cardinal directions. The southern orientation of the scheme prominently marked at its top[16] corresponds to the southern orientation of the turtle's head on face A of the tablet. This scheme, therefore, may also be regarded as directly related to the image of the turtle.

Despite the fact that the Yinwan divinatory manuscript dates from a period considerably later than that when the Shang inscriptions were produced, and, moreover, from a qualitatively different stage of Chinese history, it, nevertheless, represents a divinatory tradition that has originated from the Shang oracular ritual, and, therefore, contains some of its traces.

For example, a divinatory inscription was laid out on a plastron in such a way that the head section of the plastron was up. This means that a plastron was arrayed in front of the "reader", or, I would rather say, "user" of its inscription in a similar way to the cardinally-oriented turtle from Yinwan, namely, head up. This way of placing the turtle by the Shang may be grasped even better from a picture found on a Shang *pan*-water basin (cf. **figure 3**). The turtle is flanked by columns of two identical characters (pictograms) engraved as mirror images with respect to the central lengthwise axis of a turtle, that is, similarly to the placement of pair charges on a plastron. These characters can be read properly when the turtle is placed head up.

It is also noteworthy that the scheme below the Yinwan picture of a turtle, the scheme that conveys its conventional image, is filled out with the cyclical signs symmetrical with respect to the central lengthwise axis of the scheme, that is, according to the basic principle of laying out the oracular charges.[17]

This way of orienting the turtle, that is, facing the south, as well as arranging the writing on a plastron in such a way that its "reader" or "user" also faces the south, conforms to the prominent place of the south in the spatial notions of the Shang (e.g., south-oriented main entrances of palaces and tombs).[18] Therefore, it seems very likely that the Shang assigned the shape of the turtle to the cardinal directions in the same way as the Yinwan divinatory manuscript.

This conclusion implies that if the turtle plastron was conceived of as a cardinally-oriented object (a model of the Earth's surface), the inscription on this plastron, as part of this object, was *a cardinally-oriented non-linear textual structure.* This structure is typologically similar to the scheme below the picture of the turtle from Yinwan.

The inscriptions on oracle bones are considered to be an extremely difficult area of research, mostly because we lack a sufficient cultural context in which to place them. On the other hand, the inscriptions have one considerable advantage over most of the pre-Han classical texts whose content seems to be much easier to grasp, that is, that since the inscriptions on oracle bones accompanied an elaborate ritual procedure, it is more or less clear *what these texts were used for.* It is necessary to emphasize here that knowing the *function* of a text is crucial to an adequate understanding of it. It is the inability to reproduce the function of an ancient text that seems to be one of the main obstacles to understanding them properly.[19]

Regular and systematic production of the inscriptions on oracle bones during a relatively long period of time resulted in a very large series of these texts that shows their variation and evolution over several centuries. The analysis of these characteristics allows one to draw the conclusion that the regular arrangement of the oracular charges, as well

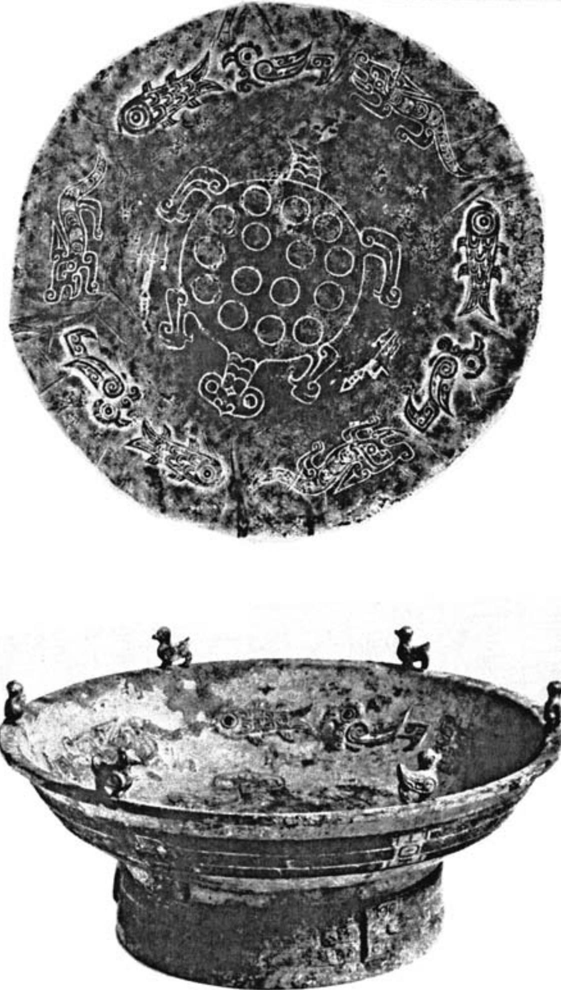


Figure 3. Shang *pan*-water basin with a turtle engraved on its bottom (Reproduced from (Allan 1991, 100)).

as the choice of an appropriate medium for such an arrangement, are closely bound up with the systematic and regular nature of the ritual activity related to these inscriptions. One can see clearly that the more uniform the ritual procedure became, the more regular became the placement of inscriptions.[20] In other words, the regular non-linear layout of the divinatory inscriptions results, at least to a very considerable extent, from the highly formalized and orderly nature of the related oracular procedure.

This, however, does not necessarily imply that any text associated with a ritual procedure automatically has a non-linear structure. Thus, the ancient Chinese inscriptions on ritual bronze vessels, bells and implements[21] have, in contrast, a prominently linear

structure. Yet, similarly to the inscriptions on oracle bones, the bronze inscriptions are distinguished by a formulaic style and remarkably regular structural pattern.[22] Moreover, these attributes tended to become more rigid towards the Late Western Zhou,[23] that is, when the related ritual became more uniform.

Consequently, ritual procedures, being "performances" of a formalized and orderly nature, tended to engender texts of similar nature, either linear or non-linear. This paper tries to identify some of the factors that favored production of the latter.

2.

The bulk of ancient Chinese texts have reached us in the form of block-printed books. There is a tremendous chronological gap between the invention of woodblock printing,[24] and the origins of Chinese writing. Nevertheless, the textual design of the block-prints is the result of the continuous development of Chinese writing over ca. 1.5 millennia, and bears clear hallmarks of the earlier stages of this evolution. For example, the division of block-printed books into sections is evidently inspired by the layout of text on bamboo slips and silk scrolls, which served as the basic writing media during the period when the body of texts known as the Chinese Classics was undergoing formation. Thus, their sections comparable with "chapters" are referred to as "bound rolls of bamboo slips" *(pian)* or "scrolls" of silk *(juan)*.[25] In some cases these textual divisions are combined in such a way that a "scroll" of silk incorporates several "bound rolls of bamboo slips".[26]

Despite these indications of ancient heritage, it is necessary to recognize that the invention of block-prints brought about a fundamental transformation of the design of texts originally written on bamboo or silk. I would like to point out here that overlooking this transformation and, consequently, the effect produced by the original writing media on the organization of a text[27] is one of the reasons why the non-linear textual structures have been mostly ignored in sinological literature.

The principal difference between block-printed books and manuscripts on such media as bamboo slips, wooden tablets[28] and silk scrolls lies in the type of connection made between textual units. Books are characterized by *an established linear sequence* of their units, whereas many of the manuscripts, by contrast, consisted *of separate units* — rolls, tablets and scrolls— and, therefore, were naturally prone to a rather loose linear order which could lead to their re-orderings and even non-linear arrangements. It is noteworthy that the succession of textual units in block-prints retains features of the original "autonomy" of "chapter-rolls/scrolls", such as independent *pagination* within each chapter and the absence of over—all pagination.[29] Indeed, quite a few finds of bamboo slips, even those which are little more than a disorganized bundle of fragments, contain slips with a special sign (a thick line at its upper end) that marks off a bound roll/set of slips, and, therefore, shows that the discovered text consisted of separate bound rolls/sets of slips.[30]

The segmentary nature and, consequently, transpositional potential of texts on bamboo slips and wooden tablets was much greater than those on silk. A bound roll/set of slips gave considerably less space for writing than a scroll of silk, and for this reason a scroll could incorporate a text written on several rolls/sets or tablets. Moreover, the binding

holding together the bamboo slips allowed for their easy rearrangement and transposition within a roll/set, as well as the shortening, expanding or further subdividing of such a roll/set.[31] In other words, the connection between slips in a roll/set was much less stable than that between columns on a wooden tablet, a piece of silk or, later, in a book. The flexibility of the bound rolls/sets of bamboo slips responded well to the needs of a still forming and evolving culture of writing.[32]

One may suggest two main variables of non-linear textual layouts that might have been favored by the nature of bamboo slips: a non-linear layout within a roll/set based on correlation between inscribed slips (microstructure level), and a non-linear layout of the bound rolls/sets themselves (macrostructure level).[33]

Unfortunately, the bamboo slips are often discovered in a jumbled or fragmentary state, as the bindings and the slips themselves rot and break after centuries buried underground. The poor state of these slips makes grasping the arrangement of the text inscribed on them a difficult task.

Nevertheless, some finds bear evident traces of a non-linear textual arrangement within a bound roll/set of slips, for example, the "almanac" (*ri shu*) from Shuihudi dating from the Qin dynasty (221–207 B.C.) that includes the so-called *ren zi* ("'human being' character") diagram (cf. **figures 4a–b**),[34] and the Fuyang bamboo "Annals" from a tomb of the Former Han Dynasty (165 B.C.) are arranged as a table (cf. **figure 4c**).[35] On the latter, the horizontal borders of the table's checkered squares are demarcated by crosswise lines on the strips. One of the divinatory texts on bamboo slips discovered in Yinwan provides an excellent example of the arrangement of a textual body into neat parallel strips divided by blank spaces (*Xingdao jixiong*, cf. **figure 4d**).[36] This type of arrangement may imply a possible correlation between the strips.

Even if bamboo slips were preserved in a perfect state, however, the probability of finding bound rolls/sets of slips in their original layout is rather low. The interconnection of the rolls/sets, whether linear or non-linear, was manifested while the text was *in use*, beyond which it existed in a "deconstructed" form. What is especially misleading, as far as the arrangement of the bound rolls/sets of bamboo slips is concerned, is that the reconstructions of the texts inscribed on them usually rely on corresponding or typologically close versions that have reached us in the form of block-printed books. This means that the starting point of these reconstructions, whether acknowledged or not, is the assumption that the rolls/sets constituted a *definitive linear succession*, like chapters and paragraphs in block-printed books.

As mentioned above, many texts on bamboo slips were later reproduced on silk,[37] and since silk provided considerably more space than a bound roll/set of bamboo slips, a silk scroll could "incorporate" several bound rolls/sets of bamboo slips. This allows one to suggest that non-linear layouts of bamboo rolls/sets of slips, if any, might have been at least in some cases "blue-printed" on silk rolls. Indeed, there is a text on an excavated piece of silk that gives some idea of how such a layout might look. This text entitled the Chu Silk Manuscript (*Chu bo shu*, cf. **figure 6a–c**) dates back to ca. the third century B.C.[38] Excavated in 1942, it still enjoys considerable scholarly interest.[39] One of the reasons for this interest is the peculiar spatial layout of the manuscript, a closer look at the structure of which suggests that it may have originated from a layout of bound rolls/sets of bamboo slips.

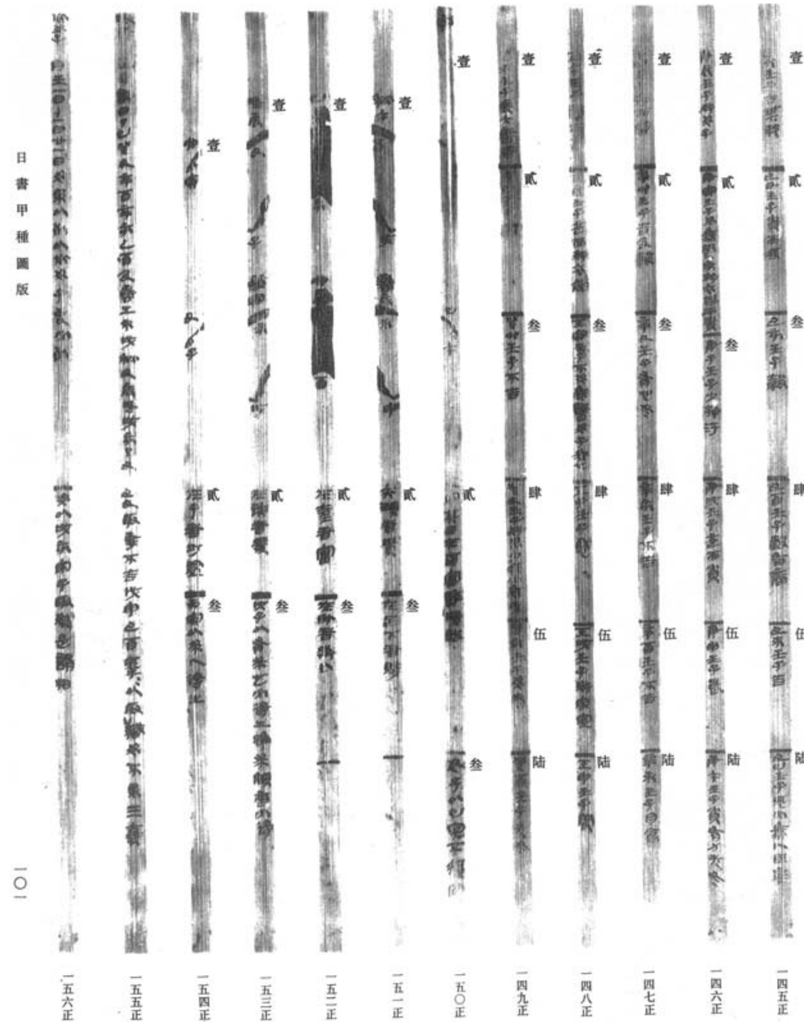


Figure 4a. An "almanac" (*ri shu*) from Shuihudi comprising the *ren zi* diagram (Reproduced from (*Shuihudi Qinmu zhujian* 1990, 101)).

Firstly, the central part of the manuscript is occupied by two relatively long textual sections[40] placed upside down with respect to each other. It is noteworthy that the length of the columns in both of these sections conforms to the numbers of characters often found on bamboo slips. From this point of view the layout of characters in these sections strongly resembles two bound rolls/sets of bamboo slips placed upside down in relation to each other.

Secondly, the central textual sections are framed by pictures and associated textual passages.[41] The pictures show twelve divinities or spirits depicted with their heads

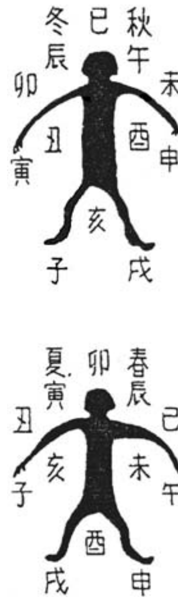


Figure 4b. The so-called *ren zi* ("'human being' character") diagram (Reproduced from (Li Ling 2000, 206)).

adjacent the central textual sections.[42] Each spirit is supplied with a name and a concise definition of its function. These are given in three characters and placed at the level of the spirit's head. Additional elucidations appended for each spirit are placed closer to the border of the manuscript. These elucidations consist of two to four short columns slightly differing in their length (full columns contain eight to thirteen characters). The framing pictures and associated textual passages form four groups —three at each side of the manuscript. The passages within a group are written along the side of the manuscript and in the same direction. All together they form a clockwise sequence. The total number of characters in columns along each side of the manuscript (in a group of framing passages) roughly conforms to the length of columns in the central sections, and, hence, to the numbers of characters often found on a bamboo slip.[43]

Finally, such a sophisticated placement of text on a silk scroll required much skill and a good eye for operating with textual passages. Although almost no other examples of similar textual arrangements on silk from the second half of the 1st millennium B.C. have been found so far, it may be that such an elaborate arrangement represented a developed practice of producing non-linear textual arrangements rather than being an exceptional case.[44] Above I advanced a supposition that the high transpositional potential of texts written on bamboo slips and wooden tablets might have served as an important favorable factor for producing non-linear textual structures. Indeed, the Chu Silk Manuscript is constituted of few parts, which, as just mentioned, correspond well to the format provided by these writing media. It seems at least possible, then, to suggest that the Chu Silk

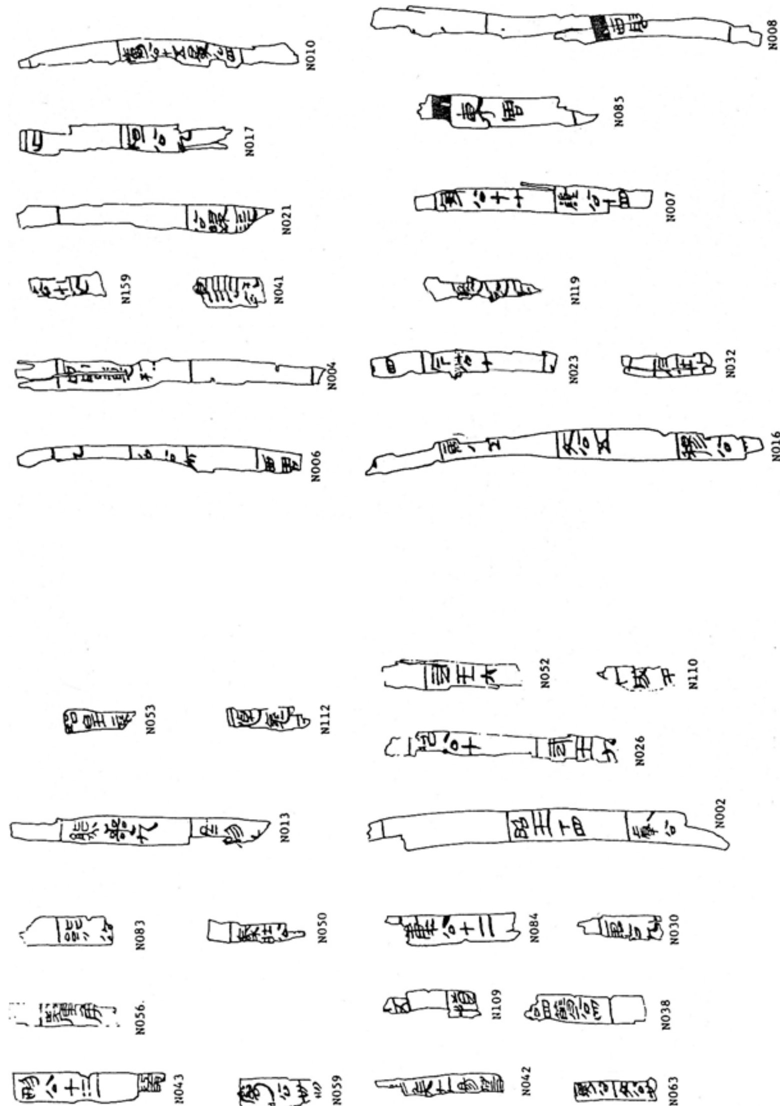


Figure 4c. Fuyang bamboo "Annals" (Hu Pingsheng 1989, 24–25).

Manuscript might have been inspired by or originated from a non-linear textual arrangement composed of six bound sets of bamboo slips (or six wooden tablets) —two in the center placed upside down with respect to each other framed by four bound sets or tablets with a strip-like textual layout, and, probably, incorporating pictures.

The spirits depicted on the manuscript represent a sort of zodiacal cycle (cf. **figure 6c**). Each spirit represents a month, and the accompanying textual passages elucidate the



Figure 4d. Divinatory text on bamboo slips from Yinwan (Reproduced from *Wenwu/*Cultural Relics 1996. 8, p. 18).

permitted or forbidden activities during the respective month. The spirits are arranged into groups of three —three spirits at each side of the frame. The spirit to the left on each side, according to the accompanying elucidation, "controls" (*si*) one of the four seasons. This spirit corresponds to the last month of a season. Therefore, a side of the manuscript comprising three spirits-months represents a season. Since the seasons correlate with the four cardinal points, the arrangement of spirits and, consequently, the entire layout of the manuscript, are then implicitly cardinally-oriented. The arrangement of the twelve pictures of spirits is complemented by pictures of four trees, which, in contrast to the former, are not accompanied by textual passages. These four pictures are placed at the corners of the cardinally-oriented frame as "separators" between the seasons and markers



Figure 5. Manuscript B of the *Lao zi* from Mawangdui (Reproduced from (*Mawangdui boshu* 1980, coloured plate 2)).

of the semi-cardinal directions.[45] The set of pictures delineates a tempo-spatial scheme —correlated structuring of time and space.

The main text placed in the center of the manuscript also deals with calendrical matters —the longer section concerns the year, the shorter the four seasons— considered in an astrological and cosmological context. When the shorter of the central textual sections is placed head up in front of the "user" it is cardinally-oriented in such a way that its top corresponds to the south. Taking into consideration of the prominent southern orientation of ancient Chinese divinatory schemes, spatial models and maps, this seems to be the probable initial position of the manuscript.[46]

Whatever order of reading of the manuscript one accepts, reading of the other central section of the text and the framing passages, which constitute a clockwise sequence,

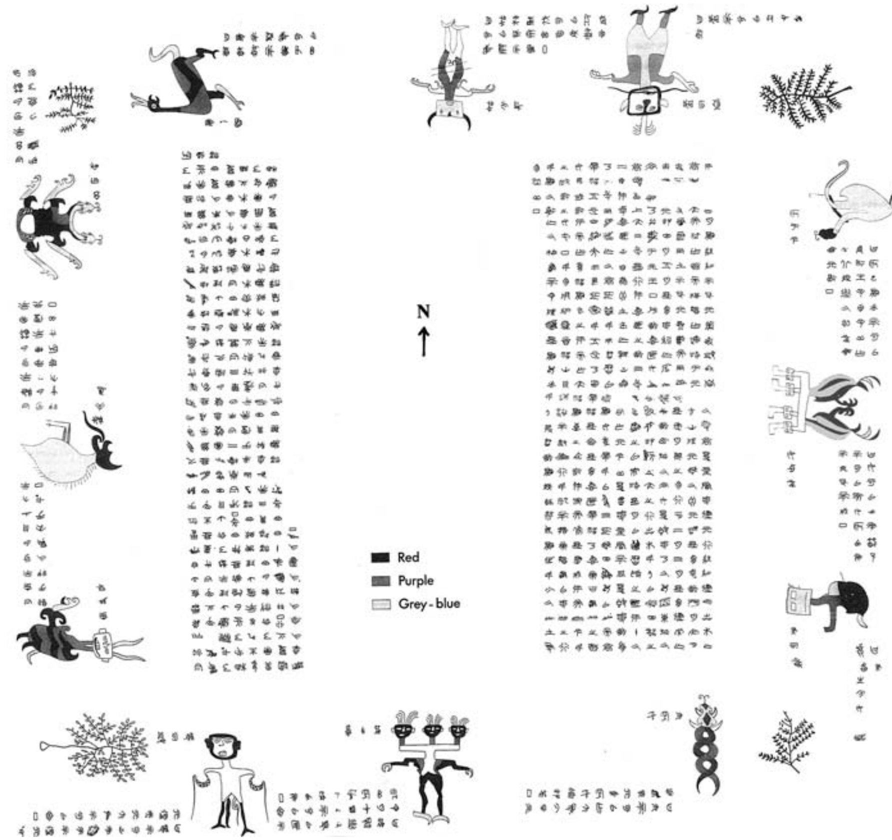


Figure 6a. The Chu Silk Manuscript (*Chu bo shu*) (Reproduced from (Barnard 1972, fig. 1, p. 2–3)).

requires either rotating the manuscript or a circular movement around it of its "user". The pictures of spirits are also designed in such a way —with the heads adjacent to the center— that their examination, similar to reading the related textual passages, also requires rotating the manuscript or a circular movement by its "user".

In this respect, the text strongly resembles rotating astronomical/astrological instruments known as *shi*, divination boards or cosmographs.[47] Moreover, it is, in fact, designed as just such an instrument, as has been pointed out by Li Ling.[48]

Shi cosmographs consist of a square board representing the Earth with a round rotating board placed on top representing the Heaven,[49] both boards supplied with several sets of degree markers. The degree markers indicate the orientation to the cardinal and semi-cardinal directions, especially prominent on the lower terrestrial part of the cosmograph. More specifically, the four cardinal points correspond to the center of each side of the bottom board, the four semi-cardinal directions to its corners. The structure derived from

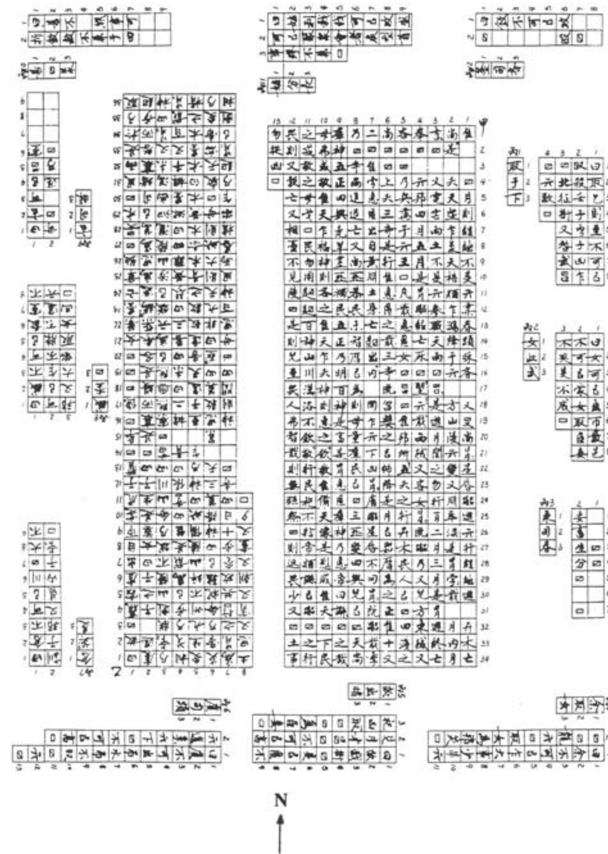


Figure 6b. Transcription of the Chu Silk Manuscript by Li Ling (Reproduced from the "Discussion" in (Lawton (ed.) 1991, fig. 14, p. 180)).

the Chu Silk Manuscript is especially similar to one of the two types of cosmographs seen in **figure 7**. It has a set of twelve degrees on the bottom board in addition to the prominent cardinal and semi-cardinal directions, and the Northern Dipper on the upper rotating part. The place of the "heavenly" part on the Chu Silk Manuscript is occupied by the main text.[50] The frame of pictures corresponds to the bottom board. Li Ling calls this arrangement of pictures a "pictorial cosmograph" (*tu shi*, cf. **figure 6c**), which in Chinese is an inversion of *shi tu* ("cosmograph design"). This elegant inversion plays with the broad meaning of the character *tu*, which designates varying types of graphic representations, and highlights the instrumental character of the Chu Silk Manuscript. This is something that is, however, given little consideration in sinological literature. As I have already noted above, taking into account how the text was used and what its original users were supposed to do with it is as important for understanding its meaning as a linguistically accurate translation of its content. Yet, scholars who are products of literary

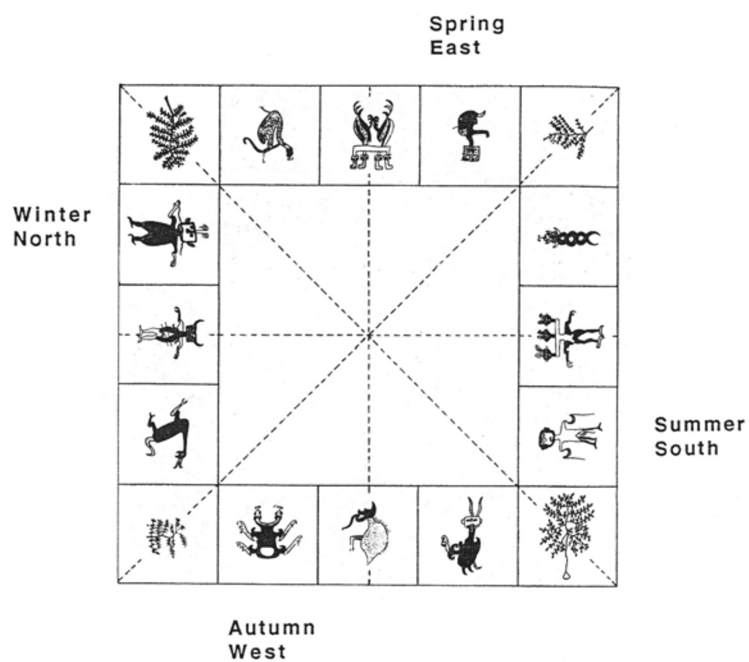


Figure 6c. The "pictorial cosmograph" (*tu shi*) by Li Ling (Reproduced from (Li Ling 2000, 180)).

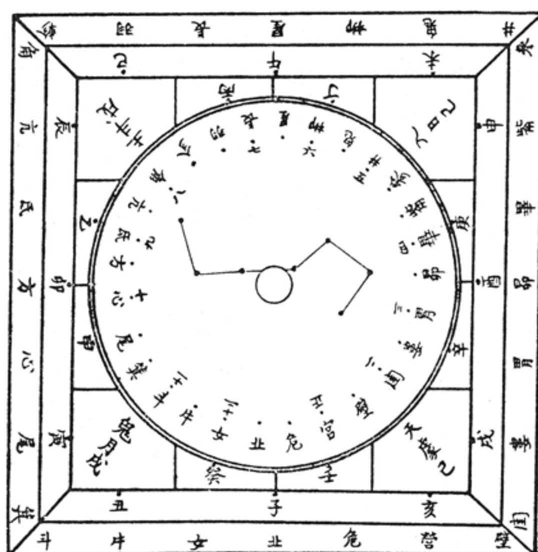


Figure 7. Divination board or cosmograph (*shi*) (Reproduced form (Field 1992, 96)).

traditions where the majority of texts are meant for reading rarely raise these questions when dealing with ancient texts.

They assume without much reflection that ancient texts are no different from contemporary texts in this respect. The major difficulty here is that there is no way one can actually see what the original readers of texts, I would rather say "users", actually did with them. We can only look for traces of their usage.

In the case of the Chu Silk Manuscript we have a rare example of a text that bears the clear stamp of a certain operational function. This text is characterized by an attribute that demands a certain action while reading it —rotating the manuscript or a circular movement by the reader or user around it, or a combination of these actions. Although we do not know the reading order of the textual passages, and whether there was supposed to be a single definitive order at all, the result of these actions is quite obvious —the tailoring of time and space according to the model given in the textual layout. Since this tempo-spatial model is "controlled" by spirits, the space created after this model is of a sacred nature. Creation of this sacred space then means establishing relationships and correspondences between parts of time-space and spirits.[51]

Seen from this perspective, the "pictorial cosmograph" is as much a tool to be used for tailoring space and time with respect to certain cosmological patterns as the cosmograph and other similar tools —divinatory schemes and "magic" mirrors. The only difference between these pure instruments and the Chu Silk Manuscript is that the latter incorporates textual passages. This difference seems, however, to be something imposed by our ideas of a tool and a text on Chinese cultural tradition, rather than being characteristic of this tradition.

This problematic is closely related to the interrelationship between text and picture in the manuscript. The textual passages of the Chu Silk Manuscript are arranged in such a way that they *form a single whole* with the pictures. Nevertheless, as with studies of the inscriptions on oracle bones and bronzes, even the most up-to-date works on the Chu Silk Manuscript are not entirely free from an exclusively philological perspective centered on its textual parts, divorcing them from the pictures. For example, Jao Tsung-i refers to the manuscript as "*a text and diagram* on matters related to astronomy",[52] as if the diagram and accompanying text would have been separate. Li Ling provides separate representations of the textual parts (cf. **figure 6b**) and the pictures (cf. **figure 6c**), thus breaking up the cohesion of the representation as found on the manuscript.[53] Hwang Ming-Chorng is more sensitive to the cohesion of the manuscript in his examination of it, but since his study is primarily focused on the spatial model conveyed by this text, he does not clearly articulate it.[54] I would like to point out here that this representation is a *single* diagram that has pictorial and textual parts, and, therefore, should be considered as a single whole.

Using silk as a writing medium is one indication of a feature of "conservation" in Chinese written culture. Thus, a text written down on a piece of silk could not be transposed or transformed as easily as if it were on a bound roll of bamboo slips. Eventually, block-printed books provided a remarkably effective means for preserving and reproducing textual versions with the precision of carbon copies.[55] It seems that the evolution of textual arrangement engendered by new writing media and multiple rewritings of texts gradually overshadowed possible network relationships between constituent elements of

non-linear textual structures.[56] Moreover, the evolution of Chinese traditions of writing was part of the general development and transformation of Chinese society and culture that resulted in considerable changes in the understanding and use of ancient texts. Indeed, quite a number of extant versions of ancient Chinese texts are known for giving an impression of being a "patch-work". This is exactly what might be expected from the transformation of a system with non-linear relationships between its units into a linear sequence, with some of the interconnections between the units no longer being in evidence.

3.

Above I proposed that if non-linear arrangements of bound rolls/sets of bamboo slips existed,[57] this arrangement could have only been displayed when the text was being used. It also seems plausible that at least some non-linear textual structures originally were *not* intended to be overtly displayed (being considered as secret knowledge). Finally, the transformation of this form of textual layout occurred gradually. Therefore, at least some of the non-linear structures that originally were laid out in a diagrammatical form and then transformed into linear sequences might still be recognized and "assembled". All this implies a knowledge of the algorithm for establishing the connections between the constituent elements of the "deconstructed" textual structures, and would require special techniques or practices for using texts. It is, however, necessary to bear in mind that there are likely to be very few traces of these techniques, since they were applied while using the text and transmitted through demonstration.

Indeed, application of these practices becomes apparent rather late —in the studies of the Classics during the Southern Song (A.D. 1127–1279) and the Yuan (A.D. 1271–1368) dynasties. These studies are distinguished by multiple and diverse diagrammatical representations of some classical texts.[58] The textual diagrams served as instruments in speculations concerning the meaning of the texts. Some of these diagrams, aimed at showing a system of interconnections between the sections of a certain text, simply delineated these sections while the entire text was not reproduced. Some incorporated its most important passages, or even all the characters of a piece of text, usually a short one.[59]

The considerable chronological gap between the diagrams and the period when the represented texts were compiled does not allow one to regard them as more than reconstructions, and, quite often they display clear elements of reinterpretation. Yet, there is some evidence to suppose that the diffusion of textual diagrams under the Southern Song and Yuan was a development of a continuous tradition of using texts, rather than the introduction of qualitatively new techniques. A good argument in favor of this supposition is found in the Genkô manuscript (Japan, A.D. 1333) of the *Wu xing da yi* ("The Compendium of the Five Elements", early 6th c. A.D.).[60] The manuscript contains a detailed textual diagram and two small supplementary diagrams (cf. **figure 8a**) featuring the structure of the *Hong fan* ("The Great Model", ca. fourth century B.C.) chapter of the *Shu jing/Shang shu* ("The Book of Documents").[61] These diagrams are said in the manuscript to be copies of Chinese originals of the Tang dynasty (A.D. 618–907).[62]

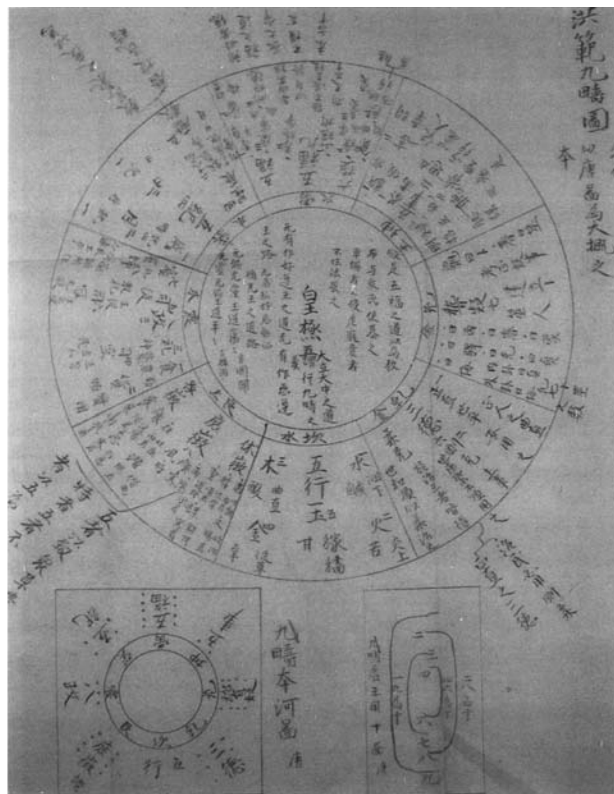


Figure 8a. Spatial layout of the nine "paragraphs" of the *Hong fan*, Gênko manuscript: "The diagram of the nine "paragraphs" of the *Hong fan*" (*Hong fan jiu chou tu*) (Reproduced from (Kalinowski 1991, coloured plate 2)).

The *Hong fan* is characterized by a clear-cut division into parts with a definitive order. It includes nine "paragraphs" *(chou)*,[63] and, in the two extant editions, each "paragraph" is supplied with a specific title and an ordinal number.[64] The main diagram entitled the *Hong fan jiu chou tu* ("The diagram of the nine "paragraphs" of the *Hong fan*") incorporates the key fragments of the "paragraphs". They are arranged according to the relative "positions" of the nine figures in the "magic square" *Luo shu*,[65] as found in its well-known representation imitating knotted cords, one that was accepted as standard in the Song dynasty (cf. **figure 8b**).[66] Thus, the first "paragraph" is placed at the "position" of the figure "1", the second at the "position" of the figure "2" and so on. The diagram is constructed as a nest of two circles.[67] The circle in the center has the fifth "paragraph" assigned to it. The peripheral ring is divided into eight sectors to provide the necessary number of "cells" for the other eight "paragraphs". It is subdivided into two layers: a thin strip enclosing the central circle contains "The Eight Trigrams" *(Ba gua)* and "The Five

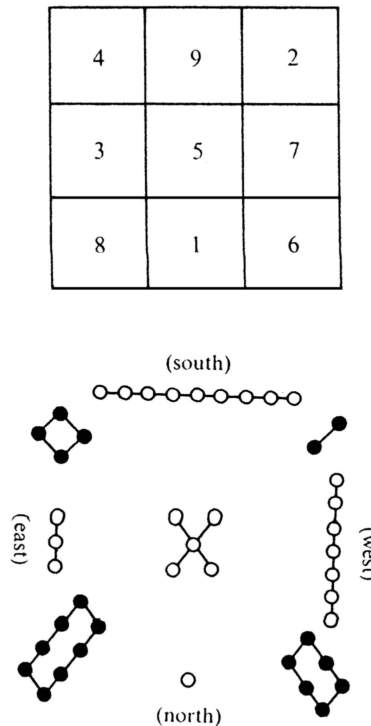


Figure 8b. *Luo shu* (“Writing from the Luo River”).

Phases”, and the rest of the space is occupied by the excerpts from the eight peripheral “paragraphs”.[68] In each “cell” the textual passages flank the title of the appropriate “paragraph”, which is given in a prominent way. Some “paragraphs” have comments provided outside the diagram.

It is especially noteworthy that the textual passages, like those of the Chu Silk Manuscript, are arranged in such a way that their reading requires rotating the diagram. Although the correspondences between the “paragraphs” and the eight cardinal directions are not provided in an explicit way, they may be derived from the placement of “The Eight Trigrams” and “The Five Phases”, and the acknowledged spatial orientation of the *Luo shu*. Consequently, the provided diagram may be regarded as having a cardinally-oriented textual layout.

The structure of the main diagram is reproduced in a simplified and slightly differing form by a small diagram at the bottom (left) of the illustration. This diagram does not contain any textual passages and its central circle is not filled up. The ring around the “empty” center contains only the names of “The Eight Trigrams”. This ring is surrounded by the titles of the corresponding peripheral “paragraphs” supplied by figures corresponding to the ordinal number of the “paragraphs”.[69] In contrast to the main diagram, the simplified

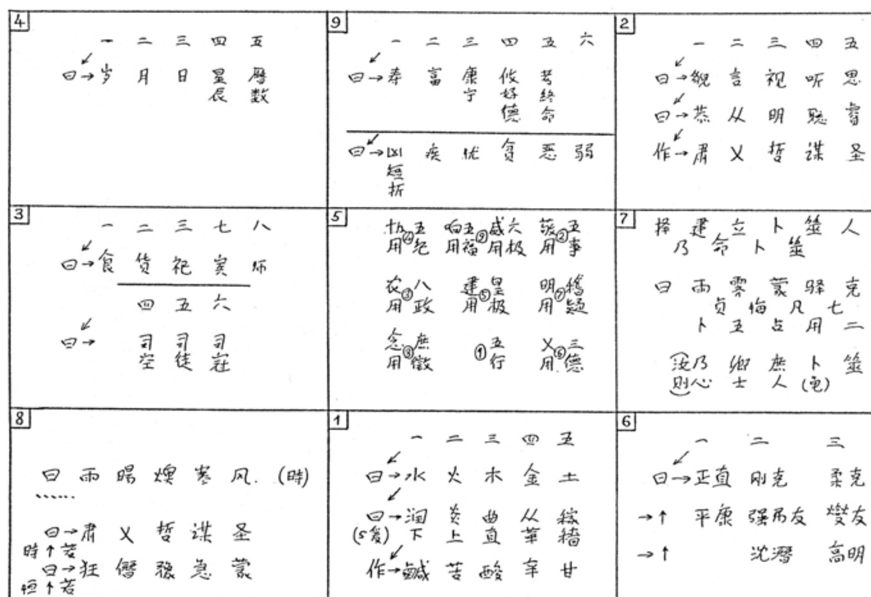


Figure 8c. Spatial layout of the nine "paragraphs" of the *Hong fan* by A. M. Karapetyants (Reproduced from (Karapiétiants 1991, 100)).

version is put in a square frame. This diagram has a title that is especially noteworthy—*Jiu chou ben He tu* ("The nine "paragraphs" are based on the *He tu*"). This means that the arrangement of "paragraphs" according to the standard representation of the *Luo shu* is referred to as *He tu*. This reference confirms the pre-Song origins of the diagrams declared in notices appended to their titles.[70] Moreover, the positions of the "paragraphs", as found on the diagrams, are listed in the *Wu xing da yi* dating from the early sixth century.[71]

Furthermore, the nine "paragraphs" of the *Hong fan* were firmly identified with the *Luo shu* by Han scholars, including Kong Anguo (ca. 156–ca. 74 B.C), Liu Xin (ca. 46 B.C.–A.D. 23) and Ma Rong (A.D. 79–166).[72] This identification allows one to suppose that the diagrammatical representation of this text might have had Western Han origins.[73] Some support for this supposition is provided by the striking typological similarity of the examined diagram to the rotating astronomical instruments of the Han epoch mentioned above. There is, however, no explicit evidence in the *Hong fan* itself that it was related to the *Luo shu*. Han scholars provided the missing link for this by stating that Yu first saw the *Luo shu* on the back of a turtle that emerged from the Luo River, then used it to compose the *Hong fan*.[74] This link allows one to establish, at least tentatively, correspondences between the diagrammatical representation of the *Hong fan* according to the *Luo shu*, and a cardinally-oriented image of the turtle (symbol of the cosmos), as it is found in the Yinwan divinatory manuscript dating from the Western Han and as has been proposed for its Shang prototype (inscribed turtle plastrons).

4.

Research on the details and significance of textual organization has become increasingly evident in Western sinological works of the last two decades.[75] Systematic exploration of formal textual structures has been carried out by Soviet sinologists since the end of 1960s.[76] However, the first step towards the recognition of the non-linear organization of some ancient Chinese texts in modern sinology was made in the middle of this century by two Chinese scholars, Wang Tingfang and Guo Moruo. Their work is of central importance, as it pinpoints the crucial characteristic of the non-linear framework of textual organization, namely, that it *has a certain meaning.*

They proposed a cardinally-oriented arrangement of a relatively extensive Chinese text, the *You guan* chapter of the *Guan zi*, a philosophical treatise compiled from earlier sources by the first century B.C. (cf. **figure 9a**).[77] This reconstruction is based on an observation that the *You guan* consists of clearly distinguished textual sections each assigned to a specific part of a spatial scheme.[78] They claimed that the scheme in question represented the *Xuan gong* ("The Dark Palace")[79], and that the arrangement of the textual sections according to their spatial references delineated the contours of the *Xuan gong*, while also describing it.

Yet, they did not offer much analysis for their reconstruction. The reconstructed scheme was then meticulously examined by W. Allyn Rickett,[80] who came to the conclusion that the current version of the *You guan* originated from its cardinally-oriented layout. Much consideration in his examination is given to the parallels between the structure of the reconstructed scheme and that of the *He tu*, which he refers to as the River Chart (note the general structure of the reconstructed chart by Rickett, **figure 9b** and compare it with the structure of the *He tu* in **figure 9c**).[81] He argues, "it would have been impossible to construct such a complicated chart on the usual narrow bamboo slips; only a relatively large area such as that provided by a silk scroll, tablet of wood or stone, or perhaps a bronze vessel would have sufficed". Further, he suggests, "that the original form of the chart may well have resembled the famous Chu Silk manuscript", but that "later, for convenience, the text must have been copied on bamboo slips in regular literary form. This may have been accompanied by a small outline of the chart showing the various geographical sections, or perhaps, because of its size, the chart was not reproduced at all".[82]

This supposition, however, overlooks the potential for reconfiguration of a text written down on a set of bound rolls/sets of bamboo slips or wooden tablets referred to above. It seems equally possible that the cardinally-oriented layout of the *You guan* could be constructed out of five bound rolls/sets of bamboo slips or wooden tablets. The five textual sections on the proposed reconstruction seem to fit this supposition quite well. Thus, the length of columns and the arrangement of the textual body into strips correspond rather well to what may be found on inscribed bamboo slips and wooden tablets, and the number of columns in a section correspond to that in a short bound roll or a set of slips. Yet, in contrast to the layout on a silk scroll (or any other medium which could provide enough space for it), "assembling" this arrangement of separate bound rolls/sets of slips or wooden tablets took place only at a specific moment in time, that is, when the text was used. Apart from this, the rolls/sets were most likely kept in a "deconstructed" form —as a pile of unarranged textual sections.

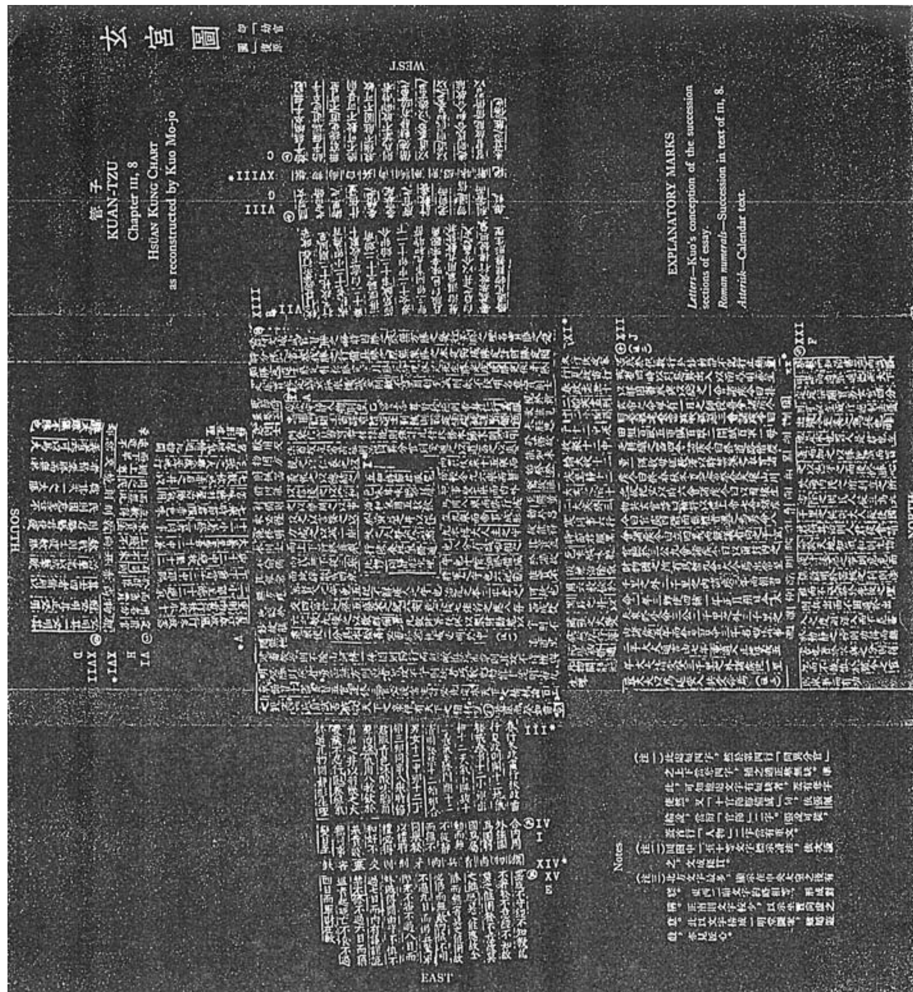


Figure 9a. Spatial layout of the *You guan* chapter of the *Guan zi* by Wang Tingfang and Guo Moruo: "The Plan of the Dark Palace" (*Xuan gong tu*) (Reproduced from (Rickett 1960, supplementary sheet)).

As far as the dating of the layout of the *You guan* in a schematic form is concerned, Rickett believes it to be most likely Western Han, and done in the wake of interest in what he refers to as "the construction of river charts" in the middle and late years of the Former Han dynasty (206 B.C.–A.D. 8), specifically, about a century before the final compilation of the *Guan zi* (ca. 26 B.C.).[83]

This argument is somewhat contradictory with the obviously pre-Han origins of the Chu Silk manuscript, which is typologically similar to "river charts", especially taking into consideration Rickett's own observation concerning the considerable resemblance

You guan Chart
Wang-Guo Reconstruction

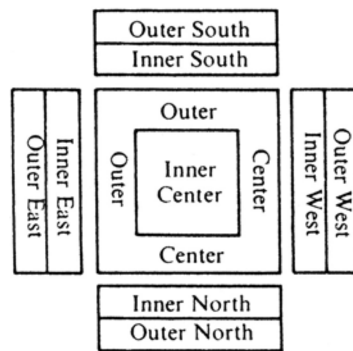


Figure 9b. General structure of the reconstructed layout of the *You guan* by W. A. Rickett (Reproduced from (Rickett 1985, 156)).

between the reconstructed layout of the *You guan* and the Chu Silk manuscript mentioned above.

Rickett focuses on the general structural characteristics of the reconstruction by Wang Tingfang and Guo Moruo, and is not interested in possible variations in the spatial layout of the *You guan*.[84] This question, in contrast, was raised by several Chinese scholars, by Wang Meng'e shortly after Wang and Guo proposed their reconstruction, and more recently by Li Ling and Hwang Ming-Chorng. All these scholars accept the ten-fold cardinally-oriented framework of the initial reconstruction, though they propose slightly differing arrangements of its ten constituent elements.[85]

More evidence of cardinally-oriented textual layouts prior to the Han is provided by the so-called Mausoleum Plan (*Zhao yu tu*) of the Zhongshan kingdom's rulers (cf. **figure 10**). It was discovered in 1978, in the tomb of King Cuo of Zhongshan (buried about 310 B.C.), and so dates to no later than the fourth century B.C.[86] The plan is engraved on a bronze plate, and the contours of the represented buildings and walls are delineated with gold and silver inlay. The contours are delineated symmetrically with respect to the south-north central axis, and the plan is oriented with south at the top, as are the majority of early Chinese maps and spatial models. The plan includes short textual passages that name the objects represented, providing their measurements and the distances between them. One relatively long passage in the southern part conveys a transcription of a decree issued by the Zhongshan king concerning the plan of the mausoleum.

However, studies on the Mausoleum Plan so far, do not give much consideration to the remarkably regular arrangement of the textual passages, an arrangement that plays a very important role in the graphic representation of the mausoleum. Most of these passages consist of columns of uniform length that form squares or rectangles filled out by characters, and, thus, reproduce the square and rectangular shapes of the objects represented by lines. Altogether, there are thirty-three squares and rectangles formed by these blocks of characters. Each of them is "complete", with no lacunae in their "filling".

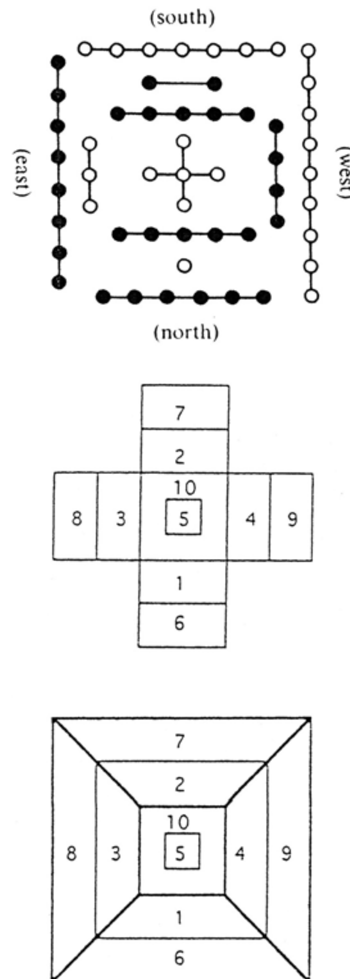


Figure 9c. *He tu* ("Plan/Chart from the He River").

In some cases the textual squares and rectangles are inscribed into the square contours of buildings delineated by gold or silver inlays. There are also single columns of characters (twenty-six) and single characters (two) on the plan. All these textual passages are placed symmetrically with respect to the south-north central axis of the plan with very few violations of perfect symmetry.[87] Finally, similar to the Chu Silk manuscript and to the reconstructed layout of the *You guan*, the textual passages of the Mausoleum Plan are placed in such a way that in the process of reading it must either be rotated or its "user" should move around it.

The same principles of constructing a plan or a map as a combination of lines and characters may be found on a map of a city, the so-called *Xiao cheng tu* ("Small map of a

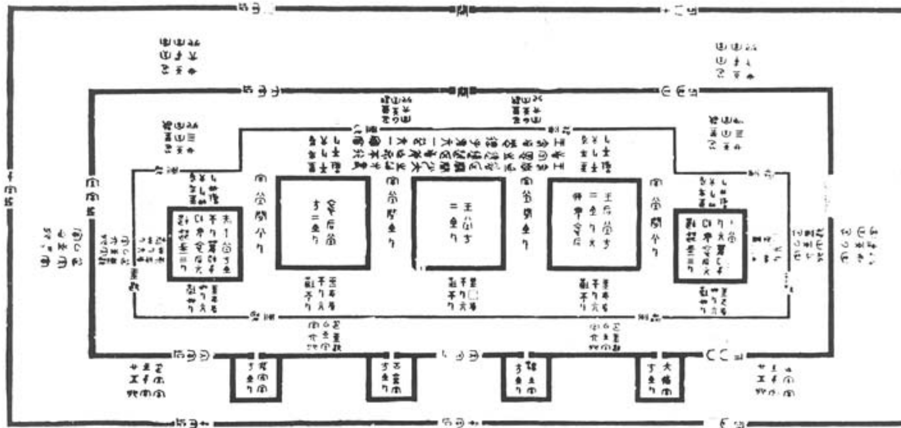


Figure 10. The Mausoleum Plan (*Zhao yu tu*, transcription) (Reproduced from (Cao Wanru, Tan Qixiang, Zheng Xihuang, Huang Shengzhang et al. [1990] 1999, figure 2)).

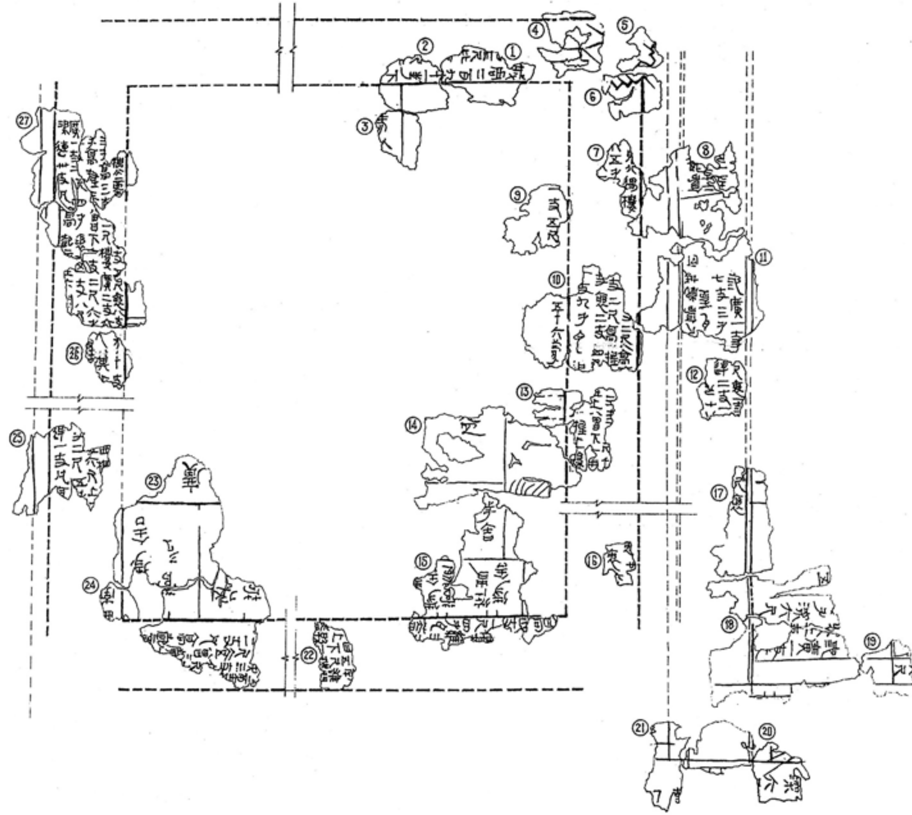
city"), dating from the Former Han dynasty, discovered at Mawangdui, but first published and studied much later than all the other Mawangdui maps (cf. **figure 11**).[88] It should be noted here that Chinese maps in general are distinguished by "textuality", that is, a strikingly high amount of textual material is included.[89]

I would also like to point out two ground-breaking conclusions by Rickett that are of central importance for analyzing textual structures.

First, he regards the later disagreements on the proper sequence of sections of the *You guan* as an indication of the originally non-linear layout of the text. He argues that such a layout, in contrast to the presentation of a text in a linear form, implied no specific order in which the text should be read, and it is therefore not surprising that there should be such disagreements.[90]

Second, on the basis of an investigation of the structure and the content of the cardinally-oriented sections of the *You guan*, he advanced the interesting idea that some textual passages might serve simply as "fillers" for the spatial scheme. Thus, he believes that the "essay" portion of the *You guan* served to fill up the contours of the scheme as the main "calendar" portion did not provide enough material for it, and that the "essay" originally was not written for the scheme.[91] This usage of the "essay", he argues, explains why the "calendar" and the "essay" portions are but barely related to each other, and why many pieces of the "essay" sometimes seem to defy any sense of order. Two interesting observations confirm his conclusions, namely, that the "outer" section of the text starts with a sentence evidently left over after "filling up" the "inner" central section;[92] and that there is one passage apparently lifted out of some other context and inserted as an additional "filler".[93] In a broad sense, the whole text of the *You guan* was used as a "filler" for the spatial scheme in the same way as the textual passages of the Mausoleum Plan, especially those filling up the delineated contours of buildings.

All the four texts—the cardinally-oriented layout of the *You guan*, the Chu Silk Manuscript, the Mausoleum Plan and the map from Mawangdui—combine a graphic



图一 马王堆三号西汉墓出土帛书中的小城图

Figure 11. "Small map of a city" (*Xiao cheng tu*) (Reproduced form *Wenwu*/Cultural Relics 1996. 6, p. 50).

representation of a model of space and an associated description. Since this textual construction is like a spatial scheme or cosmogram, it may be referred to as a *textual cosmogram.*[94]

The analysis of the spatial layout of the *You guan* and the Chu Silk Manuscript has highlighted a whole series of ancient Chinese texts whose structure emulates the conventional tailoring of space or time-space. Rickett provides an annotated list of "calendars" organized according to this principle.[95] The best known of these "calendars" is found with some variations in three texts:

- the *Ji* section (Part I) of the *Lü shi chun qiu* ("The Springs and Autumns of Mister Lü", compiled before 239 B.C.);[96]
- the *Shi ze* ("Seasonal Rules") chapter of the *Huai nan zi* ("The Philosophers of Huainan", compiled before 139 B.C.),[97] and
- the *Yue ling* ("Monthly Ordinances") chapter[98] of the *Li ji* ("The Records on Rituals").[99]

This "calendar" is often referred to as the *Yue ling*, after its officially recognized version.[100] The resemblance of the *You guan* to the *Yue ling* is detailed in the collation of citations by Chinese scholars provided in the *Guanzi jijiao* ("The Philosopher Guan with Collected Collations").[101] One of these scholars, Zhang Peilun, noted that the *Yue ling* chapter also has a longer title, namely, the *Ming tang yue ling* ("Monthly Ordinances in the Luminous Hall"),[102] and that the chart of the *You guan* was apparently typologically similar to that of the *Ming tang*.

This statement, however, seems to refer to the structure of the schemes rather than to imply that the *Yue ling* "calendar" could have originally been laid out in a non-linear form. Rickett did not suggest that the calendar texts could have originated from non-linear representations either, as, by contrast to the *You guan*, these texts do not contain straightforward indications as to such an arrangement. These indications, however, seem to be more of an exception than a rule.

CONCLUSIONS

It seems tempting to suggest, by analogy with the *You guan* and the finds of manuscripts referred to above, that at least some of the multiple ancient Chinese texts (or parts of texts) whose structure emulates spatial models could have originated from non-linear layouts, or, in other words, from textual cosmograms.[103] The only way to verify this supposition is to examine these textual structures in the same way as the *You guan*. The first step in this analysis is to determine what spatio-temporal or tempo-spatial model is emulated by a specific text. These models as such are rather simple constructs based on the principle of global symmetry and oriented to four or eight cardinal directions. As may be seen from the concise annotations to the "calendars" provided by Rickett in his list, it is generally more or less clear which specific model the text in question employs. The second step is to analyze how the body of the text is tailored to this spatial model, and then to propose a cardinally-oriented layout of the text, if it seems plausible. This analysis is, however, far beyond the scope of this paper.[104]

CNRS-EHESS

GLOSSARY

Ba gua	八卦
Chou	疇
Cu bo Shu	楚帛書
Da huang	大荒
Guo feng	國風
Guan zi	管子
du ci shu pang hang	讀此書旁行

Hai wai	海外
Han shu	漢書
He tu	河圖
Hong fan jiu chou	洪範
Huai nan zi	淮南子
Ji	紀
Jia gu wen	甲骨文
Jin wen	今文
Jing shang pian	經上篇
Jing shang pian pang hung ju du	經上篇旁行句讀
Jing shuo shang pian	經說上篇
Jing shuo xia pian	經說下篇
Jing xia pian	經下篇
Jiu chou ben He tu	九疇本河圖
juan	卷
Lao zi	老子
Li ji	禮記
Liuguo nianbiao	六國年表
Liujia yinyang shu	六甲陰陽書
Lü shi chun qiu	呂氏春秋
Luo shu	洛書
Ming tang	明堂
Ming tang wei	明堂位
Ming tang yue ling	明堂月令
Mozi	墨子
nan fang	南方
Tian nai xi Yu Hong fan jiu chou	天乃錫禹洪範九疇
Tianwen qixiang zazhan	天文氣象雜占
Tian yuan di fang	天圓地方
tu	圖
tu shi	圖式
pian	篇
ren zi	人子

Shan hai jing	山海經
Shen gui zhi fa	神龜之法
Shu jing/Shang shu	書經 尚書
Shi	式
Shi'er zhuhou nianbiao	十二諸侯年表
shi tu	式圖
Shi ze	時則
Shuo	説
si	司
Wu xing da yi	五行大義
Wu xing zhi	五行志
Xiao cheng tu	小城圖
Xingdao jixiong	行道吉凶
Xuan gong	玄宮
Zhao yu tu	兆域圖
Zhou bi suan jing	周髀算經
Zuo zhuan	左傳
Yao dian	堯典
You guan	幼官
Yue ling	月令

NOTES

[1] The first draft of this paper was written under the auspices of *the Maison des Sciences de l'Homme* Foundation. It was presented at the workshop *Geschichte der Wissenschaft, Geschichte des Textes* hosted by Einstein Forum, Potsdam & *Wissenschaftskolleg zu Berlin*, Berlin-Wannsee, Germany (March 30–April 2, 1995) which I attended thanks to the financial support of the Einstein Forum and the *Wissenschaftskolleg zu Berlin*. The revision of the earlier draft was completed under the auspices of the *Alexander von Humboldt* Foundation. I would like to thank Karine Chemla, Michael Lackner and Rémi Mathieu for their helpful comments on the earlier draft of this paper. I also owe a debt of gratitude to my colleagues who participated in the discussion of this work at the *Ostasiatisches Seminar der Universität Göttingen* on the 6th of June, 1997. This paper reproduces with some up-dates my earlier publication in the first issue of the *Göttinger Beiträge zur Asienforschung* (Göttingen Asiatic Studies) journal (2001). I am truly thankful to the editors of the journal for having granted their permission for its reproduction. My special thanks to John Moffett for corrections of my English, as well as for his valuable comments. Any mistakes found in this paper are my own responsibility.

[2] Varying types of graphic representations were apparently not distinguished clearly from each other and formed a single class. For the implications of the character *tu*, cf. (Lackner 1990, 1992, 1996, 2000), (Reiter 1990), and the paper by Craig Clunas in this collection. This term has been the topic of three recent scholarly meetings:

– Panel "*Tu* (Diagrams, Charts, Drawings) in Traditional Chinese Culture", annual conference of the AAS (The Association for Asiatic Studies, 29.03.1998, Washington D.C.);
– Panel "Illustrations (*tu*) in Traditional Chinese Science, Technology, and Medicine: Comparative and Cross-cultural Perspectives", the *9th International Conference on the History of Science in East Asia* (23–27.08.1999, Singapore);
– EUROPEAN AND NORTH AMERICAN EXCHANGES IN EAST ASIAN STUDIES CONFERENCE: *From Image to Action: The Dynamics of Visual Representation in Chinese Intellectual and Religious Culture* (3–5.09.2001, Paris).

Selected papers presented at these conferences are to be published under the title *The Power of Tu: Graphics and Text in the Production of Technical Knowledge in Chin*a (Leiden: Brill).

[3] I understand as elucidations here any textual passages relevant to the *tu*.

[4] E.g., a *tu* may emulate a spatial model thus conveying its image. Examples of such *tu* are focused on in this paper.

[5] In the following discussion much consideration will be given to the influence of the early writing media used in ancient China on structuring texts. By writing media I mean oracle bones, bronze vessels, bamboo slips, wooden tablets, silk scrolls, etc.

[6] E.g., the first Chinese Emperor, Qin Shi Huangdi (r. 221–210 B.C.), effectuated a series of measures aimed at consolidating the newly founded empire, such as reforming the Chinese script so that it would be uniform throughout the empire, and instituting a standard system of weights and measures. He is also said to have ordered the destruction of many texts, old and new, extant at the time. It goes without saying that this reform of the script and the later restoration of some of the texts he had had destroyed led to considerable transformations and even distortions of their original form. These changes also apparently resulted from further systematic editing, collating and compiling of ancient texts later in the Former Han dynasty.

[7] A brief review of these studies will be provided below.

[8] The earliest samples date from ca. the fourteenth century B.C. For the seminal work on the subject, cf. (Keightley 1985).

[9] Regular layout is particularly characteristic of many inscriptions dating from ca. 1200–1050 B.C., the period beginning from the reign of the King Wu Ding (r. ca. 1200–1181 B.C.). For metaphysical assumptions underlying the placement of the inscriptions, cf. D. Keightley 1988. He has also pointed out the *tu*-like properties of the inscriptions in his comments on papers presented at the Panel "*Tu* (Diagrams, Charts, Drawings) in Traditional Chinese Culture" organized in the framework of the annual conference of the AAS (The Association for Asian Studies, 29.03.1998, Washington D.C.).

[10] Parallel characters are also engraved as mirror images.

[11] (Allan 1991, 75–111). This hypothesis is, however, criticized by Keightley (2000, 93–96).

[12] This text (*Liujia yinyang shu*) comes from tomb no. 6 dated to the Yongshi (16–13 B.C) and Yuanyan (12–9 B.C) reign periods of Emperor Cheng of the Former Han. For a preliminary comment on the Yinwan manuscripts, cf. ("A Quantity of Bamboo and Wooden Slips . . . " 1995). For a description of the Yinwan tombs, cf. (*Lianyungang shi bowuguan/*Museum of Lianyungang City 1996a, for dating, cf. 23–24). For a description of the wooden boards and bamboo slips, cf. (Teng Zhaozong 1996). For a transcription of selected manuscripts including the divinatory text, cf. (*Lianyungang shi bowuguan/*Museum of Lianyungang City 1996b).

[13] The south-west is wrongly written here with the character *zheng* ("right", "true") being used instead of *xi* ("West"). The correct character for this line is, however, evident from the context.

[14] The cyclical signs were necessary attributes of the Shang inscriptions on oracle bones. They designated the day on which the divination took place.

[15] The name "TLV" is given according to the same principle as the T-O maps designating a distinctive type of maps in medieval cartography. The constituent elements of the scheme have the form of the letters T, L and V of the Latin alphabet. For interpretation of the TLV design of the "magic" mirrors and divinatory schemes in the context of ancient Chinese spatial conceptions, cf. (Hwang Ming-Chorg 1996, 91–118), (Kalinowski 1998–1999), (Li Ling 2000, 89–176).

[16] Two rather big characters—*nan fang*—designating the south or the Southern Quadrate are placed at the top of the scheme.

[17] However, there are some differences between the arrangement of the cyclical signs and oracular charges. Due to the evolution of writing, the cyclical signs are not inscribed as mirror images, and, in contrast

to the oracular charges, their arrangement is also symmetrical with respect to the central crosswise axis.

18 For Sarah Allan a plastron is just a symbol of a cross —with no difference between its head and tail part.

19 This problematic is raised in a remarkable work by (Sementsov 1981) concerned with the interpretation of *Upaniṣads*. He argues convincingly that these texts were created as ritual texts accompanying a ritual procedure, and, therefore, approaching them from a philosophical perspective gives rise to an inadequate interpretation.

20 (Keightley 1985, 106–8; 112).

21 Bronze inscriptions originated in the Late Shang, and became especially important and wide spread under the Western Zhou (1046/45/40–771 B.C.), cf. (Shaughnessy 1991).

22 For an analysis of their structural pattern and its variations, cf. (von Falkenhausen 1993, 152–61). Lothar von Falkenhausen also draws attention to the highly important problematic of interpreting the bronze inscriptions, which is relevant to some aspects of interpreting ancient texts in general discussed above. In particular, he argues convincingly that in order to understand the full meaning of the bronze inscriptions they should be considered in conjunction with the ritual function of the inscribed object and with its form and ornament, and criticizes the common tendency to regard the inscriptions as exclusively philological entities. Cf. (von Falkenhausen 1993, 146–52). This tendency can also be noticed in the studies of the inscriptions on oracle bones.

23 Cf. (von Falkenhausen 1993, 155).

24 According to the study by (Pan Jixing 1997), woodblock printing was invented in China sometime around the early seventh century A.D.

25 Cf. (Tsien Tsuen-hsuin 1962).

26 It is necessary to note here that bamboo is a more ancient writing medium than silk, and many of texts on silk copied those on bamboo slips. A piece of silk provided, as a rule, enough space to incorporate several "bound rolls of bamboo slips". A good illustration of this is provided by the study of the Commentaries of the silk manuscript of the *Yi jing* from Mawangdui (Former Han dynasty) by (Li Xueqin 1995). He shows that this silk manuscript apparently originated from a version on bamboo slips.

27 An excellent example of this effect is the inscriptions on turtle plastrons. The influence of the original "materiality" of ancient texts on its received form is focused on by Erik W. Maeder in his study of the composition of the "Core Chapters" of the *Mo zi*, a philosophical treatise composed about the fourth-third centuries B.C., cf. (Maeder 1992). Maeder demonstrates how the organization of the extant version of the text was determined by its original arrangement on bamboo slips.

28 Wooden tablets were used as a writing medium simultaneously with bamboo slips, but were not so widely used as the former. A wooden tablet may be compared to a bound roll/set of bamboo slips.

29 A good example of the loose order of textual parts is provided by the *Guo feng* (literally "The Winds (=Directions) of the Principalities/Kingdoms") section of the *Shi jing* ("The Classic of Songs", ca. eleventh-seventh centuries B.C.). Thus, the order of subsections of the *Guo feng* as given in the extant version of the classic (*Mao shi*) differs from the one found in the list of these subsections in the *Zuo zhuan* ("The Zuo Narrative") listed under the 29th year of Earl Xiang (cf. *Chunqiu Zuozhuan zhengyi*, *Sibu beiyao* ed., 39/5a–7b). I suggest that this set of textual sections conveys an ideal image of Zhou principalities in my paper published in the issue of *Extrême-Orient—Extrême-Occident* featuring Soviet studies of textual structures, cf. (Dorofiéiéva-Lichtmann 1991).

30 This may be clearly seen on the bamboo version of the *Shi jing* from Fuyang, cf. illustrations showing the bamboo slips in (Hu Pingsheng and Han Ziqiang 1988).

31 Cf. (Maeder 1992, especially 27–29, 81–82). The transpositional potential of bamboo slips can be clearly seen from its negative consequences—cases of apparent "migrations" of slips to a wrong place. It seems most likely that this happened when the bindings of a roll/set broke and the slips had to be re-assembled into a consistent text, for examples of such cases cf. (Shaughnessy 1986, 165–80) and (Li Xueqin 1995, 370–72).

32 Cf. (Maeder 1992, 28).

33 I would like to point out that this supposition does not mean that *every* text on bamboo slips was non-linearly arranged, merely the possibility of such an arrangement in some cases.

34 It is especially interesting that this divinatory table incorporates a diagram—two sketches of a human body with twelve cyclical signs and the four seasons assigned to its parts. The diagram is entitled

ren zi, cf. (Li Ling 2000, 204, 206). For the divinatory texts from Shuihudi, cf. (Jao Tsung—i and Zeng Xiantong 1982), (Li Ling 2000, 197–216).

[35] The Fuyang "Annals" are similar in their arrangement to the *Shi'er zhuhou nianbiao* ("The yearly table of the twelve *zhu hou* (rulers of principalities) [houses]" and *Liuguo nianbiao* ("The yearly table of the six kingdoms") of the *Shi ji* ("The Records of Historian") by Sima Qian (ca. 145–ca. 86 B.C.). Thus, years mark the horizontal columns and kingdoms the vertical columns, cf. (Hu Pingsheng 1989, 7 and illustrations).

[36] Cf. (*Lianyungang shi bowuguan*/Museum of Lianyungang City 1996a, 18). There is serious evidence to believe that the so-called *jing* ("canons") part of the *Mo zi* (chapters 40–41, the *Jing shang pian* and the *Jing xia pian*; and chapters 42–43 containing the "elucidations" (*shuo*) to the "canons", the *Jing shuo shang pian* and the *Jing shuo xia pian*) was originally arranged in a similar way—as two strips. This original structure was lost when the strips were rewritten in a linear sequence. Yet, there are clear indications to these strips at the end of chapters 40 and 43 (*du ci shu pang hang*—"read this text [according to] the rows at the [upper and lower] sides [of the bound roll of bamboo slips]") and (*jing shang pian pang hang ju du*—"[as far as] the *Jing shang pian* [chapter is concerned], [its] phrases [should] be read [according to] the rows at the [upper and lower] sides [of the bound roll of bamboo slips]", cf. *Mozi xiangu*, *Zhuzi jicheng* ed., vol. 4, pp. 195 and 234, respectively). These phrases are supplied with extensive commentaries arguing for the two—stripped arrangement of the text. A reconstruction of the original arrangement of chapters 40–41 was proposed by the Qing scholar Sun Yirang, the editor of the *Mozi xiangu* (cf. *Mozi xiangu*, *Zhuzi jicheng* ed., 4: 235–42).

[37] Some silk manuscripts from Mawangdui, e.g., the manuscript B of the *Lao zi*, give an impression of being inspired by a layout of the text on bamboo slips. Thus, clearly delineated columns and the way the borders of the manuscript are trimmed seem to reproduce how the inscribed bamboo slips looked, cf. **figure 5**.

[38] This dating is proposed by Donald Harper, cf. (Loewe and Shaughnessy, eds. 1999, 845).

[39] For seminal studies of the Chu Silk Manuscript, cf. (Barnard 1972–1973); (Jao Tsung-i (Rao Zongyi) and Zeng Xiantong 1985); (Li Ling 1985); (Li Xueqin 1994, 37–91). For the main problems raised in the studies of the Chu Silk Manuscript, cf. the "Discussion" in (Lawton, ed. 1991, 176–83). For a survey of studies of the manuscript followed by its transcription, cf. (Li Ling 2000, 178–96). For translations of this text, cf. (Barnard 1973) and (Li Ling and Cook 1999, 171–176).

[40] The shorter of these two central textual sections contains eight columns of thirty-six characters, the longer section thirteen columns of thirty-four characters, cf. transcription of the Chu Silk Manuscript by Li Ling (reproduced on **figure 6b)** supplied with numbers of characters in rows and columns (cf. also transcription in (Barnard 1972, 10)). In both cases the last columns are not filled out completely. The textual sections include 263 and 412 characters, respectively. They are referred to as text A and B by Noel Barnard, and *vice versa* by Li Ling.

[41] These passages are referred to in scholarly literature as texts C.

[42] For exploration of appearances and functions of these spirits, cf. (Barnard 1988), (Hwang Ming-Chorng 1996, 72–85).

[43] Somewhat similar compounds of pictures and textual passages elucidating them and are also found on a silk manuscript from Mawangdui that deals with similar matters—astronomy, astrology and divination (*Tianwen qixiang zazhan*). Here all the combinations of pictures (showing creatures and astronomical symbols) and accompanying textual passages are arranged into strips. This arrangement resembles the strip—like layout of text on bamboo slips mentioned above, especially the "almanac" from Shuihudi with incorporated pictures (the *ren zi* diagram, cf. **figure 2a–b**).

[44] This supposition, however, can only be confirmed by new finds of similar texts.

[45] The symbolism of the four trees in Chinese cosmology is extensively discussed in (Hwang Ming-Chorng 1996, 328–402), for the four trees depicted on the Chu Silk Manuscript, cf. (Hwang Ming-Chorng 1996, 330–334).

[46] An example of a TLV scheme oriented to the south (Yinwan wooden tablet, face B, cf. **figure 2b**) was given above. A list of other examples of southern orientation is provided by (Hwang Ming-Chorng 1996, 67–71) in his discussion of the reading order of the Chu Silk Manuscript. He also provides some other convincing reasons in favor of such an initial position of the text. It should be noted that Li Ling, as one can see from his transcription of the text reproduced on **figure 6b**, nevertheless proposes to start the reading of the manuscript from the opposite position—beginning from the longer of the

central sections placed head up in front of the "user". The top of the text in this case corresponds to the north.

47 The earliest of the discovered cosmographs date from the Former Han dynasty. For the cosmograph, cf. (Harper 1978–1979), (Cullen 1980–1981), (Field 1992), (Li Ling 1991; 2000, 89–176), (Major 1993, 39–43; 1999, 141–42), (Li Ling and Cook 1999, 172).

48 Cf. (Li Ling 1991; 2000, 180; 190–191), (Li Ling and Cook 1999, 172).

49 The earliest occurrence of the classical formula "Heaven [corresponds to a] circle, Earth [corresponds to a] square" (*Tian yuan di fang)* is found in the philosophical treatise *Huai nan zi* compiled shortly before the 139 BC, cf. *Huai nan zi*, SBBY ed, 3/9a, cf. also *Huai nan zi*, 3/1b; 15/3a. It is also found in the opening section of the astronomical and mathematical classic *Zhou bi suan jing* (compiled ca. 50 B.C.–A.D. 100), cf. (Qian Baocong, ed. 1963, 4).

50 For a list of similarities between the cosmograph and the layout of the Chu Silk Manuscript, cf. (Li Ling 2000, 190–91).

51 Elsewhere I discuss a cosmograph-like representation of terrestrial space in the *Shan hai jing* ("The Itineraries of Mountains and Seas", compiled no later than the beginning of the first century B.C.). I define the nature of this representation as a *sacred* or *spiritual landscape*, and point out its typological similarity with the cardinally-oriented system of spirits of the Chu Silk Manuscript, cf. (Dorofeeva-Lichtmann 2003). I further develop on this issue, in particular on the instrumental aspects of both texts, in (Dorofeeva-Lichtmann forthcoming 2003, part II). My study is primarily concerned with the first part of this text, the *Shan jing* ("The Itineraries of Mountains"). Similar conclusions are drawn by Hwang Ming-Chorng as the result of his exploration of its other parts—"Outside the Seas" (*Hai wai*) and "The Great Wilderness" (*Da huang*). He argues that these parts of the SHJ originated from some types of cosmograph, cf. (Hwang Ming-Chorng 1996, 494–509, especially 502–506 for "Outside the Seas"; and 537–677, especially 666–677 for "The Great Wilderness"). He believes that these cosmographs were comprised of spatially arranged pictures, which would make them strongly resemble the Chu Silk Manuscript.

52 Cf. the "Discussion" in (Lawton 1991, 176).

53 The textual parts and the pictures are not only divorced from each other by Li Ling, but even their layouts are given from different angles. Moreover, his study concerned with the "pictorial cosmograph" contains a transcription of the textual parts without showing their placement in the manuscript. This is especially surprising, as when pointing to the structural similarity between the "pictorial cosmograph" of the Chu Silk Manuscript and cosmograph design, Li Ling argues that the two central textual sections are placed at the position of the Big Dipper or Taiyi (Li Ling 2000, 190, §5), thus recognizing the structural importance of the textual parts.

54 Especially in his discussion of the reading order of the Chu Silk Manuscript (Hwang Ming-Chorng 1996, 67–71).

55 As was mentioned above, bamboo slips corresponded well to the needs of a living tradition. The use of silk and of block-prints, in their turn, also responded well to the need of the moment —"conserving" and reproducing the systematized and "canonized" cultural heritage.

56 E.g., although the columns in block-printed books originate from bamboo slips, the former are considerably shorter. Many texts on bamboo slips and wooden tables were arranged into strips, as mentioned above. This arrangement is no longer in evidence in block-prints. All these factors inevitably lead to some transformations of the original textual arrangement.

57 The same may be supposed with respect to wooden tablets.

58 Cf. (Lackner 1990, 1992, 1996, 2000). It should be noted that the diagrams are the "end products" of these practices. The principles of deriving them, that is, the principles of the commentarial practice are almost never explicit, cf. (Lackner 1996, 35).

59 As may be seen from the diagram discussed below, these two modes of representing textual organization may be easily transformed one into another.

60 Cf. (M. Kalinowski, trans. 1991).

61 The *Hong fan* is one of the most important ancient Chinese texts, and considered to be the *locus classicus* for the theory of "The Five Phases". For translations of this text, cf. (Legge 1865,3/4:320–44), (Couvreur [1899] 1913, 194–209), (Karlgren 1950, 28–35). For its comprehensive study, cf. (Nylan 1992). According to the introductory part of the *Hong fan* it was granted by Heaven to the mythical emperor Yu the Great (*Tian nai xi Yu Hong fan jiu chou* —"Heaven then granted to Yu the *Hong fan* in

nine 'paragraphs' ", cf. (Legge 1865,3/4:323, §3), (Couvreur [1899] 1913, §3); (Karlgren 1950, original text p. 28, §3; trans. p. 30, §3).

62 The main diagram is referred to as the "Tang diagram" (*Tang tu*) in the notice appended to its title (see the upper right corner of the illustration on **figure 8a**). The character "Tang" is appended, as a short notice, to the titles of the two small diagrams (see the bottom of the illustration).

63 Literally *chou* means "a ploughed up field". For this reason referring to them as "paragraphs" is a conventional translation showing that they are relatively short pieces of text (the list of total numbers of characters in each "paragraph", as found in the *Jin wen* ("New Writing") version of the *Hong fan*, is provided by Artemy M. Karapetyants (cf. (A. M. Karapiétiants 1991, 105), and are as follows: 58, 48, 30, 20, 254, 84, 172, 178 (158), 45 correspondingly, 889 (869) in sum).

64 The ordinal numbers and titles are first given separately, as a "table of contents", and then as headings of the "paragraphs".

65 For the highly important role of the pair of "magical" patterns known as the *Luo shu* ("The Writing from the Luo River", cf. **figure 8b**) and the *He tu* ("The Plan or Chart from the He River", cf. **figure 9c**) in Chinese thought, cf. (Granet 1934, 177–208), (Needham 1959, 56–59), (Cammann 1961; 1962; 1985, 231–35), (Henderson 1984, 82–87), (Major 1984, 145–52), (Saso 1978). The ritual origins of these patterns, the *He tu* in particular, are pointed out by (A. Seidel1983, 297–302). She came to the conclusion that the term *He tu* in ancient sources referred not to one definite object but rather to a genre of power objects or magic texts. For the list of references to these patterns in pre-Han sources, cf. (Karlgren 1946, 273).

66 There are no earlier graphic representations or reliable identifications of the *Luo shu* and *He tu*.

67 The standard representation of the *Luo shu* is often associated with the 3×3 square grid (cf. **figure 8b**). It should be pointed out here that it can equally be inscribed into a circle, as done on the examined diagram.

68 Each trigram and phase corresponds to a certain peripheral "paragraph".

69 It is noteworthy that the figures are given according to the same principle as those found on the standard representation of the *Luo shu*. The small diagram at the lower right corner of the illustration is designed to point out that the figures associated with the "paragraphs" arrayed across the center give a constant sum—10 ($1+9$; $2+8$; $3+7$; $4+6$).

70 For the confusion of the *Luo shu* and *He tu* in the pre-Song sources, that is, before their standard representations appeared, cf. (Reiter 1990, 321). The example from the Genkô manuscript shows how this confusion was transmitted into Japan. Indeed, there are some examples of confusing the *Luo shu* and *He tu* in Japanese and also Korean sources after this no longer occurred in China.

71 Cf. (M. Kalinowski, trans. 1991, 182–83). It is especially noteworthy that the positions are defined according to their orientation with respect to the cardinal points. This supports regarding the diagrams as cardinally-oriented structures.

72 Cf. (Henderson 1984, 84 and reference 86).

73 The reconstruction of the arrangement of the *Hong fan* according to the *Luo shu* inspired by these references was proposed by Karapetyants, cf. (Karapiétiants 1991, 100–19, the reconstruction is given on p. 100, cf. **figure 8c**). Since the author of the reconstruction did not know yet about the diagram from the Genkô manuscript, it seems interesting to compare them. Although they differ in shape (the reconstruction is inscribed into 3×3 grid) and all the textual passages are facing the "reader" so that it need not be rotated, there is impressive typological similarity between them.

74 According to (Seidel 1993, p. 302), the earliest reference to this link is probably the one in the *Wu xing zhi* ("Treatise on the Five Phases") of the *Han shu* ("History of the Han Dynasty") by Ban Gu (A.D. 32–92), (*Han shu* 1975, *Ershisi shi* 1972–1977 ed., p. 1315).

75 A series of of relevant studies were published in the following issues of the journal *Extrême-Orient—Extrême-Occident*:

- *La Canonisation du Texte: aux origines d'une tradition* (n° 5, 1984);
- *Effets d'ordre dans la civilisation chinoise—Rangements à l'oeuvre, classifications implicites* (n° 10, 1988);
- *L'art de la liste* (n° 12, 1990);
- *Regards obliques sur l'argumentation en Chine* (n° 14, 1992);
- *Disposer pour dire, placer pour penser, situer pour agir* (n° 18, 1996).

Apart from four papers by Michael Lackner mentioned above (two of them published in *Extrême–Orient—Extrême–Occident*), cf. (A. Plaks 1977), (B. Elman 1984), (R. G. Wagner 1986), (D. J. Keegan 1988), (J. B. Henderson 1991, especially 106–115), (G. R. Hardy 1992), (L. Kohn 1992), (E. W. Maeder 1992), (L.Vandermeersch 1994, 235–275). Issues related to textual organization are being raised more and more often at international conferences of sinologists, cf. (Le Blanc 1994), (Mayer 1996), and especially, the series of scholarly meetings focused on the concept of *tu* (cf. footnote 2).

76 For the general state of Soviet researches in this area, cf. *Extrême-Orient Extrême-Occident* 13 (1991): *Modèles et structures des textes chinois anciens. Les formalistes soviétiques en sinologie*. It includes translations of papers or parts of books previously published in Russian, as well as articles written for this publication. An introductory paper by Alexei Volkov provides a survey of the Soviet studies of non-linear textual structures along with a comprehensive bibliography. A concluding paper by the same author summarizing the results of these studies contains important insights into the problematic.

77 Cf. (Guo Moruo, Wen Yiduo, and Xu Weiyi 1956, vol. I, facing p. 140).

78 Each section is supplied with an ending indicating its place within the scheme, for example, "the preceding is situated on the outer side of the western part of the chart."

79 The *Xuan gong* or *Xuan tang* ("The Dark Hall") designated the northern section of the ideal plan of the Zhou king's palace. Wang Tingfang and Guo Moruo claim that the characters *you guan* were written by mistake instead of *xuan gong*.

80 Cf. (Rickett 1960; 1965, 180–219; 1985, 149–192).

81 Cf. (Rickett 1985, 154–58). (Major 1984, 150) notes that apart from the striking resemblance of the reconstructed chart to the *He tu*, pointed out by Rickett, the chart may also be represented as a nest of concentric squares. It is, however, surprising that he does not recognize that the idea of concentric squares is obviously conveyed by the *He tu*. By contrast, he proposes an arrangement of the *He tu* in a 3×3 grid, cf. (Major 1984, 151).

82 Cf. (Rickett 1985, 152).

83 Cf. (Rickett 1985, 169).

84 Similar approach is found in the study by (Sarah Allan 1991, 75–111) who uses the reconstruction of the *You guan* as an example of the division of space into five quadrates (*fang*) oriented according to the cardinal directions and the center.

85 For a critical survey of proposed layouts, followed by his own reconstruction, cf. (Hwang Ming-Chorng 1996, 85–91). Cf. also (Li Ling 2000, 136–37).

86 Zhongshan kingdom was situated in the north-east of China. For detailed description of excavations of the King Cuo's tomb, cf. (*Hebei sheng wenwu yanjiusuo/The Hebei Provincial Institute of Cultural Relics* 1995). For a large-scale photo of the Mausoleum Plan bronze plan, and a copy of its characters and transcription, cf. (Cao Wanru, Tan Qixiang, Zheng Xihuang, Huang Shengzhang et al. [1990] 1999, figures 1, 2 and 3, respectively). For examinations of the Mausoleum Plan, cf. (Fu Xinian 1980), (Harley and Woodward (eds.) 1994, 36–37). Petra Klose provides a meticulous exploration and translation of its textual passages, with helpful schemes aimed at highlighting the structure of the designed plan and the reading order of the incorporated text (Petra Klose 1984). I would like to express my thanks to Alexander Mayer who called my attention to this work.

87 There are contours of three big squares in the central part of the plan. The left square contour contains two columns of characters, whereas the right one contains three. However, this slight violation of symmetry is "outweighed" by the symmetry of the contours. The passage with the transcript of the king's decree is arranged in such a way that its columns go from right (west) to left (east), as well as two single columns of characters in the middle of the of the lower part of the plan (north), but they are perfectly symmetrical with respect to the south-north central axis from the point of view of the number of characters.

88 Cf. (Fu Xinian 1996). Unfortunately, the poor state of this map, in contrast to the Mausoleum Plan, does not allow us to examine its structure.

89 The "textuality" of Chinese maps is pointed out by Cordell Yee, cf. (Harley and Woodward (eds.) 1994, 93–127), and by (Smith 1996, 3–4). It should be noted, however, that the other Mawangdui maps are, in contrast, impressively conventional and do not contain extensive textual interpolations, cf. (Harley and Woodward (eds.) 1994, 37–46).

90 Cf. (Rickett 1985, 152–54).

91 Cf. (Rickett 1985, 164–65).

[92] Cf. (Rickett 1985, 164, 185, reference 162).

[93] Cf. (Rickett 1985, 164–65).

[94] For Western traditions of making schemes out of texts, cf. (Schmitt 1989), also reference 13 in (Lackner 1992). For examples of cardinally-oriented texts in Western tradition, cf. (*Cartes et figures de la terre* 1980, 167–78). For interrelations between space and text discussed within a broad cultural context, cf. (Toporov 1983). For the spatiality of textual layout with respect to Judeo–Christian tradition of representing of space, cf. (Desreumaux and Schmidt (eds.)1988, 99–154, "L'espace de la page" section).

[95] Cf. (Rickett 1985, 158–63). One can add to this comprehensive list a part of the *Yao dian* ("The Laws of Yao", ca. fourth-third centuries B.C.) chapter of the *Shu jing/Shang shu* ("The Book of Documents") concerned with the brothers Xi and He (for a reconstruction of its tempo-spatial model, cf. (Hwang Ming-Chorng 1996, 624–28). For translations of this part of the *Yao dian*, cf. (Legge 1965,3/1:18–21, §§3–7), (Couvreur [1897] 1971, 3–6, §§3–7), (Karlgren 1950, 2–3, §§3–7).

[96] For translations of this section, cf. (Wilhelm 1928, 1–156), (Kamenarovic 1998, 29–189), (Knoblock and Riegel 2000, 59–273), (Tkachenko 2001, 71–181), its title (*Ji*) is translated as "Aufzeichungen", "Traités", "Almanacs", and "*Zamety*" ("Notes"), respectively.

[97] For translations of this chapter, cf. (Major 1993, 217–68), (Le Blanc and Mathieu 2003, 205–47).

[98] For translations of the *Yue ling*, cf. (Legge 1885.27: 248–310), (Couvreur [1899] 1913, 330–410).

[99] An anthology of ritual rules compiled during the Former Han Dynasty and recognized as one of the "Classics" (*jing*). The extant edition of 49 chapters is most likely traced to the 1st c. A.D. The origins of this text and a more precise dating, as well as of each of its sections, are still subjects of dispute, cf. (Riegel 1993, 294–95).

[100] For its outline and a comparison of its versions, cf. (Hwang Ming-Chorng 1996, 44–47). He uses the *yue ling* as a general term, and points out the *yue ling* aspects of the Chu Silk Manuscript, cf. ibid., 71–72.

[101] Cf. (Guo Moruo, Wen Yiduo, and Xu Weiyi 1956,1:104–105).

[102] The *Ming Tang* ("The Luminous Hall") is supposed to occupy the southern part of the ideal plan of the palace of the kings of the Zhou dynasty (1046/45/40–256 B.C.). The main reference for the concept of the *Ming tang* is now the dissertation by (Hwang Ming Chorng 1996). This remarkably comprehensive work discusses a broad range of primary sources (a large number of texts describing the *Ming tang*, various archeological finds), and is supplied with almost exhaustive bibliography of the issue. It also regards the concept of the *Ming tang* in the wider context of Chinese cosmology. For other studies of the *Ming tang* in Western languages, cf. (Maspero 1948–1951); (Soothill 1951, 84–96); (Sickman and Soper 1956, 212); (Bilsky 1975, 290–299; 324–330); (Henderson 1984, 59–87; 1994, 212–16); (Shatzman Steinhardt 1999, 15–16).

[103] An attempt to reconstruct a text-cosmogram behind the *Yue ling* "calendar" found in the "Observations" of the *Lü shi chun qiu* and in the "Monthly Ordinances" of the *Li ji* was done by a Russian sinologist (G. A. Tkachenko 1986), but, unfortunately, this work has never been published. Short remarks on the composition of the first twelve chapters of the *Lü shi chun qiu* are found in his earlier paper, cf. (Tkachenko 1976). This paper was translated into French, cf. (G. A. Tkatchiénko 1991), in the special issue of *Extrême-Orient—Extrême-Occident* featuring Soviet studies of textual structures mentioned above.

[104] I discuss the structure of the textual body of the *Shan hai jing* in (Dorofeeva-Lichtmann 1995, 69–71) and develop on this issue in the second part of my forthcoming paper, cf. (Dorofeeva-Lichtmann forthcoming 2003). In particular, I investigate the question of the so-called "lost" maps related to the *Shan hai jing*, and I show that this idea appears quite late, at the end of the eighteenth century. I advance a hypothesis that the original spatial layout of this text would have combined the properties of a *tu* (scheme-map) and elucidations (*shuo*) on this *tu*, and constituted a *textual cosmograph*. In this case maps seem simply not to be needed, as the textual body served the function of a map. I further suggest that, at least in some cases when the "loss" of ancient "global" maps is reported, no maps accompanying texts were lost or ever existed. The loss concerned the spatial layouts of the texts and practices of using these texts.

I propose a reconstruction of another text combining the properties of a spatial scheme and its elucidation (the *Ming tang wei*—"Positions in the Luminous Hall" chapter of the *Zhou li* —"Zhou Rituals", compiled about the fourth-third centuries B.C.) in the article prepared for the collection of papers on *tu* mentioned in footnote 2.

REFERENCES

"A Quantity of Bamboo and Wooden Slips Excavated from Han Tombs at Yinwan, Lianyungang City." 1995. *Early China News* 8: 21 (after *Zhongguo wenwu bao*, 29 October 1995).

Allan, Sarah. 1991. *The Shape of the Turtle: Myth, Art and Cosmos in Early China.* Albany: State University of New York Press.

Barnard, Noel. 1972–1973. *The Ch'u Silk Manuscript: Scientific Examination* (Part I, 1972), *Translation and Commentary* (Part II, 1973). Monographs on Far Eastern History, series 5. Canberra: Australian National University.

Barnard, Noel. 1988. "The Twelve Peripheral Figures of the Ch'u Silk Manuscript." *Zhongguo wenzi* 12: 453–513.

Bilsky, Lester James. 1975. *State Religion of Ancient China*. In Asian Folklore & Social Life Monographs, edited Lou Tsu-k,uang, Vol. 70–71. Taibei.

Cammann, Schuyler. 1961. "The Magic Square of Three in Old Chinese Philosophy and Religion." *History of Religions* 1.1 (June): 37–80.

Cammann, Schuyler. 1962. "Old Chinese Magic Squares." *Sinologica* 7.1: 14–53.

Cammann, Schuyler. 1985. "Some Early Chinese Symbols of Duality." *History of Religions* 24.3 (February): 215–54.

Cao Wanru, Tan Qixiang, Zheng Xihuang, Huang Shengzhang *et al*. [1990] 1999. *Zhongguo gudai dituji (The atlas of ancient Chinese maps)* [Warring States—Yuan Dynasty]. Beijing: Wenwu chubanshe.

Cartes et figures de la terre. 1980. Paris: Centre Georges Pompidou, CCI.

Couvreur, Séraphin, trans. (comprises original text). [1899] 1913. *Lî kî ou mémoires sur les bienséances et les cérémonies*, vol. 1. Ho Kien Fou: Imprimerie de la Mission Catholique.

Couvreur, Séraphin, trans. (comprises original text) [1897] 1971. *Chou king* (texte chinois avec une double traduction en français et en latin, des annotations et un vocabulaire). Taibei: Ch'eng Wen publishing company.

Cullen, Christopher. 1980–1981. "Some Further Points on the *Shih." Early China* 6: 31–46.

Desreumaux, Alain, and Schmidt, Francis (eds.) 1988. *Moïse géographe: recherche sur les représentations juives et chrétiennes de l'espace.* Paris: Librairie Philosophique Vrin.

Dorofiéiéva-Lichtmann, Viéra. 1991. "Les 'Vents des royaumes' (*Guo feng*): un schéma géographique." *Extrême-Orient—Extrême Occident* 13 (*Modèles et structures des textes chinois anciens. Les formalistes soviétiques en sinologie*): 58–92.

Dorofeeva-Lichtmann, Vera. 1995. "Conception of Terrestrial Organization in the *Shan hai jing*." *Bulletin de l'Ecole Française d'Extrême Orient* 82: 57–110.

Dorofeeva-Lichtmann, Vera. 2003. "Mapping a 'Spiritual' Landscape: Representing Terrestrial Space in the *Shan hai jing*." In *Political Frontiers, Ethnic Boundaries, and Human Geographies in Chinese History*, edited by Nicola di Cosmo and Don Wyatt, 35–79. London: Curzon—Routledge.

Dorofeeva-Lichtmann, Vera. forthcoming 2003. "Text as a Device for Mapping a Sacred Space: A Case of the *Wu zang shan jing* ("Five Treasuries: The Itineraries of Mountains")." In *Creating and Representing Sacred Spaces*, edited by Michael Dickhardt and Vera Dorofeeva-Lichtmann. *Göttinger Beiträge zur Asienforschung* 2–3.

Elman, Benjamin. 1984. *From Philosophy to Philology*. Harvard University Press.

Falkenhausen, Lothar von. 1993. "Issues in Western Zhou Studies: A Review Article." *Early China* 18: 139–226.

Field, Stephen. 1992. "Cosmos, Cosmograph, and the Inquiring Poet: New Answers to the 'Heaven Questions'." *Early China* 17: 83–110.

Fu Xinian. 1980. "Zhanguo Zhongshan wang Cuo mu chutu de 'Zhaoyu tu' ji qi lingyuan guizhi de yanjiu." (A Study of the Mausoleum Map unearthed from the tomb of the King Cuo of the Warring States period's Zhongshan kingdom and the planning of the mausoleum). *Kaogu Xuebao* 1980.1: 97–118.

Fu Xinian. 1996. "Ji Gu Tiefu Xiansheng fuyuan de Mawangdui sanhao mu boshu zhong de xiao chengtu." (Some notes on a small city-map on a piece of silk from the Mawandui Tomb 3 restored by Mr. Gu Tiefu). *Wen wu/Cultural Relics* 1996.6: 49–55.

Granet, Marcel. 1934. *La pensée chinoise.* Paris: La Renaissance du livre.

Guo Moruo, Wen Yiduo, and Xu Weiyu. 1956. *Guanzi jijiao* (The *Guanzi* with Collected Collations). Beijing: Kexue Chubanshe.

Hardy, Grant. R. 1992. "Form and Narrative in Ssu-ma Ch'ien's *Shih-chi*." *Chinese Literature: Essays, Articles, Reviews* 14 (December): 1–13.

Harley, John B. and Woodward, David (eds.) 1994. *Cartography in the Traditional East and Southeast Asian Societies* (The History of Cartography, Vol. II.2). Chicago—London: The University of Chicago Press.

Harper, Donald J. 1978–1979. "The Han Cosmic Board *(Shih)*." *Early China* 4: 1–10.

Hebei sheng wenwu yanjiusuo/The Hebei Provincial Institute of Cultural Relics. 1995. *Cuo mu—Zhanguo Zhongshanguo guowang zhi mu/Tomb of Cuo, the King of the Zhongshan State in the Warring States Period*, 2 vols. Beijing: Wenwu chubanshe.

Henderson, John. B. 1984. *The Development and Decline of Chinese Cosmology.* New York: Columbia University Press.

Henderson, John B. 1991. *Scripture, Canon, and Commentary: A Comparison of Confucian and Western Exegesis.* Princeton: Princeton University Press.

Henderson, John B. 1994. "Chinese Cosmographical Thought: The High Intellectual Tradition." In *Cartography in the Traditional East and Southeast Asian Societies* (*The History of Cartography*, Vol. II.2), edited by John B. Harley and David Woodward, 203–27. Chicago—London: The University of Chicago Press.

Hu Pingsheng. 1989. "Some Notes on the Organization of the Han Dynasty Bamboo 'Annals' Found at Fuyang." *Early China* 14: 1–26.

Hu Pingsheng and Han Ziqiang. 1988. *Fuyang Han jian Shijing yanjiu* (A Study of a Han version of the *Shijing* on bamboo slips from Fuyang). Shanghai: Guji Chubanshe.

Hwang Ming-Chorng. 1996. "Ming-Tang: Cosmology, Political Order and Monuments in Early China." Ph. D. Diss., Harvard University.

Jao Tsung-i and Zeng Xiantong. 1982. *Yunmeng Qinjian rishu yanjiu* (A Study of Almanac Texts on Qin Dynasty Bamboo Slips from Yunmeng), Hong Kong: Xianggang Zhongwen Daxue Chubanshe.

Jao Tsung-i (Rao Zongyi) and Zeng Xiantong. 1985. *Chuboshu* (The Chu Silk Manuscript). Hong Kong: Zhonghua Shuju.

Kalinowski, Marc, trans. 1991. *Cosmologie et divination dans la Chine Ancienne. Le compendium des cinq agents (Wuxing dayi, VIe siècle).* Paris: Publications de l'Ecole Française d'Extrême-Orient, vol. CLXVI.

Kalinowski, Marc. 1998–1999. "The *Xingde* Texts from Mawangdui." *Early China* 23–24: 125–202.

Kamenarovic, Ivan P., trans: 1998. *Printemps et automnes de Lü Buwei*. Paris: Les Editions du Cerf.

Karapiétiants (Karapetiants), Artiémiî (Artemy) M. 1991. "Modèle universel *(Hong fan)* et classifications chinoises antiques à cinq et à neuf termes." *Extrême-Orient—Extrême-Occident* 13 (*Modèles et structures des textes chinois anciens. Les formalistes soviétiques en sinologie*): 100–119.

Karlgren, Bernhard. 1946. "Legends and Cults in Ancient China." *Bulletin of the Museum of Far Eastern Antiquities*18: 119–365.

Karlgren, Bernhard. 1948–1949. "Glosses on the Book of Documents." *Bulletin of the Museum of Far Eastern Antiquities* 20 (1948): 39–315; vol. 21 (1949): 63–206.

Karlgren, Bernhard, trans. 1950. "The Book of Documents." *Bulletin of the Museum of Far Eastern Antiquities* 22: 1–81.

Keightley, David N. [1978] 1985. *Sources of Shang History. The Oracle-Bone Inscriptions of Bronze Age China.* Berkeley, Los Angeles, London: University of California Press, 1985.

Keightley, David N. 1988. "Shang Divination and Metaphysics." *Philosophy East and West* 38.4 (October): 367–97.

Keightley, David N. 2000. *The Ancestral Landscape: Time, Space and Community in Late Shang China (ca 1200–1045 BC)*. Berkeley: Institute of East Asian Studies, University of California; Center for Chinese Studies (China Research Monograph 53).

Keegan, David J. 1988. "The Huang-di nei-ching: The Structure of the Compilation; The Significance of the Structure." Ph.D. Diss., University of California, Berkeley.

Klose, Petra. 1984. "Die Geschichte König Ts'o's von Chung-shan und die Rekonstruktion seines Grabes." M.A. Thesis. Ruprecht-Karls-Universität Heidelberg (Philosophisch-Historische Fakultät, Fachbereich Kustgeschichte Ostasiens).

Knoblock, John, and Jeffrey Riegel, trans. (comprises original text). 2000. *The Annals of Lü Buwei: a complete translation and study.* Stanford: Stanford University Press.

Kohn, Livia. 1992. "Philosophy as Scripture in the Taoist Canon." *Journal of Chinese Religions* 20 (Fall): 61–76.

Lackner, Michael. 1990. "Die Verplanung des Denkens am Beispiel der *tu*." In *Lebenswelt und Weltanschauung in frühneuzeitlichen China*, edited by Helwig Schmidt-Glintzer, 134–56. Stuttgart: Franz Steiner Verlag.

Lackner, Michael. 1992. "Argumentation par diagrammes: une architecture à base de mots. Le *Ximing* (l'Inscription Occidentale) depuis Zhang Zai jusqu'au *Yanjitu*." *Extrême-Orient—Extrême-Occident* 14: 131–168.

Lackner, Michael. 1996. "La position d'une expression dans un texte: explorations diagrammatiques de la signification." *Extrême-Orient—Extrême-Occident* 18: 35–49.

Lackner, Michael. 2000. "Was Millionen Wörter nicht sagen können: Diagramme zur Visualisierung klassischer Texte im China des 13. bis 14. Jahrhunderts." *Semiotik* 22.2: 209–37.

Lawton, Thomas, ed. 1991. *New Perspectives on Chu Culture During the Eastern Zhou Period.* Arthur M. Sackler Gallery (Smithsonian Institution, Washington, D.C.), distributed by Princeton University Press.

Legge, James, trans. (comprises original text). 1865. *The Chinese Classics, Vol. 3 (Shoo King).* Hongkong—London: London Missionary Society's Printing Office.

Legge, James, trans. 1885. *The Lî Kî.* In Müller, Max F. (ed.), *The Sacred Books of the East*, vol. 27–28. Oxford: Clarendon Press.

Le Blanc, Charles. 1994. "Le texte comme dispositif: démontage de deux unités littéraires du *Huainan zi*, VI," paper presented to the International Conference "Philosophie et pensée chinoise" (21–22.10.1994, Paris, Université Paris VII, France).

Le Blanc, Charles, and Rémi Mathieu, trans. 2003. *Philosophes taoïstes, II, Huainan zi*, Paris, Gallimard (collection de "la Pléiade").

Li Ling. 1991. "Chuboshu yu 'shitu' " (The Chu Silk Manuscript and Cosmograph Design), *Jiang Han kaogu* 1991.1: 59–62.

Li Ling. [1993] 2000. *Zhongguo fangshu kao* (A Study of Chinese Magical Techniques). Beijing: Dongfang chubanshe.

Li Ling and Constance A. Cook. 1999. "Translation of the Chu Silk Manuscript." In *Defining Chu: Image and Reality in Ancient China*, edited by Constance A. Cook and John S. Major, 171–76. Honolulu, University of Hawai'i Press.

Li Xueqin. 1994. *Jianbo yiji yu xueshu shi* (The History of Lost Texts on Bamboo and Silk and [Their] Scholarship). Taibei: Shibao Wenhua Chuban ltd.

Li Xueqin. 1995. "Basic Considerations on the *Commentaries* of the Silk Manuscript *Book of Changes*." *Early China* 20: 367–80.

Lianyungang shi bowuguan/Museum of Lianyungang City. 1996a. "Jiangsu Donghai xian Yinwan Han muqun fajue jianbao" (Short report on the excavation of Han tombs at Yinwan, Donghai, Jiangsu), *Wenwu/Cultural Relics* 1996.8: 4–25.

Lianyungang shi bowuguan/Museum of Lianyungang City. "Yinwan Hanmu jiandu shi wenxuan" (Commented selection of texts on bamboo slips and wooden boards from Han tombs at Yinwan), *Wenwu/Cultural Relics* 1996.8: 26–31.

Loewe, Michael and Edward L. Shaughnessy (eds.) 1999. *The Cambridge History of Ancient China: From the Origins of Civilization to 221 B.C.* Cambridge: Cambridge University Press.

Maeder, Eric W. 1992. "Some Observations on the Composition of the 'Core Chapters' of the *Mozi*." *Early China* 17: 27–82.

Major, John S., trans. 1993. *Heaven and Earth in Early Chinese Thought (Chapters Three, Four and Five of the Huai nan zi).* Albany: State University of New York Press.

Maspero, Henri. 1948–1951. "Le Ming-t'ang et la crise religieuse avant les Han." *Mélanges chinois et bouddhiques* 9: 1–71.

Mawangdui boshu/Silk manuscripts from Mawangdui, vol. 1. 1980. Beijing: Wenwu chubanshe.

Mayer, Alexander. 1996. "Some Observations of the Classificatory Schemes *(kepan)* Used in Chinese Buddhist Commentarial Literature," paper presented at the 11th European Association of Chinese Studies Conference (4–7.09.1996, Universitat Pompeu Fabra, Barcelona, Spain).

Needham, Joseph, ed. *Science and Civilisation in China*, vol. 3. Cambridge: Cambridge University Press, 1959.

Nylan, Michael. 1992. *The Shifting Center: The Original "Great Plan" and Later Readings.* Monumenta Serica Monograph Series 26, Netteral: Steyer Verlag.

Pan Jixing. 1997. "Yinshuamu de qiyuan di: Hanguo haishi Zhongguo?" (The Birthplace of Printing: Korea or China?). *Ziran kexueshi yanjiu/Studies in the History of Natural Sciences* 16.1: 50–68.

Plaks, Andrew H. 1977. "Conceptual Models in Chinese Narrative Theory."*Journal of Chinese Philosophy* 4.1: 25–47.

Qian Baocong, ed. 1963. *Suan jing shi shu (Ten Mathematical Classics)*, vol. 1. Beijing: Zhonghua Shuju.

Rickett, W. Allyn. 1960. "An Early Chinese Calendar Chart: *Kuan-tzu*, III, 8 (*Yu Kuan).*" *T'oung Pao* 48.1–3: 195–251.

Rickett, W. Allyn. 1965. *Kuan-tzu, a Repository of Early Chinese Thought.* Hong Kong: Hong Kong University Press.

Rickett, W. Allyn. 1985. *Guan zi. Political, Economic, and Philosophical Essays from Early China (a study and translation).—vol.* 1. *chapters* I, 1—XI, 34, *and* XX, 64—XXI, 65–66. Princeton: Princeton University Press.

Riegel, Jeffrey K. "Li chi." 1993. In *Early Chinese Texts: Bibliographical Guide*, edited by Michael Loewe. Early China Special Monograph Series 2, 293–97. Berkeley: The Society for the Study of Early China and The Institute of East Asian Studies, University of California.

Reiter, Florian C. 1990. "Some Remarks on the Chinese Word *T'u 'Chart, Plan, Design'*." *Oriens* 32: 308–27.

Roth, Harold D. 1992. *The Textual History of the Huai-nan Tzu.* Ann Arbor: The Association for Asian Studies.

Saso, Michael. 1978. "What is Ho-t'u." *History of Religions* 17.3-4: 399–416.

Shaughnessy, Edward L. 1986. "On the Authenticity of the Bamboo Annals." *Harvard Journal of Asiatic Studies* 46.1: 121–48.

Shaughnessy, Edward L. 1991. *Sources of Western Zhou History: Inscribed Bronze Vessels*. Berkeley and Los Angeles: University of California Press.

Schmitt, Jean-Claude. 1989. "Les images classificatrices."*Bibliothèque de l'Ecole des Chartes* 147: 311–41.

Seidel, Anna. 1983. "Imperial Treasures and Taoist Sacraments." In *Tantric and Taoist Studies in Honour of R. A. Stein*, vol. 2, edited by Michael Strickmann. *Mélanges Chinois et Bouddhiques* 21: 291–371.

Sementsov, Vladimir S. 1981. *Problemy interpretatzii brakhmanitcheskoi prozy (The Problems of Interpretation of the Brahmanic Prose).* Moscow: Nauka.

Sickman, Laurence, and Soper, Alexander. 1956. *The Art and Architecture of China.* Harmondsworth: Penguin.

Smith, Richard J. 1996. *Chinese Maps.* Oxford University Press: Hong Kong—Oxford—New York.

Soothill, William E. 1951. *The Hall of Light: A Study of Early Chinese Kingship.* London: Lutterworth Press.

Shatzman Steinhardt, Nancy. [1990] 1999. *Chinese Imperial City Planning*. Honolulu: University of Hawai'i Press.

*Shuihudi Qinmu zhujian/*Bamboo slips from the Qin tomb at Shuihudi 1990. Bejing: Wenwu chubanshe.

Teng Zhaozong. 1996. "Yinwan Hanmu jiandu gaishu" (General account of bamboo slips and wooden boards from Han tombs at Yinwan). *Wenwu/Cultural Relics* 1996.8: 32–36.

Tkachenko, Grigory A. 1976. "O kompozitsii '12 zamet' v 'Lyuy shi chun' tsyu' ('Vesny i oseni Lyuy Buveya')" (About the composition of the "12 Notes" of the *Lü shi chun qiu* (Springs and Autumns of Lü Buwei.) In *7-aya Nauchnaya konferentsiya "Obshchestvo i gosudarstvo v Kitae." Tesisy dokladov* (The 7th Conference "Society and State in China". Abstracts of Papers), vol. 1, 51–56. Moscow: Nauka, GRVL.

Tkachenko, Grigory A. 1986. "The *Lü shi chun qiu*," paper presented on the 6th of March, 1986, at the seminar "Structural Researches in Chinese Classics," Institute of Oriental Studies, Academy of Sciences of the USSR, Moscow.

Tkatchiénko (Tkachenko), Grigorî (Grigory) A. 1991. "Sur la composition du *Shi'er ji* dans le *Lü shi chunqiu* (Printemps et automnes de Lü shi)" *Extrême-Orient—Extrême Occident* 13 (*Modèles et structures des textes chinois anciens. Les formalistes soviétiques en sinologie*): 121–26.

Tkachenko, Grigory A., trans. 2001. *Lyshi chun'tsyu: Vesny i oseni gospodina Lyuya* (*Lü shi chunqiu*: Springs and autumns of Mister Lü). Filosofskoe nasledie (Philosophical heritage) series, vol. 132. Moscow: Mysl', 2001.

Toporov, Vladimir N. 1983. "Prostranstvo i tekst" ("Space and Text.") In *Tekst: semantika i struktura (Text: Semantics and Structure)*, edited by T. V. Ziv'yan. Moscow: Nauka.

Tsien Tsuen-hsuin. 1962. *Written on Bamboo and Silk. The Beginnings of Chinese Books and Inscriptions.* Chicago: University of Chicago Press.

Vandermeersch, Léon. 1994. *Etudes sinologiques* (Collection "Orientales"), Paris: PUF.

Volkov, Alexeï. "La structure des textes chinois anciens: quelques remarques." *Extrême-Orient—Extrême-Occident* 13 (*Modèles et structures des textes chinois anciens. Les formalistes soviétiques en sinologie*): 155–61.

Wagner, Rudolf G. 1986. "Wang Bi: 'The Structure of the Laozi's pointers' *(Laozi weizhi lilüe)*." *T'oung Pao* 62: 92–129.

Wilhelm, Richard, trans. 1928. *Früling und Herbst des Lü Bu We*. Jena: Eugen Diderichs.

Part II

THE CONSTITUTION OF SCIENTIFIC TEXTS: FROM DRAFT TO *OPERA OMNIA*

EBERHARD KNOBLOCH

LEIBNIZ AND THE USE OF MANUSCRIPTS: *TEXT AS PROCESS*

ABSTRACT

Text played a crucial role in Leibniz's scientific thinking. This article describes four aspects of this interrelation. First of all, text served the art of invention. Tables, illustrations, and figures enabled Leibniz to find rules, laws, and regularities. This will be shown by means of examples taken from additive number theory and combinatorics. Secondly, text served the purpose of visualization of thoughts, theorems, and proofs. The examples concern the theory of prime numbers and of infinite series. Thirdly, Leibniz used text to fix certain results and insights, to detect errors, to elaborate treatises, and to generalize theories. These practices are illustrated by his studies on symmetric functions, on life annuities, on elimination theory, on conic sections, and on financial mathematics. Finally, Leibniz's texts reflect his monologues or dialogues with fictitious interlocutors, in other words his argumentation.

INTRODUCTION

"Those who know me on the basis of my publications, don't know me" said Leibniz about his scientific production. Indeed, the number of his handwritten items is close to 50,000, whereas the number of his publications is very small. Leibniz's posthumous writings provide a unique insight into his intellectual workshop and verify the above remark in a two-fold way: they not only reveal a tremendous amount of hitherto unknown scientific results and achievements but also show how Leibniz obtained them by thinking in writing. For him thinking was thinking in writing. Text was his instrument of thinking. I would like to describe this intellectual practice by dealing with the following four of its aspects:

1. Text serving the art of invention: the dynamic inductive role of his tables, illustrations, and figures
2. Text serving the visualization of his thoughts, theorems, and proofs
3. Text used to fix insights and to elaborate treatises
4. Text as discussion and argumentation: thinking by writing.

K. Chemla (ed.), History of Science, History of Text, 51–79.

1. TEXT SERVING THE ART OF INVENTION: THE DYNAMIC ROLE OF TABLES, ILLUSTRATIONS, AND FIGURES

Tables, illustrations, and figures play a crucial role in Leibniz's mathematical thinking. They enable him to find rules, laws, and regularities; in other words, they serve the art of invention, sometimes successfully, and sometimes not. Let us first consider two examples concerning additive number theory.

1.1 Tables

Leibniz looks for the number of additive partitions of a natural number into 2, 3, 4 or more terms, whose sum is equal to that natural number.

First example[1]

He aims at a universal rule believing that he will find it by generalizing the case of partitions into three terms. These partitions can be produced by means of partitions into two terms which are ordered according to the magnitude of the first term.

He exemplifies his method by using the number n = 8. There are n − 1 = 7 classes or partitions into two terms: 7 + 1, 6 + 2, 5 + 3, 4 + 4, 3 + 5, 2 + 6, 1 + 7. This first step implies repetition.

The partitions of their first term into two terms lead to the partitions into three terms. From class to class there is an increasing number of repetitions. Only the m-th partition of the m-th class provides a new partition into three terms. The first up to the (m−1)th partition is contained in the first up to the (m−1)th class respectively. Hence if the m-th class has (m−1) partitions, then it contains only repetitions. All m-th classes, which do not contain more than (m−1) partitions must also be cancelled. If one class must be cancelled, then all the following classes must also be cancelled. This is so because the first term continuously becomes smaller and admits fewer partitions.

In our case the first class, 7 + 1, provides 6 + 1 + 1, 5 + 2 + 1, 4 + 3 + 1, the second class, 6 + 2, provides 5 + 1 + 2, 4 + 2 + 2, 3 + 3 + 2. The first partition 5 + 1 + 2 of the second class is a repetition of 5 + 2 + 1 of the first class. Only the second partition 4 + 2 + 2 of the second class provides a new partition into three terms.

The third class, 5 + 3, has two partitions. Hence it contains only repetitions. The same applies to all the following classes.

In this way Leibniz deduces inductively a rule which is at once formulated for an arbitrary number n and an arbitrary number k of terms: Look for all partitions p_m^{k-1}, $1 \leq m \leq n - 1$, into $(k - 1)$ terms of the numbers (n − 1) down to 1, but subtract from P_m^{k-1} at a time the number of the preceding numbers, that is, n − $(m + 1)$.

Let us look again for all partitions of n = 8 into k = 3 terms. We must look for all partitions of
m = 7 into k −1 = 2 terms: we get 3 partitions;
m = 6 into two terms: we get 3 partitions;
m = 5 into two terms: we get 2 partitions.
We need not go further because we must subtract $8 - (7 + 1) = 0$ from 3, $8 - (6 + 1) = 1$ from 3, and $8 - (5 + 1) = 2$ from 2.



Figure 1. LH XXXV 12,1 sheet 15r; Leibniz 1976a, no. 40.

The rule itself has been cut off from the sheet of paper, that is, Leibniz separated the result from its origin, so that it is no longer contained in illustration 1 but on another sheet of paper.[2] The rule is indeed valid for partitions into three terms, but not for partitions into arbitrarily many terms:

$$P_8^3 = P_7^2 + \left(P_6^2 - 1\right) + \left(P_5^2 - 2\right) = 3 + (3 - 1) + (2 - 2) = 5.$$

But $P_8^4 = 5$, while the false recursion formula leads to

$$P_8^4 = P_7^3 + \left(P_6^3 - 1\right) = 4 + (3 - 1) = 6$$

In other words there are five partitions of n = 8 into k = 4 terms, while the formula provides the false result of six.

His added remark is very instructive and revealing: “Quaerere summam discerptionum sine Tabula, id ego aliis relinquo,” “I leave it to others to look for partitions without table.”[3]

Second example[4]

This time his aim is a recursion formula. He writes down all partitions of the numbers 1–12 into one, two, three, and four terms, whereby the terms are ordered according to their magnitude. Partitions which differ only by the order of the terms are counted just once. His list is distorted by some errors, so that it is all the more difficult for him to find any formation rule or regularity. Hence his first reaction is: “Ex his apparet progressionem Numeri discerptionum esse valde perplexam.” “From this (that is, from this table of partition) it is clear that the progression of the numbers of partitions is very confused.”[5]

However, by analyzing the table, he realizes that every number of partitions of a given number v into e terms is composed of the numbers of partitions of numbers smaller than v into (e − 1) terms. From a certain number onward partitions must be excluded because of the principle of order.

For example, if we are looking for the number of partitions of v = 10 into four terms, then we must add to the first term 7, 6, 5, 4 all partitions into three terms of the numbers x = 3, 4, 5, 6. If x = 7, then the partitions into three terms $5 + 1 + 1, 4 + 2 + 1$ are useless, and that obviously because the sums of the last two terms $1 + 1$, $2 + 1$ (the partitions into two terms) behind the first terms 5 or 4 are too small: Leibniz tries to calculate the number of partitions into e terms by means of the number of partitions into (e − 1) and (e − 2) terms. In this way he deduces a recursion formula. But this formula is false.[6] He creates a new notation $\overline{\text{v}\,\boxed{\bar{\text{e}}\ \text{sect.}}}$ for the number of partitions of the number v into e terms and remarks: “Mirabilia calculandi specimina”, “Wonderful specimens of calculation”.[7]

In terms of this notation his false equation reads as follows:

$$\overline{\text{v}\,\boxed{\bar{\text{e}}\ \text{sect.}}} = \text{sum.}\ \overline{\text{v}-\text{x}\,\boxed{\overline{\text{e}-1}\ \text{sect.}}}$$
$$-\text{sum.}\ \overline{\text{v}-2\text{x}-\text{y}\,\boxed{\overline{\text{e}-1}\ \text{sect.}}}$$

Here “sum” means the sum of the numbers of all relevant partitions of v − x into e-1 terms or of v − 2x − y into e − 2 terms, respectively.

Further manipulations lead to an even more clumsy equation.

Leibniz ends by saying:

> “Sufficiet talia ad calculum revocasse ... Diligentius considerandum.” “It suffices to have reduced such things to calculation ... this must be considered more diligently”[8].

Perhaps he suspected that something was wrong with his results. In other words, tables are a tool which should be replaced by rules of calculation. The same applies to the title of

Figure 2. LH XXXV 12,1 sheet 232r; Leibniz 1976a, no. 46.

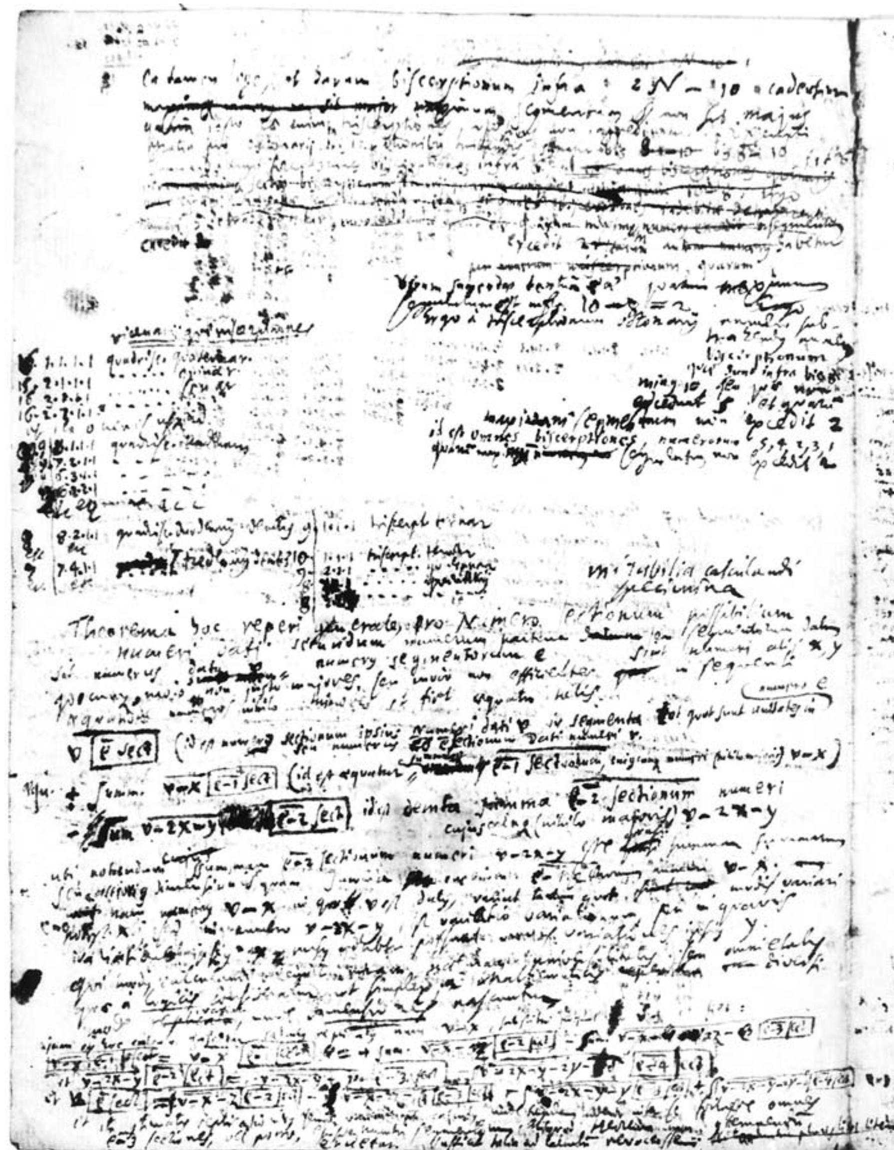


Figure 3. LH XXXV 12, 1 sheet 232v; Leibniz 1976a, no. 46.

his most important treatise on infinitesimal geometry written in 1675/76: “Arithmetical quadrature of the circle, the ellipse, and the hyperbola, which results in a trigonometry without tables.”[9] “Trigonometry without tables” means the use of certain infinite series. But while tables were necessary conditions for the text, the rule, or the recursion formula

to come into being with regard to the number theoretical manuscript, tables play another role in this treatise. We will comment on this role somewhat later.

1.2. The role of illustrations or figures

The role of illustrations or figures in Leibniz's mathematical thinking becomes evident when he calculates the powers of polynomials such as $(a + b + c)^4$.[10] The result consists of terms such as a^4, a^3b, a^2bc etc. multiplied by special coefficients. These coefficients are, as he knows, the numbers of permutations of such expressions. For example, there are 12 different transpositions of the term a^2bc.

Third example[11]

Leibniz tries to illustrate these 12 permutations by means of a scheme, a figure. At first he draws four unsuitable figures. They are crossed out, because they do not reveal the symmetries underlying the problem.

The most satisfying figure provides a completely symmetrical figure which, as he says, makes evident to the observer, that there are as many possible transpositions as there are numbered ways of producing the same term a^2bc by means of which we can come from the supreme points to the lowest, touching those in between.

In other words, the completeness and symmetry of the figure enables him to check and to illustrate the solution of the permutation problem.

2. TEXT SERVING THE VISUALIZATION OF THOUGHTS, THEOREMS, AND PROOFS

Thoughts and theorems cannot be heard or seen. But they can be made visible by certain representations. To this end, Leibniz uses special characters which lead to certain "apparitions".

2.1. Visualization as a tool for the art of invention

The following example is meant to demonstrate how such a visualization made possible certain mathematical insights in the true sense of the word. I would like to discuss Leibniz's inquiry into the law of distribution of prime numbers.[12] He aims at making visible the relations between prime and composite numbers by elaborating a figure. Thus, in this case, we might say, that visualization is a kind of geometrization of thoughts. He elaborates suitable figures in order to detect the law of distribution.

The first manuscript is entitled "Figura numerorum ordine dispositorum et punctatorum ut appareant qui multipli qui primitivi", "A figure of numbers arranged and punctuated in an order so that it becomes clear which are multiples and which are prime numbers." Leibniz constructs the figure as follows.

(a) He draws arbitrarily many dotted horizontal lines under the sequence of natural numbers. The first line begins below 2, the second below 3, the n-th below $(n + 1)$. In the first line every dot corresponds to a multiple of two, in the second to a multiple of three, and in the n-th line to a multiple of $(n + 1)$.

Figure 4. LH XXXV 4,5 sheet 18r; Leibniz 1976a, no. 56.

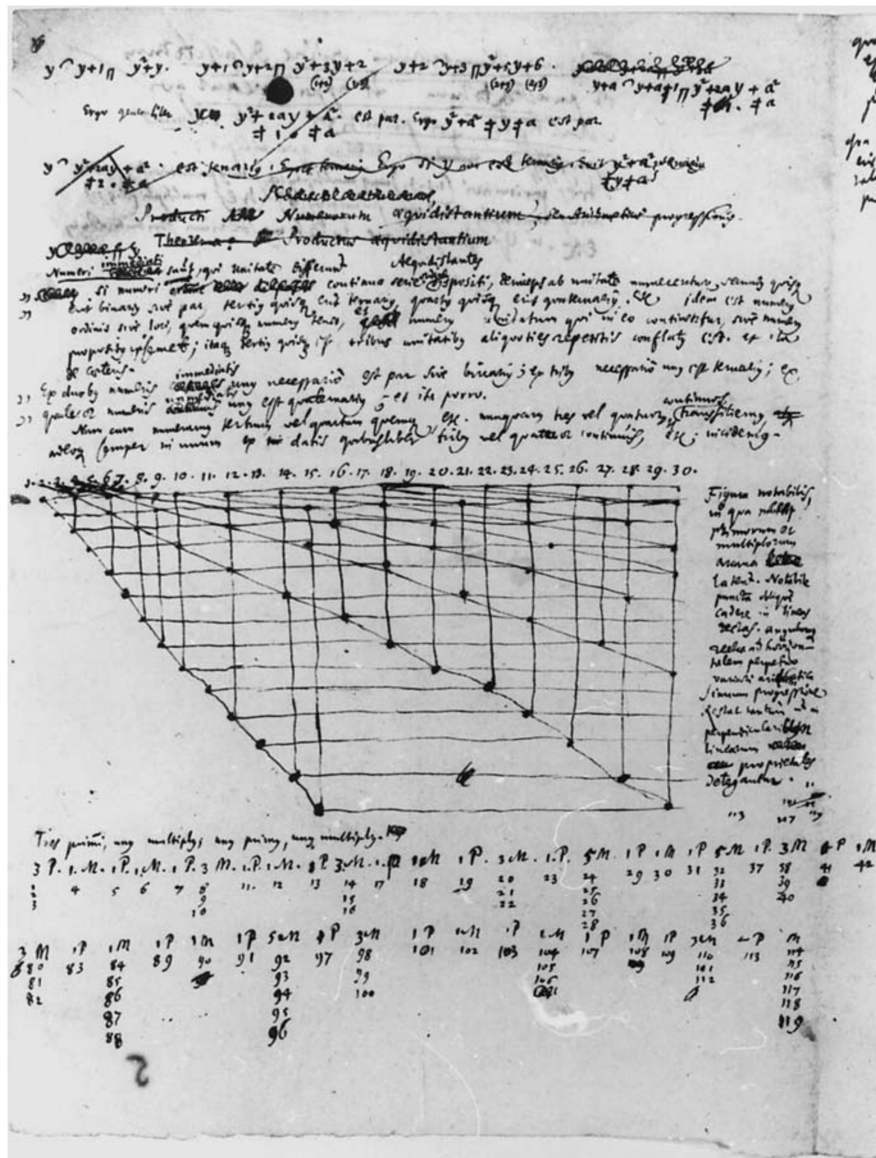


Figure 5a. LH XXXV 4,17 sheet 5v-6r; Leibniz 1990, no. 87.

(b) He draws inclined lines which connect the multiples of one, two, three, four, etc. belonging to different horizontal lines: Let us assume that a dot lies on the n-th inclined line and the k-th horizontal line. Then we have to take a step of n units in order to find the next dot on it which lies at the same time on the (k + 1)-th horizontal line.

Figure 5b. (*Continued*)

(c) He draws perpendicular lines which connect dots lying one beneath the other and he comments: “Figura notabilis, in qua primorum et multiplorum arcana latent. . . . Restat tantum ut in perpendicularibus linearum proprietates detegantur”, “A notable figure, in which the secrets of primes and multiples lie hidden. It only remains to detect the properties of the lines on the perpendiculars.”[13]

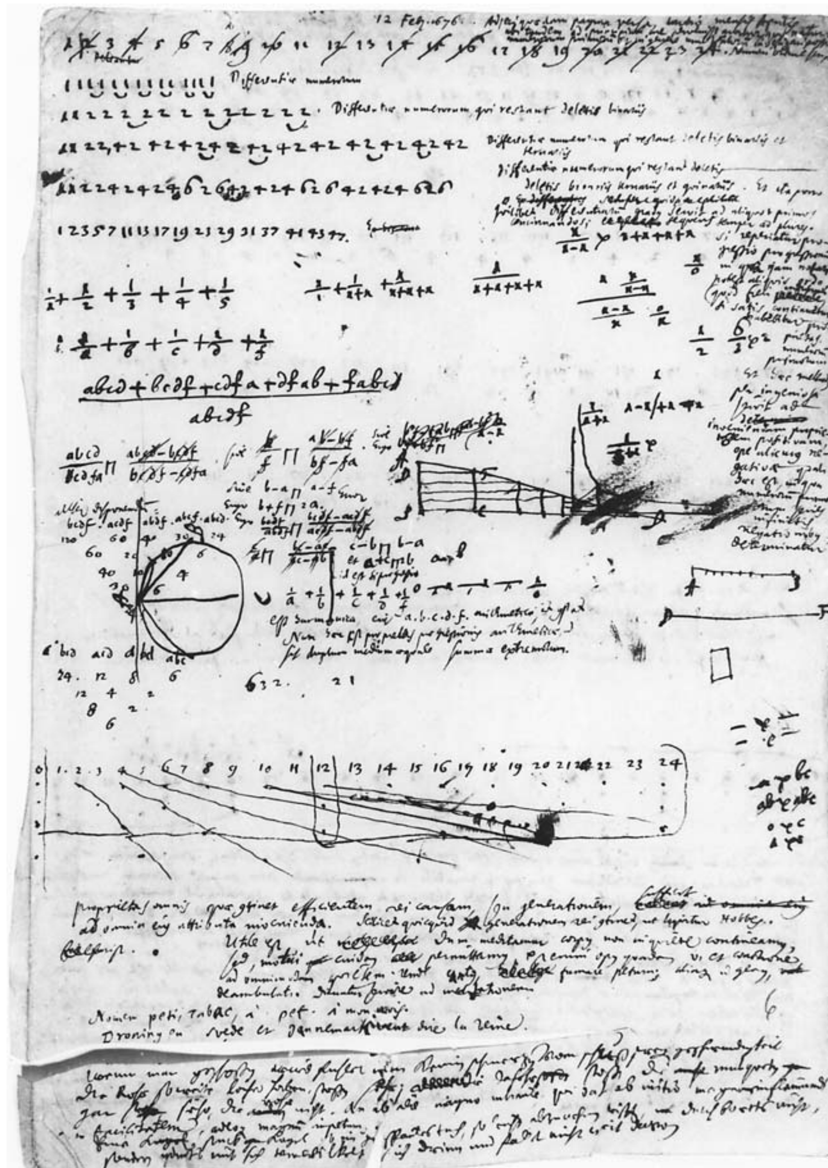


Figure 6. LH XXXV 3B, 15 sheet 6r; Leibniz 1990, no. 92.

Some weeks later he uses this approach once more.

This time only the inclined lines, together with their dots, are drawn. At the very top of this page he remarks that the addition of zero enabled him to see something miraculous, which he did not see before. By this only addition it shines forth at once, at first sight.[14]

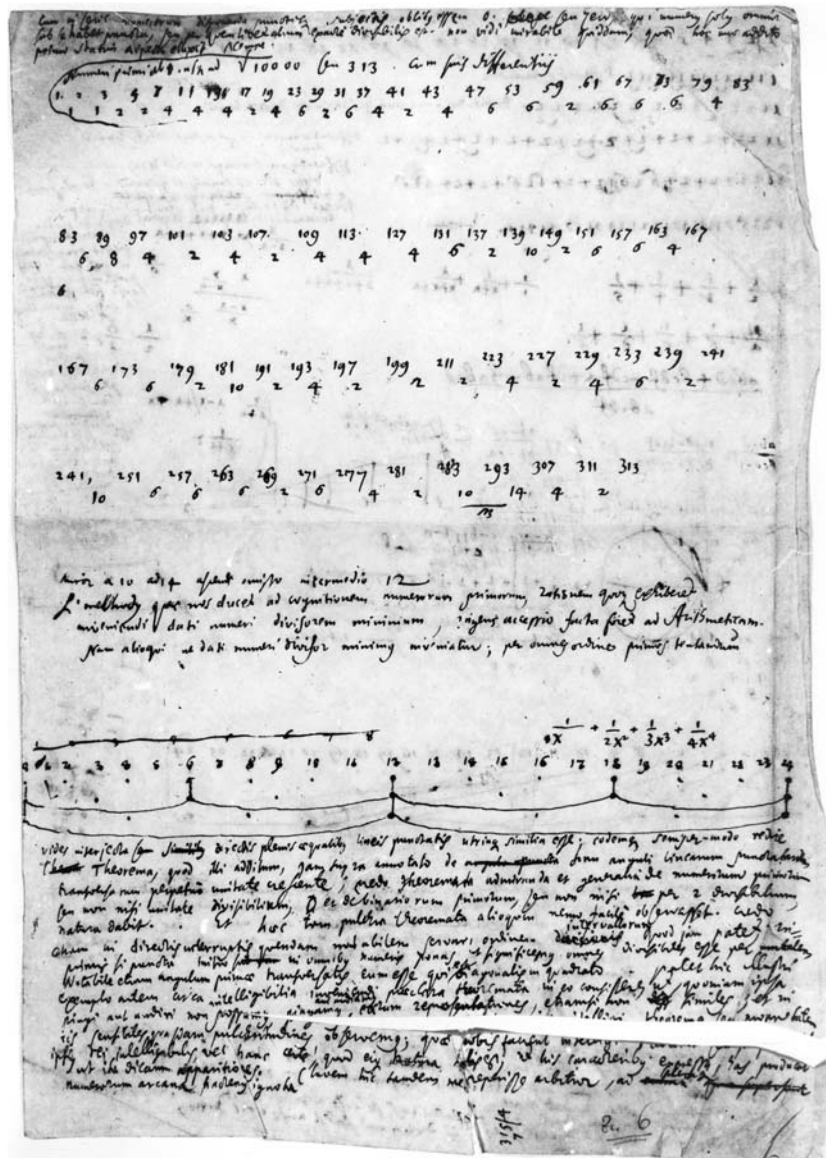


Figure 7. LH XXXV 3B, 15 sheet 6v, Leibniz 1990, no. 92.

He has in mind two observations (one true and one false): We see that the spaces between drawn perpendicular lines which are equal to the dotted perpendicular lines are similar on both sides. (This is true.) This series of the angles of inclinations of the inclined lines form an arithmetical progression. (This is false.) And he concludes: "Et haec tam

pulchra theoremata alioquin nemo facile observasset. . . . Patet hic illustri exemplo artem circa intelligibilia inveniendi praeclara theoremata in eo consistere, ut, quoniam ipsa pingi aut audiri non possunt, pingamus aut audiamus earum repraesentationes . . . et in iis sensibiles quasdam pulchritudines observemus; quae nobis facient intelligi theorema seu proprietatem ipsius rei intelligibilis, vel hanc certe, quod eius natura talis est, ut his caracteribus expressa, has producat ut itam dicam apparitiones. Clavem hic tandem me reperisse arbitror, ad pleraque numerorum arcana, hactenus ignota." "And otherwise, nobody would have easily observed such beautiful theorems. It is evident here by an illustrious example that the art of inventing famous theorems, regarding the intelligible, consists in painting or hearing their representations, because they themselves cannot be painted or heard. . . . And in observing some sensible beauties in them. They will enable us to understand a theorem or the property of the intelligible thing, or at least that which is of such a nature that it produces, so to say, these apparitions if it is expressed by these characters. I believe, that I finally found here the key to most of the hitherto unknown secrets of numbers."[15]

Leibniz says, that we observe theorems. It goes without saying that he overestimated the capacity, the efficiency, of such geometrical means, of such devices. But we note once more that visualization was a necessary condition for Leibniz's text to come into being. It is a kind of self-generation of the text.

2.2. Visualization as a tool for illustrating known intellectual relations

Let us consider proposition 26 of his treatise mentioned above on the arithmetical quadrature of conic sections, which was written in 1675/76.[16] The sum s of a geometrical series which decreases to infinity is to the first term a as the first term is to the difference between the first and the second term aq, q being the quotient between two terms following one another:

$$s : a = a : (a - a.q) \text{ or } s = a.1/1\text{-}q$$

Leibniz uses a figure in order to prove this proposition by means of two similar triangles:

He says explicitly that he chose that proof among the many available which puts the problem before the eyes in a certain way ("quae rem quodammodo oculis subicit"). In other words, illustrative proofs are easier to understand than other kinds of proofs.

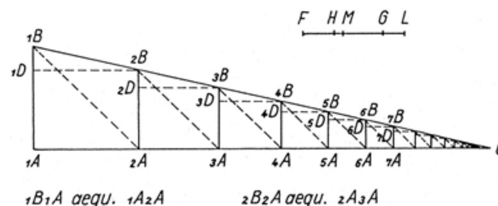


Figure 8. Leibniz 1993, 72.

3. TEXT AS FIXATION OF INSIGHTS AND ELABORATION OF TREATISES

I would like to distinguish six aspects:

3.1. The static, fixing role of tables

While the dynamic role of tables, discussed in section 1, consists in serving the art of invention, the static role of tables serves to avoid repeated calculations at future times. According to Leibniz, such a table is the best way of fixing the results which have been calculated once and for all. The tables can be used to find a needed value. While the purpose of the first use of tables, as we were told, is to reduce tables to calculations, the purpose of the second use of tables is to replace calculations by tables.

As a consequence, Leibniz elaborates, for example, multiplication tables for forms or symmetric functions[17].

Illustration 9 consists of three matrices (in modern terminology): the single terms a_i of the first column are multiplied by the single terms b_k of the first row. The element c_{ik} of the matrix is the product of a_i times b_k. Only some calculations of the products have been carried out.

Let us consider two examples:

First matrix:

$$a_1 = lmn,\ b_3 = l^3m^2.\ \text{Hence}\ a_1.b_3 = c_{13} = l^4m^3n$$

Third matrix:

$$a_9 = l^2m^2n,\ b_3 = l^4m^3n.\ \text{Hence}\ a_9.b_3 = c_{93} = l^6m^5n^2$$

If these terms are considered as terms which designate a symmetric function, then such multiplications become far more difficult.[18] In order to avoid unnecessary multiplications of terms we must know the number of terms which constitute a particular symmetric function. This number depends on the number of variables in the function. If, say, there are four variables l, m, n, o, then the function lm reads:
lm+ln+lo+mn+mn+no, giving six terms. Obviously, this is just a combinatorial question. Leibniz elaborated a table for the numbers of terms in an arbitrary form:

The different forms are enumerated in the first column. The numbers behind a form are the increasing numbers of terms of this form if there are 1, 2, 3 etc. variables.

Example:

Let there be three variables l, m, n and let l^2m be the form. We get:

$$l^2m = l^2m + lm^2 + l^2n + ln^2 + m^2n + mn^2,$$

that is, the form consists of six terms. We find this result in Leibniz's table (third row, third column).

These are the same kinds of tables about which Leibniz speaks in the title of his treatise on conic sections quoted earlier.

While this first aspect applied to the fixing of certain, reliable results, only such fixings of insights by means of schemes and figures enabled Leibniz to detect errors (aspect 3.2),

Figure 9. LH XXXV 14,1 sheet 293v; Leibniz 1976a, no. 24.

and to modify and improve chosen formulations. This implies a potentially infinite process (aspects 3.3 up to 3.6). I discuss the following examples in order to explain these five other aspects.

3.2. Clarifications or the detection of errors.[19]

Our example is related to Leibniz's inquiries into life annuities. He takes four living beings. They are supposed to belong to a species which dies after four years at the latest. He looks for the presumed life span of an arbitrary pair of such beings. It is obvious, he says, that they can be equally easily combined either when both live less than a year, or when one lives less than a year, the other 1, 2, or 3 years, or when one lives 1 year, the other one 2 or 3 years, as if we had two tetrahedra. He concludes: "Eruntque paria

Figure 10. LH XXXV 14,1 sheet 293r; Knobloch 1973, folding sheet between pp. 248 and 249.

possibilia numero 16, ut apparet in schemate adjecto", "There will be 16 possible pairs, as becomes evident from the adjoined scheme"[20]:

Scheme of pairs			
0.0	0.1	0.2	0.3
(0)	(1)	(2)	(3)
1.0	1.1	1.2	1.3
(1)	(1)	(2)	(3)
2.0	2.1	2.2	2.3
(2)	(2)	(2)	(3)
3.0	3.1	3.2	3.3
(3)	(3)	(3)	(3)

The life span of an association is the upper limit of the individual life spans of its members: an association survives until the death of its last member. In the foregoing example the associations consist of two persons (unarranged pairs) chosen at random in a group of four persons A, B, C, D. They are characterized by the individual life spans. According to the assumption, there are four such individual life spans, namely 0, 1, 2, or 3 years.

Leibniz considers arranged pairs: 2.1 means that the life span of the first person is two years, and the life span of the second person is one year. 1.2 means the opposite. In either case the life span of the pair is two years (2 is the upper limit of 1 and 2).

The numbers in brackets give the maximal life span of a pair. Leibniz's first explanation reads as follows: Unequal pairs (like 0.1, 1.2, etc.) occur always twice, because every number of years of life (0, 1, 2, 3) can be combined with every possible number of this kind.

His judgment is based on an analogy with a game with two unbiased six-faced dice where there are 36 possible combinations. If we look for the probability of throwing two numbers whose sum is two or three, then we must distinguish between (1, 2) and (2, 1). The probability of throwing the sum 3 is $2/36 = 1/18$, while the probability of throwing 2 is $1/36$, that is, it is twice as large. After finishing this argument Leibniz concludes that it cannot be transferred to the case of life spans, i.e., that he has detected an error. Hence he abandons the scheme of 16 pairs and replaces it by another triangular scheme adding another explanation:

Scheme of pairs			
0.0	0.1	0.2	0.3
(0)	(1)	(2)	(3)
	1.1	1.2	1.3
	(1)	(2)	(3)
		2.2	2.3
		(2)	(3)
			3.3
			(3)

"Sophisma hic notabile", "Here is a notable sophism", he says, and continues at first: "Et facilitas tantum", "only the facility", still adhering to the language of probability: facility is the degree of probability. Then he corrects to "Et possibilitas tantum hedrarum fuit consideranda aeque enim facile cadere potuere hae duae, quam aliae duae", "Only the possibility of the faces had to be considered, because these two faces could be cast as easily as two other faces".[21]

Indeed, insofar as the maximal life span of a pair is concerned, it does not matter whether we consider the possible pair (n, m) or (m, n)—m and n can be unequal—because the maximal life span is an upper limit. Obviously, fixing a thought enabled Leibniz to modify this solution. But it matters, of course, if equally possible outcomes of two cast tetrahedra are considered. His "improved" explanation leads astray.[22]

3.3. Condensation—augmentation

For Leibniz, editing a text meant canceling or adding passages. Some of the passages involved were long. We will call such a transition from one status of the text to another a condensation or an augmentation.

Leibniz's treatise on conic sections might serve as an example.[23] Leibniz explains the limited results and achievements of his predecessors Fermat and Wallis. He mentions Roberval, who told him of their work. After lengthy, but very interesting, historical explanations he turns to the quadrature of the circle. While he at first continues the text by formulating proposition 27, he later on prefers to insert two corollaries. After reading once more his manuscript, he crosses out the whole historical passage and the two inserted corollaries as well. Thus he conceals valuable historical information. This example, however, must not lead to the conclusion that Leibniz usually concealed from the reader historical information about contemporary mathematical studies. On the contrary, somewhat later, in the same manuscript, he adds a very long and interesting scholium in which he explains Nicolaus Mercator's method of division and then says: "Sed haec clarissimum virum Isaacum Neutonum ingeniose ac feliciter prosecutum, nuper accepi, a quo praeclara multa theoremata expectari possunt", "But I learned recently that the most famous man Isaac Newton ingeniously and successfully achieved that, [Newton], from whom many excellent theorems can be expected."[24]

3.4. Generalization or a new treatment from a higher standpoint

Leibniz studied the elimination problem, that is, he tried to eliminate the common variable x from two algebraic equations, for example, cubic equations:[25]

$$\text{equ. 1}\quad 10x^3 + 11x^2 + 12x + 13 = 0$$
$$\text{equ. 2}\quad 20x^3 + 21x^2 + 22x + 23 = 0$$

His fictive numerical coefficients denote not natural numbers but double-indexed indeterminate coefficients. The first figures, 1 or 2, refer to the equations. The second figures, 0, 1, 2, 3, when added to the exponent of x, form a constant sum. In the case of the cubic equation the sum in question is 3.

In order to eliminate step by step the common variable x, Leibniz multiplies equation 2 by the first coefficient of equation 1, that is, by 10, and equation 1 by the first coefficient of equation 2, that is, by 20. Then the multiplied equation 1 is subtracted from the multiplied equation 2. Thereby the highest power of x is eliminated. The result is

$$\begin{gathered} 10\cdot 21x^2 + 10\cdot 22x + 10\cdot 23 \\ -11\cdot 20X^2 - 12\cdot 20x - 13\cdot 20 = 0 \end{gathered}$$

or

$$(20)x^2 + (21)x + (22) = 0$$

The new coefficients (20), (21), (22) can be described by means of the old coefficients, the old coefficients are, as Leibniz said, "unfolded" (explicati):

$$(20) = 10\cdot 21 - 11\cdot 20,\ (21) = 10\cdot 22 - 12\cdot 20,\ (22) = 10\cdot 23 - 13\cdot 20$$

This "unfolding" can be repeated again and again. According to the rule

$$2n = 10\cdot 2(n+1) - 1(n+1)\cdot 20,\ 0 \le n \le 8$$

Leibniz does not use the letter n nor does he discuss the question of what will happen to the right numerals if the unfolding should be repeated more often than nine times, or if the degree of the original equation is higher than 9. The mechanical unfolding (of the old) original coefficient 20 leads to a dichotomic table:

$$20$$

$$10\cdot 21 \qquad\qquad -11\cdot 20$$

$$10\cdot 22 - 12\cdot 20 \qquad\qquad 10\cdot 21 - 11\cdot 20$$

$$10\cdot 23 - 13\cdot 20 \quad 10\cdot 21 - 11\cdot 20 \qquad\qquad 10\cdot 22 - 12\cdot 20 \quad 10\cdot 21 - 11\cdot 20$$

etc.

If these substitutions are really carried out, then the third line reads:

$$\begin{aligned} &10^3\cdot 23 - 10^2\cdot 13\cdot 20 - 10^2\cdot 12\cdot 21 + 10\cdot 11\cdot 12\cdot 20 - 10^2\cdot 11\cdot 22 + 10\cdot 11\cdot 12\cdot 20 \\ &\quad + 10\cdot 11^2\cdot 21 - 11^3\cdot 20 \end{aligned}$$

It is this form of the dichotomic table which is calculated in a manuscript dating from 1693/94.[26]

The factors of an expression are vertically written one below the other. There is an obvious similarity between this table and a genealogical tree. Therefore Leibniz calls the terms of the tree father, first-born, second-born, etc. He begins to describe the rule of unfolding the terms by looking at his table. Then he cancels most of his explanations saying: "Sed rem omnem altius repetendo exponere placet". "But it is good to explain the whole problem by going further back." That is, he decides to elaborate a more general, systematical unfolding theory, based on the use of fictive numerical coefficients and comprising 34 rules (canones).

Let us consider a numerical example. In the example given above 20 is the "father", 10.21 is his first-born, −11.20 his second born. The left number of the first-born is 10.

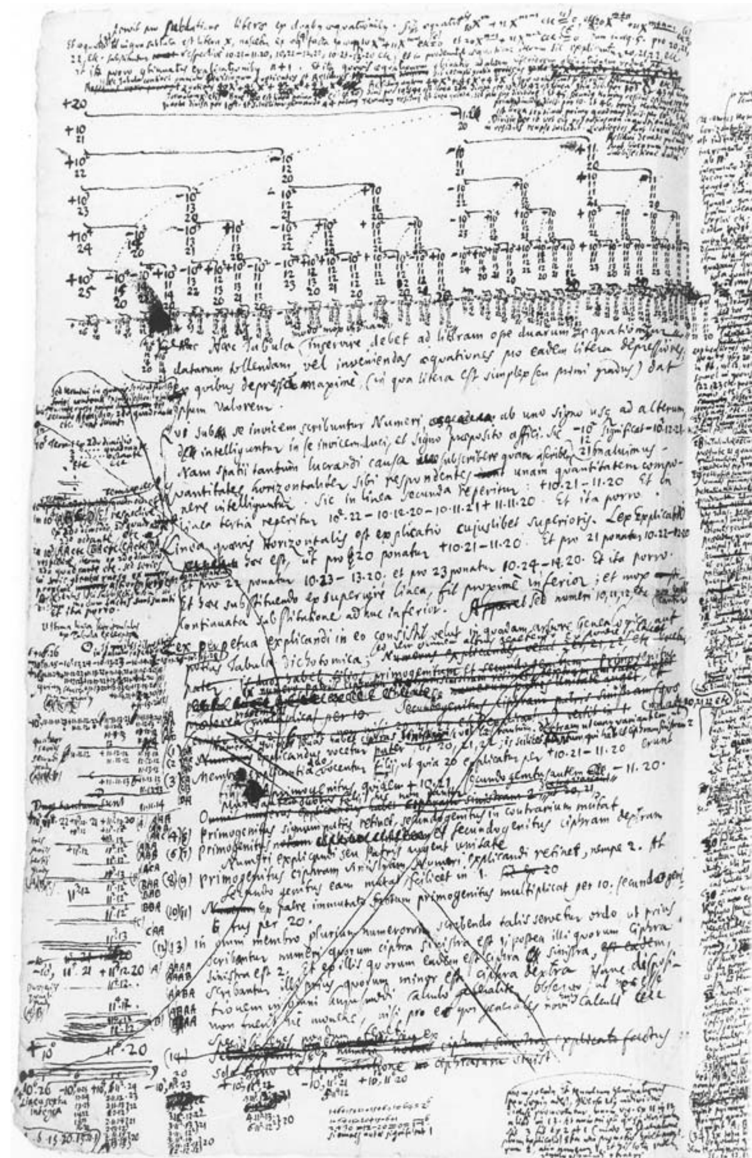


Figure 11. LH XXXV 14,1 sheet 167v; Knobloch 1974.

The right number 21 of the first-born is the number of the father enlarged by one, that is 21. The second born −11.20 permutes the right figures 0, 1 of the numbers of his brother.

The rules explain how the coefficients are formed, how to use the table in order to eliminate a common variable from two algebraic equations, the occurrence of the signs



Figure 12. LH XXXV 14,1 sheet 168r; Knobloch 1974.

of the terms, the relations between terms of the same and of different lines, etc. The many consequences make him continue the text in the left margin of the page. It becomes more and more difficult to read this text[27]. In a word, only after writing down his first ideas was Leibniz able to generalize and to systematize them.

3.5. Fixing of insights

The text reflects the process of recognition and of finding, and especially the elaboration of figures. In the first long version of his treatise on the arithmetical quadrature of the circle Leibniz outlines nine rough drafts of the figure relating to the parabola.[28] Again and again he is not content with his drawing. Sometimes it is the size of the drawing, and sometimes the curvatures, and sometimes something else fails to satisfy him.

He formulates the following problem: "Figuram analyticam simplicem quamlibet quadrare". "To square an arbitrary simple analytical figure." He calls a figure "simple"

Figure 13. LH XXXV 2,1 sheet 155v.

if the relation between the ordinates and the abscissas can be expressed by an equation consisting of only two terms.

After introducing some notations relating to a (necessarily specialized) geometrical figure, he interrupts his considerations and concludes: "Necesse est ut per singulos procedamus casus particulares, sed methodo quae possit esse universalis", "It is necessary that we proceed by particular cases, but by a method which could be universal." This is at the same time an example of the dialogical or monological. At any rate, it exemplifies the discursive character of his text. Now he distinguishes between figures for the parabolic, elliptic, and hyperbolic cases, but formulates the general theorem 13 which relates to a simple analytical curve in general.

What is important here is Leibniz's methodological insight and scientific intention. He must consider particular cases but he is interested in a universal method. The fruitfulness of his method of indivisibles, in his interpretation of infinitely small quantities, led him to elaborate a treatise on the arithmetical quadrature not only of the circle but of conic sections. Consequently, we must say a few words about his elaboration of treatises.

3.6. The elaboration of treatises: Text production as a process

Whenever Leibniz published an important article, we can be sure that he elaborated many, more or less different, drafts. Whether he actually published one of them is another question. I would like to mention two illustrative examples:

(1) "Treatise on the arithmetical quadrature of the circle, the ellipse and hyperbola which results in a trigonometry without tables." We touched on his treatise several times. It was first published in 1993. The introduction to its edition describes the history of its origins.[29] By "arithmetical quadrature" Leibniz meant convergent infinite series of rational numbers. His arithmetical quadrature of the circle was his alternating series $1 - \frac{1}{3} + \ldots$ for $\frac{\pi}{4}$, of which he informed Christiaan Huygens in the summer of 1674. In October 1674 he worked out for him a first version of his treatise.[30] By the end of 1675 he prepared two other versions for Jean Paul de La Roque[31] and Jean Gallois but did not send them. A further, hitherto unpublished, extensive version "Arithmetical quadrature of the circle" was elaborated by Leibniz in 1675. By and large, its content is identical with that of the first 25 theorems of the last version, published in 1993. Originally, Leibniz wanted to write only a comprehensive treatise about the quadrature of the circle. But the generality of his proofs and the fecundity of his principles induced him to extend the treatise to all conic sections and to the logarithmic curve.

(2) "Meditatio juridico-mathematica de interusurio simplice", "A legal-mathematical memoir on the simple rebate". This memoir appeared in 1683[32].

We know of five preliminary versions which differ considerably in style, content, length, and title. None of them has been published. Leibniz published a completely altered version. The manuscript used by the printer does not exist any longer. What exists is an untitled manuscript whose text is nearly identical with that of the printed article.[33] The titles of these five preliminary versions read as follows:[34]

(2.1) "Meditatio juridico-mathematica quanto plus petere intelligatur qui plus tempore petit seu de resegmento anticipationis, vulgo Rabat", "Legal-mathematical memoir:

Figure 14. LH II 5,1 sheet 9r; Leibniz 2000, no. II.8.

By how much is somebody understood to claim more who claims more by time or on the allowance of advanced payment, popularly called rebate."[35] "More by time" means "prematurely."

(2.2) "Meditatio juridico-mathematica quanto plus petere intelligatur qui plus tempore petit seu de Resegmento Anticipationis vulgo Rabat, sive interusurio", "Legal-mathematical memoir: By how much is somebody understood to claim more who claims

more by time or on the allowance of advanced payment, popularly called rebate, or on the interest of an intervening period."[36]

(2.3) "Meditatio juridico-mathematica de Interusurio sive Resegmento anticipationis vulgo Rabat", "Legal-mathematical memoir on the interest of an intervening period or on the allowance of advanced payment, popularly called rebate."[37]

(2.4) "Meditatio juridico-mathematica de interusurio seu de Resegmento Anticipationis vulgo Rabat, de aestimando jure percipiendi praestationes annuas certo annorum numero definitas, et Reditus ad vitam; ac de licitatione rei quae sub hasta distrahitur, oblata solutione particulari", "Legal-mathematical memoir on the interest of an intervening period or on the allowance of an advanced payment, popularly called rebate, on the estimation of the right to receive annual benefits assessed for a certain number of years and life annuities and on the bid for a thing which is sold by auction after presenting a particular solution."[38]

(2.5) "De interusurio seu Resegmento Anticipationis, vulgo Rabat, de aestimando jure percipiendi praestationes annuas certo Numero definitas, et Reditus ad vitam; ac de Licitatione rei quae sub hasta distrahitur, oblata solutione particulari", "On the interest of an intervening period or on reduction in the case of advanced payment popularly called rebate, on the estimation of the right to receive annual benefits, assessed for a certain number of years and life annuities, and on the bid for a thing which is sold by auction after presenting a particular solution."[39]

Already the titles allow us to form the following groups: versions (2.1) and (2.2); (2.3); (2.4); and (2.5). What is more, version (2.2) depends on version (2.1). Up to the word "differat" (line 31) it is a copy of version (2.1) (see above and below illustrations 14 and 15) for these reasons: Firstly, version (2.2) at once takes into account all additions and modifications of version (2.1) but leaves aside its cancelled passages. For example, version (2.1) adds "re aut" in line 3. These words are inserted into the text of version (2.2). Secondly, version (2.1) cancels some lines after "petere" which are left out in version (2.2) where the words "at de aestimanda" follow directly after "petere", etc. Thirdly, version (2.1) adds a paragraph "ab Jurisconsultis, quia ad eam inveniendam . . . " Version (2.2) inserts this paragraph into the text but leaves out the words "ab Jurisconsultis" and adds "accurate" between "eam" and "inveniendam". In the same way it can be proved that version 5 depends on version 4: sheets 19–20 are an improved, modified copy of the sheets $1 + 17 + 13 - 14$. It is very likely that version 3 precedes version 4, because it has a long paragraph at the very beginning which is not taken into account in version 4. But version 4 begins with a passage from version 3, which comes after this paragraph.

Leibniz himself says that the inquiry into the rebate problem led him to the question of life annuities. As a consequence, the titles of the later versions take this subject into account. There are four unpublished versions of an article on life annuities.[40]

4. TEXT AS A TRANSCRIBED DISCUSSION AND ARGUMENTATION

Leibniz's texts reflect his thinking as thinking while writing. Every idea, every question, every doubt, every access and optimism, every provisional result, every plan or intention is written down.

Figure 15. LH II 5,1 sheet 15r; Leibniz 2000, no. II.9.

The essential difference between his literary bequest and that of other mathematicians and scientists is based on this peculiarity. The text reflects his monologue or a dialogue with a fictitious interlocutor. At times, the interlocutor who utters doubts or objections is Leibniz himself, that is, Leibniz answers himself. Hence his style is very personal. He is accustomed to writing in the first person singular or plural:[41]

Investigavimus,	we shall investigate;
supponamus,	let us suppose;
investigemus,	let us investigate;
interea fateor,	in the meantime I confess;
puto idem sic solvi posse,	I think that the same can be solved in the following way;
possum ergo problema solvere hoc modo,	thus I can solve the problem in the following way;
ut inveniamus,	so that we find;
lucrati ergo sumus,	thus we have gained;
ut obiter dicam,	what I would like to say by the way
considerabimus,	we shall consider;
Ergo id dari posse non dubito,	hence I do not doubt that this can be given;
utemur,	we shall use;
videamus,	let us see
video,	I see;
quod ita ostendo,	what I demonstrate in the following way
poteram dividere,	I could have divided;
credo,	I believe.

Three examples illustrate his dialogical style.

1) He discusses a construction of curves and says that a special problem cannot be solved by the intersection of curves. The fictive interlocutor says: "At inquies hoc habet commodum constructio per intersectionem curvarum, ut per puncta describi possint curvae, saltem mechanice, cum hic opus sit motu. . . . Respondeo hic quoque designari posse quaesitum tentando." "But you will say that the construction by means of the intersection of curves has the advantage that the curves can be described by means of points, at least mechanically, because here a motion is needed. . . . I answer that here too what is looked for can be designated by trying."[42]

2) He is looking for a solution of a number-theoretical problem and says that most of the superfluous quantities must be set equal to one. "At inquies, cum superfluae sint, poterant ab initio omitti. Ita est; si divinare possemus; nunc quando homines sumus; satis nobis esse debet artem reperisse, quae eas in progressu ostendat." "But you will say: 'Because they are superfluous, they could have been omitted from the beginning.' 'That's the case, if we could prophesy. Because we are human beings, as things now stand, we must be content to have found an art which shows these quantities, if we get ahead.' "[43]

3) He recommends the test of nine in order to check calculations and refutes a whole series of potential objections:[44] "Dicet aliquis", "Somebody will say"; "Dicetur", "It will be said"; "At si inquiet . . . Respondeo." "But if somebody will say . . . I answer."

CONCLUSION

The examples discussed above bear overwhelming witness to the mathematical thinking of Leibniz. It is inseparably intertwined with the genesis of his mathematical texts. To a certain extent thinking and writing are for him two sides of the same coin, of his tremendously creative intellect.

T.U. Berlin

ACKNOWLEDGEMENT

I would like to thank Abe Shenitzer for polishing my English.

ABBREVIATION

LH Leibniz-Handschriften der Niedersächsischen Landesbibliothek Hannover

NOTES

[1] Leibniz 1976a, n. 40 (ca. August 1673).
[2] Leibniz 1976a, 261.
[3] Leibniz 1976a, 260.
[4] Leibniz 1976a, no. 46 (ca. 1678–1684).
[5] Knobloch 1973, 190; Leibniz 1976a, 275.
[6] Knobloch 1973, 190–94.
[7] Leibniz 1976a, 280.
[8] Leibniz 1976a, 281.
[9] Leibniz 1993.
[10] Knobloch 1973, 234.
[11] Leibniz 1976a, no. 56 (ca. 1676 or 1700).
[12] Leibniz 1990, no. 87 (ca. January 2, 1676), no. 92 (ca. February 12 and April 1976).
[13] Leibniz 1990, 580.
[14] Leibniz 1990, 597.
[15] Leibniz 1990, 598.
[16] Leibniz 1993, 71.
[17] Leibniz 1976, no. 24 (1677/78); Knobloch 1973, folding sheet between pp. 248 and 249.
[18] Knobloch 1973, 114–21.
[19] Leibniz 2000, no. III.11, part B (1680–1683).
[20] Leibniz 2000, 479.
[21] Leibniz 2000, 478.
[22] Leibniz 1995, 339–351.
[23] Leibniz 1993, 71 and 140f.
[24] Leibniz 1993, 77.
[25] Knobloch 1974.
[26] Knobloch 1974, 162f.
[27] Knobloch 1974.
[28] LH XXXV 2,1 sheet 155v.
[29] Leibniz 1993.
[30] Leibniz 1976b, no. 39.
[31] Leibniz 1976b, no. 72.
[32] Leibniz 1683; Leibniz 2000, no. II.22.

[33] LH II 5,1 sheets 33–34.
[34] Leibniz 2000, nos. II.8 – II.12.
[35] LH II 5,1 sheet 9.
[36] LH II 5,1 sheets 15–16.
[37] LH II 5,1 sheets $11 - 12 + 6 - 10$.
[38] LH II 5,1 sheets $1 + 17 + 13 - 14$.
[39] LH II 5,1 sheets $19 - 20$.
[40] Leibniz 2000, nos. III.3 – III.5, III.8.
[41] Leibniz 1990, 28; 38; 43, 67; 104; 285; 293; 294; 295; 297; 196; 177; 196; 196 etc. used constantly; 154, 193, 277, 280 etc.; 388; 387; 155, 277 etc.
[42] Leibniz 1990, 122.
[43] Leibniz 1990, 282.
[44] Leibniz 1990, 530.

REFERENCES

Knobloch, E. 1973. *Die mathematischen Studien von G.W. Leibniz zur Kombinatorik. Auf Grund fast ausschließlich handschriftlicher Aufzeichnungen dargelegt und kommentiert*. Wiesbaden: Steiner (Studia Leibnitiana Supplementa XI).

Knobloch, E. 1974. "Unbekannte Studien von Leibniz zur Eliminations—und Explikationstheorie." *Archive for History of Exact Sciences* 12: 142–73.

Leibniz, G.W. 2000. *Hauptschriften zur Versicherungs—und Finanzmathematik*. Ed. by E. Knobloch and J.-Matthias Graf von der Schulenburg. Berlin: Akademie Verlag.

Leibniz, G.W. 1683. "Meditatio juridico-mathematica de interusurio simplice." *Acta Eruditorum* October 1683: 425–32. Reprinted in: Leibniz, G.W. 1849–63. *Mathematische Schriften*, 7 vols. Ed. by C.I. Gerhardt. Berlin-Halle: Asher-Schmidt (Reprint Hildesheim 1962), vol. 7, 125–132.

Leibniz, G.W. 1976a. *Die mathematischen Studien von G.W. Leibniz zur Kombinatorik*, Textband. Ed. by E. Knobloch. Wiesbaden: Steiner. (Studia Leibnitiana Suppl. XVI).

Leibniz, G.W. 1976b. *Sämtliche Schriften und Briefe*, series III, vol. 1. Ed. by J.E. Hofmann. Berlin: Akademie Verlag.

Leibniz, G.W. 1990. *Sämtliche Schriften und Briefe*, series VII, vol. 1. Ed. by E. Knobloch and W.S. Contro. Berlin: Akademie Verlag.

Leibniz, G.W. 1993. *De quadratura arithmetica circuli ellipseos et hyperbolae cujus corollarium est trigonometria sine tabuli*s. Kritisch herausgegeben und kommentiert von E. Knobloch. Göttingen: Vandenhoeck & Ruprecht. (Abhandlungen der Akademie der Wissenschaften in Göttingen, Mathematisch-physikalische Klasse 3. Folge Nr. 43).

Leibniz, G.W. 1995. *L'estime des apparences, 21 manuscrits de Leibniz sur les probabilités, la théorie des jeux, l'espérance de vie*. Texte établi, traduit, introduit et annoté par M. Parmentier. Paris: Vrin.

Leibniz, G.W. 2004. *Quadrature arithmétique du cercle, de l'ellipse et de l'hyperbole et la trigonométrie sans tables trigonométriques qui en est le corollaire.* Introduction, traduction et notes de M. Parmentier. Texte latin édité par E. Knobloch. Paris: Vrin.

MICHAEL CAHN

OPERA OMNIA: THE PRODUCTION OF CULTURAL AUTHORITY

ABSTRACT

This essay argues for a more reflexive understanding of collected works in the history of science, and the history of the book more broadly. It touches upon a large number of cases, which show that the significance of collected editions is not understood when they are considered purely as purveyors of editorially purified texts. They can be monuments of national pride, an attempt by a publisher or editor to increase his status, or typographical reference objects. By introducing the juxtaposition of opera and opuscula the paper also argues that the special status of collected works is best understood as a phenomenon of post-Gutenberg print culture.

1. INTRODUCTION

We know much more than we know. We understand much more about books than we can explain. Years, generations among books have taught us many things about their significance, about the implications of their exterior appearance. Yes, you need to know the letters, but beyond the letters, there lies a different alphabet of significance. Those who plan to replace printed publications with electronic documents severely underestimate the extent to which, when it comes to books, we all judge by appearances. Nor are these judgments necessarily misleading. Before we read a book, we look at it. And while we read it, we see more than only letters and words. Don MacKenzie has recently reported on an almost Platonic experiment he undertook with his students.[1] He handed them a book without a word printed in it, and it took them only a very short time to identify the decade when it would have been printed, its potential subject matter, and perhaps even the size of the edition depending on the kind of paper being used. Format, the varieties of binding, the size, the quality and color of the paper, all this allows us to read a book before we have read the first word.

Much of this knowledge about books is anecdotal or intuitive. It helps us find our way in a book-shop or in a library, but there is little conscious awareness of it. An antiquarian bookseller will be much better equipped to judge a book from its covers than a literary scholar who often is interested in the text only. In what follows I would like to advance the understanding of what it means to know a book before opening it.

Collected works represent one important area in which such a raw and spontaneous bibliographical knowledge occurs. *Opera Omnia*, the number of volumes, their all-important size, the “festive” nature of presentation and print, all this speaks to us

K. Chemla (ed.), History of Science, History of Text, 81–94.

with an alluringly simple immediacy and tells us something about the author and his supposed standing, his authority in short. But when we try to explain these meaningful perceptions, we quickly run into problems. While we all have an intuitive understanding of the cultural (and political) aspirations associated with the creation of collected works, we encounter an almost total silence when we try to lay bare the cultural semantics of this genre of print. Go through the standard bibliographies or reference works in order to locate the scholarly work which has been done in this area and you will invariably draw a blank. Of the few titles a search will bring up, most refer to the collected papers of bibliographers themselves. Indeed, if my own bibliographical attempts are anything to go by, very little has been published.[2] This is indeed strange. Even on a very superficial scanning of the typographical universe, be it with regard to literary or to scientific *oeuvres*, the phenomenon of Collected Works appears to be very prominent indeed. While they stand out among the multitude of books, they seem to be virtually invisible to the scholars of print: The knowledge of the grammar that governs collected works is never made explicit. All great authors speak to us from the elevated pedestal of their collected works. Students are taught to quote from these editions, libraries feel obliged to buy them, authors are eager to have their own writings collected in one, and publishers apply for support from third parties when they undertake them. Of all the books that have been printed, those collected editions enjoy an additional and special privilege of permanence and of importance. That is why reprint-publishers find it so profitable to reprint them. Librarians reflect this status in their cataloging rules which make special provisions for filing them: Collected Works come first, and this position is indeed a fit expression of their prominence. But in bibliographical research, they come last, if they come at all. Why should Collected Works receive such a careless handling by the bibliographic community? Children's books, certainly less important one may think, have been the object of many studies, ephemera and miniature books are all very well catered for by book historians, even unfinished books have been thoroughly researched. One reason why collected works have remained in the shadow for so long could be the general orientation of modern historical bibliography, which tends to focus on the single book, the original event of a first edition, which can be assigned a precise date.[3] Collected works do not fall into this class; for the bibliographer they are secondary, they are reprint-phenomena, removed from the scene of original publication. In what follows, I shall try to unravel the alphabet of print by making this genre of re-print my starting-point.

2. THE BEST TEXT

The first collected edition of Shakespeare's plays "Comedies, Histories and Tragedies," posthumously edited by Heminge and Condell in 1623, offers a good example for the way a collected edition is approached today. This interest is directed towards a specific editorial goal: to contribute to the establishment of a better text of Shakespeare's plays. If I can identify patterns in the work of a particular composer who set Shakespeare's text for this edition, then I can arrive at a better estimation of what the authorial MS might have looked like. More precisely, such a study of Shakespeare's collected plays is interested not in the folio of his plays as an event in the world of books, but as a stepping-stone

towards a new edition, presumably a new collected edition. Bibliographers study collected editions with the aim of producing better collected editions. If anything, this seems to be a somewhat limited perspective. Perhaps we should pause and think about what we are doing. Or rather, we should pause and think about what the editions themselves are doing.

This interest in the text as opposed to the event of the text is most evident in a very valuable publication which must be mentioned in this context: Waltraut Hagen's *Handbuch der Editionen*.[4] Frau Hagen is the editor of the supplementary volumes to Goethe's Collected Works which were published in the former GDR. In these volumes she has presented splendid source material regarding the publishing history of Goethe's writings during his lifetime. Working on Goethe, she is very much aware of the significance of the extended romance of publishing in which Goethe was involved during most of his career as a writer. She offers fascinating material on the complex negotiations which precede the creation of a number of his collected editions. But if we open the *Handbuch der Editionen*, which covers about 500 German language authors, most of this historical background suddenly disappears. The *Handbuch* presents a descriptive bibliography of the editions of the major authors in the German language. Rather than looking at the *Gesamtausgaben* as a cultural event, she considers them under seven categories (Text, Erläuterungen, Entstehungsgeschichte, Textgeschichte, Wirkungsgeschichte, Literaturhistorische Einschätzung, Register) and judges them according to their relative usefulness. This is a grand project of evaluation, often concluding with the formula: "A collected edition which could satisfy the demands of a modern student is not available." Indeed, for many authors this is a sorry state, and the *Handbuch* impresses its reader with the necessity to put more money and more editorial manpower into the production of collected editions. The reiterated demand for more and better collected editions projects the vision of a literary heritage in which all collected editions have been completed and a national literature has been constituted once and for all. Overestimating the importance of the literary heritage, and underestimating the historicity of any collected edition, this is a truly absurd vision. Chasing after editions which would deliver the best text, Hagen's work exhibits no sense of the intrinsic interest that is connected with collected works, even if they do no longer satisfy the refined philological requirements of the present. It is symptomatic that Hagen thinks of collected editions only in terms of their philological reliability, as best texts, and that she remains reluctant to appreciate them as historically situated cultural artifacts in themselves, as reprint events which document the reception of an author, the constitution of his *oeuvre*, and the production of authority. In her perspective, the *Gesamtausgabe* is the textual ideal which is exempt from history. For us, it is the typographical device which effects the conversion of an author into an authority. She wants to guide the reader to the edition with the best text, and she forgets all that in which a collected edition is more than the source of the best text.

The demand for collected editions has often been justified with reference to one reliable, authoritative text, to which the scholarly community could make reference, a firm basis without which scholarship seems unable to flourish. No doubt, many words have been set right by critical editors who found meaning where beforehand no one saw any, and improved it where it needed improvement. But any short review of the history of collected editions will reveal that this work has as much to do with scholarly ingenuity

and achievement, as with literary politics, with status, with power within academic disciplines. Editors play a central role in the history in which texts change their meanings in the hands and minds of their readers. While they like to think of themselves as the policemen of textual correctness, they are indeed the tyrants of textual manipulation, creating and re-creating not only texts, but authors too. The cultural advancement effected whenever a book is reprinted as part of a collected edition is the work of the editor and his collaborators. Every publication presents its text in a certain way, and each way of publication throws a certain light on the text. Publication is always "publication as": As a first edition, as paperback, as journal essay, published at the expense of the author, on laid paper, as the production of a certain publisher, as employing a certain typeface, as an anonymous publication, as carrying a certain dedication, as unauthorized publication, or as part of a collected edition, associated with the name of a certain editor, etc.

Competing collected editions erected upon the same corpus offer some indication of the influence of the editor, which goes far beyond creating the best text. At the same time it should be kept in mind that "the best text" is not an ideal which presents itself spontaneously, but that this notion of textual perfection itself depends upon the history of collected editions in which it has been cultivated as a practical goal. Is it not precisely the genre of collected editions that has planted *the idea of the best text* into our philological culture?

If we start to reflect upon the uses to which collected works were put in the past, then we might recognize that the scholarly editions of our own time are possibly much less of an objective philological enterprise than we like to believe. Historicizing collected editions, focusing on the vanity and the pride which they can articulate so powerfully, reading the heterogeneous cultural messages they can take on, that could force on us the insight that more is here at stake than a timeless, a historical act of philological expertise. For instance, it would allow us to read collected editions as political, in particular as national events.

My first example is the French philosopher Malebranche. What happened with his works seems a freak case, but represents rather well the diverse cultural values which over determine the business of Collected Works. At the meeting of the *Académie de Sciences* in Paris on March 3, 1917, the proposal by Monsieur Boutroux was accepted to publish a collected edition of the writings of Malebranche. The reason for this proposal: Such an edition, says Mr Boutroux, would be "la meilleure réponse à la critique que Wundt a faite aux Français de n'avoir pas la tête métaphysique."[5] Here the collection of the writings of an author is nothing less than the opening of an editorial theater of war. With this edition, a certain group of French academics wanted to counter the humiliating allegation of a German philosopher who denied that they had "a metaphysical mind".

Philology, hunting after the "best text", seemed far removed from politics and war. The case of Malebranche is an indication that even editorial work on a philosophical *oeuvre* can become part of a dispute which mirrors the political and military conflict between Germany and France. A historicizing reflection of collected works, which focuses on their import as cultural events, will help us to question the notion of neutral philological work.

Seemingly innocent gestures such as a teacher's advice to the student to always quote from collected editions offer a glimpse of their academic importance. But such advice,

if looked at more closely, is far from innocent. It is based on a set of ideas about what the literary text is, where to go for the best version, and finally on the idea that such a best version can be created, a text purified from the accidents of its own time. One way to contextualize the notion of the pure text is offered by a legal perspective. The law of most countries affords special treatment for collected editions. The law focuses on collected works as being made up of independently published texts, and raises the question how a collected edition differs from a sum of single editions. This locates the problem of a collected edition in the relationship between author and publisher and their agreements. If a publisher holds the rights over every single title the author has written, he still lacks the additional right to undertake a collected edition.[6] He must seek this additional agreement from the author before he can bring out a collected edition. This possibly is why a German edition of the plays of Bernard Shaw in fourteen volumes is being advertised by Suhrkamp with the odd condition: "Die Bände sind nur einzeln lieferbar." The law insists that even if the publisher only inserts a half title or adds a numbering to the spine, he must obtain the explicit authorization to do so. The legal system recognizes a difference here for which it is rather difficult to find a justification within the traditional universe of philology. If the text is the same, if the text is precisely the same, word for word, letter for letter, why should it be a different publication? Why should it be a different event? Evidently, in the realm of print something can happen to texts which cannot be explained if we look at the text only, if we only look at its words and letters. Collected works are an interesting case where the conditions of publication can affect a text without affecting its words.

3. "*. . . Everyman, that prints, adventures*"[7]

To better understand this non-textual level of publication, we will have to give a passing glance at the system of typographical reproduction of texts. Printing is more than communicating a text to readers. Printing is an adventure of communication with definite social implications. In Britain in the Sixteenth and Seventeenth Century, an author of social standing would always feel the need to justify his (ad-)venturing into print. To have one's works printed was thought inappropriate by some, because the world of print was perceived to annihilate social distinction. Print means the reign of hacks and reading maids, as Swift often complained. On the cart of the book merchant, the hack writer and the Lord were suddenly, and very inappropriately, equals. In some circles, this might have lead to what has been called "the stigma of print".[8] In Seventeenth Century Britain, for example, we have many cases of authors protesting that they did not really want to have their book published. They affirm to have been forced into publication by a unscrupulous publisher who had laid his hands on a manuscript which was meant to circulate privately only among friends. Book production, even though it is always also the production of distinction and of differences, does contain egalitarian elements which become more pronounced when the mass of books increases, when authorship loses its distinction, and books penetrate all levels of the social fabric. Collected Works are situated precisely in this context. They offer a mode of publication which counteracts the egalitarian implications of print. Collected Works allow to draw a sharp distinction in the realm of books, they represent a bibliographical distinction which reliably reflects social

distinction. They divide the anonymous literary foot soldier, the hack, from the heroes of writing, those whom we call the classical authors. *Opera Omnia* create a divide among all books in general: A few and important *Opera Omnia* on the one hand, and the multitude of mere *Opuscula* on the other hand.

I would argue that collected works, as they effect such a hierarchical division, are best understood as a typographical genre. Before print, e.g. for the classical authors of Greece and Rome, we possess many tantalizing catalogues which list the works of one author that were once known or ascribed to him, and also some instances of actual editorial collecting. Any such collections confers distinction, but the full rhetorical impact of *Opera Omnia* will only be found after Gutenberg, in the domain of print. After Gutenberg it is no longer the shortage of texts and their doubtful survival, but the excess of publications, which governs the significance of collected editions. They now address the delicate danger of a symbolic loss of status in the sheer mass of texts considered insignificant and a-canonical. The distinction between *Opera Omnia* and *Opuscula* has its place in the business of print, or, to be more precise, collecting works is an activity which takes place in the world of re-printing.

Re-printing is indeed a fundamental structure of the typographical universe, and it is much more characteristic for the medium than the first printing which has such a wide following among the collectors of first editions. Re-printing a book offers many clear advantages for a publisher, and the law of copyright has sought to regulate these. The re-print publisher caters to a well-tested market, he can copy the editorial labor of his predecessor without having to pay for it, and he might even, with little a effort, improve on the work he is copying. Collected Works are a special case of reprinting, in which the author is the dominant principle of selection.

While collected works fit tightly into the economical structures of printing, they are of course much more than only a successful marketing strategy in the business of publishing. They are the books with the greatest status, not least because of the precedent of the classical authors, for which considerable effort has been spent on constituting such texts. Collected works are the mode of publishing the classics, and whenever a modern author is thought to achieve a comparable status in his own time, a complete edition would become due. *Opera Omnia* therefore mark the highest state of any writerly existence. Who would want to bestow *Opuscula* to posterity when he or she could possibly leave the world a veritable *Opus*? Sadly, there has not yet been invented a way to write a collected edition. The production of such an edition takes more than the sharp pen and the quick mind of an author. Books can be written, but collected works must be edited. Collected works, just as social status and rank, are not a purely authorial or subjective production.[9] They involve literary status, which, like social status, is conferred in a network of acknowledgement and collaboration as the result of complex negotiations. Status cannot be produced by the mere work of pen and eraser, even if this is a necessary precondition. The additional role of the editor is crucial for such collections, and it is just as important that most of the *oeuvre* has been published beforehand in different form. The function of the editor can be assumed by a student of the master, it can be the son or the mother, or it can even be the personal doctor, as in the case of Grillparzer. It can be a philologist, a publisher, "ein Verein von Freunden des Verewigten" in the case of Hegel, members of the family for Lichtenberg and finally, since the Nineteenth Century, national institutions such as

scientific academies. And as national institutions tend to publish national authors, the whole business of collected editions becomes deeply entangled in nationalistic issues, contributing significantly to the production of national consciousness.[10]

The context of Collected Works is the expansive and ever growing sea of printed books. If Gutenberg's machine has, in the long run, led to a devaluation of the book by sheer numbers, then the multi-volume collected editions of our (would-be) best authors attempt to counter this loss of authority. In this context of plenty, where every single book is quickly lost in a large ocean of books or remainders, they fight for cultural survival. And how do they do it? They use the magic word on the title, and they are produced in a consistent manner. Works are not printed in bigger type, not printed on better paper, not bound in a special way, but they are standardized. Standardization is the magic wand which can turn books into works. It belongs to the exterior of the book, to the book-physics. It covers the identical design for the spine, in the more desperate cases it might include a design of the spines which unites the composite volumes into a greater unit. Standardization also implies the same height of the spine, and possibly even the same thickness of the single volumes. Well, you might think, who but a mad book-designer would insist on the identical thickness of the single volumes in a collected edition? Among others, Goethe insisted on it, and Goethe certainly was not a mad book-designer. 1786, in a letter to Goeschen with whom he negotiated a new edition of his works to date, Goethe writes:

> "Es wäre sehr zu wünschen, daß alle Bände einerley Bogenzahl hätten. Ich glaube daß jeder Band bequem ein Alphabet füllen wird, beym Werther kommts auf die erste Anlage an, was für Lettern man nehme, und wie man das Ganze eintheilen will. Die übrigen dramatischen Schriften kann man ohne dies mehr oder weniger ausdehnen oder zusammenrücken."[11]

On Goethe's authority, collected works are not only about the text, they are also about the thickness of the single volumes. Collected works are a mode of re-presentation, they present the author and his status, and volumes of the same size are apparently better adapted for this task of representation.

The external identity of the single volumes in a multi-volume edition is important. This can be gathered from an interesting case of abuse to which this criteria has given rise. In 1709, Nicholas Rowe published a edition of Shakespeare's plays in octavo, in 6 vols. He only published the plays, leaving out, for his own good reasons, presumably, Shakespeare's occasionally more frivolous and doubtful poetry, in particular the Sonnets. It took less than a year, and some literary operator, whose name has not yet been established with certainty, changed that. Not only did he publish an edition of the poetry Rowe left aside, he also published it in precisely the same format and design as Rowe's volumes, in effect adding an illegal, subversive seventh volume to Rowe's edition. This case of abuse shows very well, I believe, the kind of cultural conflicts connected with the genre of collected works. The anonymous editor of the seventh volume evidently has a precise agenda: He wanted to "create" a different Shakespeare than Rowe had in mind. At the same time he reminds us with appropriate force that a complete edition is often contested: not least because it is never really complete, and the decision what to leave out, the exclusion every collection effects, represents a significant cultural intention. Rowe's

partiality as editor is not a thing of past. Still today, the most revealing way to describe a collected edition is a list of what is not included in it. At the end of the Nineteenth Century, collected works of scientific papers adhere to the convention to leave out all polemical pieces. They were thought to be irrelevant and transient. But today's historian of science will be very interested in precisely these polemical exchanges, and will draw the lines of exclusion differently.

The production of Rowe's irregular seventh volume involved not much more than some moderate typographical expertise. As both the legal and the "illicit" part was sold unbound, it was not necessary to feign a similarity in the binding. This would make such a project slightly more difficult today. Only in the Nineteenth Century does the publisher take full possession of the spine and uses it to design the book and hence to design the association of a series of books. At that moment the uniformity of collected works reaches a new stage. Given the fact that today the production of such an edition often takes many years, this demand for uniformity can indeed pose serious problems. Efforts are made to secure an identical appearance for volumes that are published as far as 30 or 40 years apart: history arrested. If the book historian looks at the *Opera* of Christiaan Huygens, published by the *Société Hollandaise des Sciences* between 1882 and 1950, he is struck by the effort that must have gone into creating the appearance of timelessness during the extended period of publication. And times of great difficulties these were: All the volumes have the same, exquisite paper, even those published in the years 1937, 1940 and in 1944. The high quality of this paper gives us a revealing indication of the effort which must have gone into the deliberate *construction* of the timelessness of these volumes. Examine the volume published 1944 for traces of its time, and you will find none. A perfectly timeless book has been created. By the testimony of its uniformity, this edition has been produced as valid for all times, a classic, untouched by such minor disturbances as two world wars. The work which has gone into the creation of an edition which remains uniform between 1882 and 1950 throws new light on the philological dream according to which collected works are devoid of history. (Hagen's Handbuch) The case of Huygens helps us to understand that this assertion of timelessness is not a natural attribute of collected editions, but something which must be produced against the adversity of the times. It is precisely the tendency of these editions to create an impression of their a-historical standard. I should think it is time to question this timelessness, and to historicize them.

4. PUBLISHING AND PUBLISHERS

From the beginning, collected editions have been adventures in publishing. Financial, philological, political and typographical requirements must be fulfilled for them to succeed. As their production required a greater investment in paper, they soon became the specialty of the great European publishers. Aldus Manutius in Venice found the format of collected works so appealing, that he even published collections of texts by different authors in this format: His collection of the writings of the Greek rhetors and that of the ancient astronomers are examples of this.

Collected editions imply cultural or political assertions, and as such they have been the object of political control. A publisher would often attempt to secure a privilege

which could protect the work for a certain period from being reprinted. More significant is the role of these publishing ventures in the self-affirmation of a cultural program: One example of this is Janus Gruter's involvement in the edition of Plautus, which was turned into a something like a political manifesto of the "geistigen Pfalz".[12] Heidelberg was a center of Protestantism, and the editions of classical authors produced there have a definite political significance in the context of the 30 Years War. The Protestant parties were very suspicious that editions published in Rome or under Roman supervision were being tampered with Patrick Young, the librarian of Charles I, explicitly spoke of "the Corruption of Scripture by Prelates of the Church of Rome".[13] In this context, the editions of the collected writings of the early Church Fathers had a definite political import, which is closely intertwined with the notion of the best text.

But these political implications are never simple and pure. In the case of Commelin's editions from Heidelberg,[14] or, even more evident, the massive Chrysostomos in eight volumes published by Henry Savile in Eton in 1610–1613, and dedicated to the King, these are also assertive gestures which place the publisher or the editor in the foreground. Savile had lost his only son and had decided to invest his wealth in this edition. He acquired a new Greek typeface in Paris, spend much time on studying the text of the surviving manuscripts, and succeeded to expend a considerable part of his estate on this lavishly produced edition. This edition not only offered a "Protestant" text of Chrysostomos, it also underpinned Savile's claim as a Renaissance humanist, and at the same time it was meant to prove the ability of England to produce great books. Only a collected edition can do all these things at the same time.

On the other side of the struggle of the early Seventeenth Century, the Jesuits too were active and maintained their own publication program. Again, it is not only the publication of small pamphlets or single books which carries their point, but also and importantly the *Opera Omnia*. When Heidelberg attempted to present Plautus as a Protestant author, then the Jesuits went for the five volume *Opera Mathematica* of Christoph Clavius, a Jesuit from Bamberg, published in Mainz in 1611/12. It would be quite naive to assess such an edition merely under the perspective of its offering the best text, even if that is what the editions explicitly maintain about themselves: "Ab auctore nunc denuo correcta & plurissimis locis aucta". Collected editions are never far from the center of the cultural battles of their time.

Political parties or cultural programs love to deal in collected works, and the presence of such an edition, just as its absence, is a very sensitive indicator of the political landscape. But this is not to say that such political intentions could exhaustively explain such editions. The situation is often much more complex. Within the political or cultural position the publisher or editor himself starts to develop his own agenda: For one publisher it is proof of his own status when he sponsors such an edition, whereas another publisher might finance a collected edition because he can turn it into a reference-object for a new typeface which he wants to promote. This is precisely what happened to Goethe, who auctioned the rights of one of his early collected editions to Unger, who was very keen to bring it out because it offered him a chance to advertise his newly developed Unger Fraktur type through the *oeuvre* of a literary lion. Goethe's very words turn into an extended type specimen: "Probe einer neuen Art deutscher Lettern".[15] For Unger, the collected edition of Goethe's writings is significant with reference to his own publishing and typesetting

enterprise. In Cambridge, between 1880 and 1920, collected works serve a similar goal: When the University Press publishes an imposing series of collected editions in physics and mathematics it effectively sheds the image of a bible and text-book printing house and assumes the status of an international academic press.[16]

6. AUTHOR

These are just a few examples of how publishers can hijack this mode of publication for their own purposes. The authors too become involved in the publication of collected works and connect their own intentions with this genre. Authors have been keenly interested in collected works. They were at all times very much aware of the elevated status of this mode of publication: Max Beerbohm's collection ironically entitled *Works* (1896), published when he was 24 years old, later expanded to *Works and More*, is just one indication of this. Collected Works are the paradise of authorship. They would finally justify all the authorial labors, they could prove to the world that their writings are indeed worthy to be kept for all times. With surprising frankness, prefaces inform us again and again that they consider such a collected edition as a valid means to assert the standing of the author, to bring him into the prominence he deserves, to finally set the record straight. Authors attempt to be their own editors and try to utilize these editions as a tool of self-fashioning. For Goethe's literary self-fashioning it was important that all volumes should have the same thickness, and Congreve believes that a sloppy presentation of his writings in print would even violate his rights as a man. He states: "It will hardly be denie'd that it is [. . .] a Right which every man owes to himself, to endeavor that what he has written may appear with as few faults as he is capable of avoiding."[17] If the presentation in print becomes a matter of "a Right which every man owes to himself," then the presentation of the author includes much more than words on the page: it encompasses a portrait, it includes fac-similes of his handwriting, perhaps even a colored engraving of the burial site of the author.

Collected works being initially the medium of the classical authors, they would of course offer an interesting battle-ground for the *Querelle des Anciens et des Modernes*. Already in the Renaissance, authors would dare to imitate the form reserved for the great classics and begin to arrange the publication of their writings into a consistent order, very much aware of the precedents. Bacon, for instance, has arranged his various writings in a way that emphasized their systematic interconnection. With a few words on the title page he transforms independently published treatises (*De Ventis*, 1622; *Historia Vitae and Mortis*, 1623) into parts of his *Instauratio Magna*. Bacon was very concerned to establish such a bibliographical unity for his works, but his problem was that he seems to have changed his mind about the title and structure of that one work, of which all his publications should be parts. This attempt at bibliographical unification is specially striking because in his own writings, Bacon is of course the most outspoken critic of book-learning. But in manipulating the unity of his own works, he was very well aware that the order of books is not as irrelevant as his criticism of a book-culture implied.

Authors, too, are generally very much aware that such editions offer more than the best text. Horace Walpole, for instance, started to publish a collected edition of his writings

with a clear idea in mind how this collection, by exclusion, can constitute his works. Walpole writes in 1768: "As I have been an author in various ways and in various forms, somebody or other might think of collecting my works. To prevent this, and at the same time to avoid having pieces attributed to me which I never wrote, and to condemn, by suppressing as far as I can, some which do not deserve publication, I have determined to leave this collection behind me." (1768)

Just as in any form of contextualization, a collection of writings affects their meaning. When Ben Jonson published his plays in a large folio volume, consciously transgressing the convention which confined plays to the more humble quarto format, he boldly calls the volume his *Works* on the title page. In doing so, Jonson is not only pretentious or arrogant. Rather, he is making use of the great cultural potential of *Collected Works* for his own status as *poeta laureatus*. And his contemporaries understood very well, how daring this gesture was. Writes one of his writer-colleagues: "Where does the mystery lurke, What others call a play, you call a worke."[18]

7. *RETOUR AU TEXTE*

I hope these examples lend credibility to my claim that *Collected Works* are a culturally over-determined form of publication which offers much more than a reliable text. What, one wonders, gave rise to the forced naiveté which lead some contemporary scholars to consider these publishing events purely in relation to the philological quality of their text, and without regard to the subtle changes in the meaning of the texts which are produced by their unification and standardization, and by their consequent monumentalization. If we look at the collected works which have been produced since the Nineteenth Century, the reductive philological perspective appears even more problematic and even more restricted. Since about 1850, this collected works of major European authors appear as *Nationalausgaben*, *édition nationale, edizione nazionale*. They enjoy the support of national academies, they flourish under royal protection, and their editors work in the firm and sincere belief that they perform nothing less than a national duty: "pour la Science et la gloire du pays",[19] we read in one mathematical collection of 1882. These national editions are the background for the war-edition of Malebranche mentioned above. As the great writers become national heroes, their editions become national events. Jacob Grimm, lamenting the fact that Schiller has not been honored with a fitting edition, explicitly speaks of the duty a nation has with regard to its greatest sons ("gleichsam eine schuld abtragend"). In the face of this all-pervasive nationalism in the publication of collected works, it does indeed take a lot of nerve, or ignorance, to approach collected works merely as containers of the best text.

In conclusion I want to re-examine the assertion that collected editions deliver the best text for still another angle. After the heterogeneous intentions and cultural values which can invade a collected edition, I shall now look at the text itself. I shall ask: How does the uniformity and extension of these editions affect their texts? How does the collected edition influence the uses a reader may make of these texts?

Every collected edition offers a structured access to its corpus. It will have to order its texts in some way, and this order invariably affects the relative significance of the texts collected. Put something in an appendix and damn it, place it in the beginning and

praise it, or hide it in the middle somewhere. Instead of a purified, castigated text, a collected edition often offers an implicit biography of its author, by offering its texts in a chronological order. The texts present material for future historians, but in the way the material is arranged, a specific history is already implied. The authors themselves often engage in this kind of biographical edition. In an edition of the mathematical writings of Sylvester the preface reads: "The object aimed at, in these volumes, has been to present a faithful record of the course of the author's thought, without such additions as recent developments of the subjects treated of might have afforded . . ."[20] Often the author himself tells his readers that he refrained from even correcting his more serious mistakes, in order to leave the biographical record intact.

The order of the texts within a collected edition often creates difficult problems for the editor. Regardless which order is being adopted, it will inevitably affect the perception and thus the meaning of the texts. Which text is to be the first to open the collection? What criterion shall govern the division of the volumes? Is the chronological order defensible when it makes an edition into an implicit biography? Is the development of the author's mind indeed paramount?

Collected works standardize all texts of one author into one single format. They cancel the historical singularity of their original modes of publication, and they cancel the differences between the texts which make it up. They murder any texts and make them all look exactly the same, all sterilized to the same degree, free from the typographical accidents of history, and divorced from contemporary debates and contexts in which these writings were first produced and later re-used. By excluding these contexts, collected works erect "walls of accessibility" around the writings of one single author, and isolate him. Accessibility is a dialectical concept: Collected works make some writings accessible, but they do it in a manner that makes related writings less accessible.

Collected works employ a number of devices with which to make a homogenized and pasteurized mass of text accessible. The index organizes a very specific access to the whole *oeuvre*, but only according to the terms or names entered in the indices. The decisions in compiling such an index can effectively hide parts of the work. At the same time, such an index, as useful as it often is, consistently privileges certain approaches: It favors either a biographical approach, or a *begriffsgeschichtliche* perspective, for example an analysis of how an author changes his views about a certain question in the course of his life. The tool of the index has no use whatsoever for a study of contexts which extend beyond the writings of the author. The collected edition cancels all historical attributes of the book and transforms it into a timeless text. Its typographical modifications, its use of modern paper, its uniform format, all that excludes history, and creates a fictive unity for the writings thus collected.

Since Gutenberg, the typographical genre of the Opera Omnia has enjoyed a massive cultural prominence. It has established the cultural hierarchy between those who only publish books and those who have their Collected Works published. Once we have started to see collected works as historical productions which can alter texts without altering their words, we also realize how much more work in the multi-facetted history of this typographical genre remains to be done.

Plurabelle Books, Cambridge

NOTES

[1] McKenzie 1993.
[2] Cf. Bogeng 1920; Halporn 1989; Speiser 1990; Cahn 1997.
[3] This perspective is of course well coordinated with the demands of a market which primarily caters to collectors. A different tradition of bibliographical research is represented by Edward Arber (1836–1912), who had a much broader perspective, produced cheaper books (Arber Reprints), was less interested in literary heroes, and envisioned a cultural history of printed matter in which even the anonymous printer would finally be recognized as the true hero of literature (See Arber 1875, Preface).
[4] Hagen 1979.
[5] Cf. Roustan 1938.
[6] Sieger 1989.
[7] Donne 1622, A3r.
[8] Cf. Saunders 1951, Bennett 1965, 292; Traister 1990.
[9] It would be instructive to look at those authors who have tried to integrate their output into a systematic structure, and who attempted to design their own collected edition.
[10] And if two states claim the same author, they are each likely to fund their own editions, which is why we had two editions of Goethe's writings when we had two German states.
[11] Letter to Goeschen, 2 May 1786, quoted by Hagen 1990, 33: "It would be very desirable that all volumes have the same number of sheets. I trust that every volume will easily accommodate one sequence of signatures, but with regard to the *Werther* it all depends upon the initial design, the size of the letters, and how it is divided. The remaining dramatic writings are easily expanded or compacted [using typographical variation]."
[12] Forster 1967.
[13] This is the title of a book published by Young (Junius) in 1625.
[14] Mittler 1986, 425–435.
[15] Kraft 1970, 123.
[16] Cahn 1997.
[17] Cf. McKenzie 1981, 81–126.
[18] Brady 1991, 114.
[19] Cauchy 1882–1974, Ser 1, 1: vi.
[20] Sylvester 1904–1912, Vol 1, Preface.

REFERENCES

Arber, Edward. 1875. *Transcripts of the Registers of the Company of Stationers of London.* London: Arber.

Bennett, Henry. 1965. *English Books and Readers 1558–1603, Being a Study in the History of the Book Trade in the Reign of Elizabeth I.* Cambridge: Cambridge University Press.

Bogeng, Gustav. 1920. "Gedanken über Gesamtausgaben." *Die Bücherstube* 1: 75–84.

Brady, Jennifer (Editor). 1991. *Ben Jonson's 1616 Folio.* Newark: Delaware University Press.

Cahn, Michael. 1997. "Wissenschaft im Medium der Typographie. Collected Papers aus Cambridge 1880–1910." In *Fachschrifttum, Bibliothek und Naturwissenschaft im 19. und 20. Jahrhundert.* Edited Christoph Meinel, 175–208. Wiesbaden: Harrassowitz.

Cauchy, Augustin-Louis. 1882–1974. *Oeuvres Complètes.* Sous la direction de l'Académie des Sciences et sous les auspices de M. le Ministre de l'Instruction publique, Paris: 1882–1974.

Donne, John. 1622. *A Sermon Upon the viii Verse of the I Chapter of the Actes of the Apostles, Preached to the Honourable Company of the Virginina Plantation, 13 Nov. 1622.* London: Th. Jones.

Forster, Leonard. 1967. *Janus Gruter's English years. Studies in the continuity of Dutch literature in exile in Elizabethan England*. Leiden: University Press; London, Oxford University Press.

Hagen, Waltraut (Editor). 1990. *Goethes Werke*, Edited for the Deutschen Akademie der Wissenschaften zu Berlin by Ernst Grumach etc. Berliner Ausgabe, Ergänzungsband 2, 1. Berlin: Akademieverlag.

Hagen, Waltraut. 1979. *Handbuch der Editionen. Deutschsprachige Schriftsteller, Ausgang des 15. Jahrhunderts bis zur Gegenwart*. München: Beck.

Halporn, Barbara. 1989. *Johann Amerbach's Collected Editions of St. Ambrose, St. Augustine, and St. Jerome.* Dissertation, Indiana University.

Kraft, W. 1970. *Editionsphilologie.* Darmstadt: WBG.

McKenzie, Donald. 1981. "Typography and Meaning: The Case of Wiliam Congreve." In *Buch und Buchhandel im 18. Jahrhundert*. Edited by Giles Barber, Bernhard Fabian, 81–126. Hamburg: Hauswedell.

McKenzie, Donald. 1993. *"What's Past is Prologue"*. London: Hearthstone 1993. (The Bibliographical Society Centenary Lecture)

Mittler, Elmar (Editor). 1986. *Bibliotheca Palatina. Katalog zur Ausstellung*. Heidelberg: Braus.

Roustan, Désiré. 1938. "La première édition des OEuvres Complètes de Malebranche." *Revue Philosophique de la France et de l'Etranger* 75:129–141.

Saunders, J. W. "The Stigma of Print. A Note on the Social Bases of Tudor Poetry." *Essays in Criticism* 1:139–164.

Shakespeare, William. 1627. *Mr William Shakespeares Comedies, Histories, and Tragedies, Published According to the True Originall Copies.* London: Jaggard and Blount.

Sieger, Ferdinand. 1989. "Der Gesamtausgabevorbehalt." *Das Buch in Praxis und Wissenschaft,* Edited by Peter Vodosek. Wiesbaden: Harrassowitz.

Speiser, D. and Radelet-de Grave, P. 1990. "Publishing Complete Works of the Great Scientists: An International Undertaking." *Impact of Science on Society* 40: 321–348.

Sylvester, James. 1904–1912. *The Collected Mathematical Papers.* Edited by H. F. Baker. Cambridge: Cambridge University Press.

Traister, Daniel. 1990. "Reluctant Virgins: The Stigma of Print Revisited." *Colby Quarterly* 26: 75–86.

HANS-JÖRG RHEINBERGER

WRITING WORKS[1]: A REACTION TO MICHAEL CAHN'S PAPER

ABSTRACT

Taking up and expanding on the topic of Michael Cahn's essay, this paper presents some observations on scientists' editions of their collected works. These editions span the time from the second half of the eighteenth to the first half of the twentieth century. The main focus is on the practice of claiming literary and scientific authority by editing one's collected works during one's own lifetime. The paper begins by briefly describing the collected works of Pierre Louis Moreau de Maupertuis and Charles Bonnet. The central section is devoted to Georges Louis Leclerc de Buffon's Natural History. Buffon's work is taken as an example not only of *editing*, but of *writing* collected works. Buffon's edited works had a long history of being expanded after their author's death. At the end, two examples of early twentieth-century collections of research papers, by Hugo de Vries and Carl Correns, are considered. The paper is a corollary to Michael Cahn's thoughts on the cultural history of texts and asks for a more general reflection on the historical development of forms and genres of scientific writing.

Michael Cahn's "*Opera Omnia*" describes a much-neglected aspect of the life of texts: "The Production of Cultural Authority." Looking at the phenomenon of collected works, he deals with what he calls an "alphabet of significance [beyond] the letters," a "cultural semantics" of editing. He shows that beyond the fiction of the "best text," that is, the "critical edition," which is itself the product of nineteenth-century philological historicism, the edition of the complete works of an author is embedded in historically varying cultural contexts that are much in need of critical examination in both their epistemological and social dimensions.

1. OEUVRES: PIERRE LOUIS MOREAU DE MAUPERTUIS, CHARLES BONNET

Throughout the eighteenth century, an author himself frequently edited his own collected works. The French mathematician, geographer and philosopher Pierre Louis Moreau de Maupertuis (1698–1759) is a good case in point. The first collected edition of his works dates from 1744, when he was forty-six, a year after he had become a regular member of the *Académie des Sciences* in Paris.[2] Printed in Amsterdam, the compilation essentially contained his papers on astronomy and physical geography. When Maupertuis was named president of the Berlin Academy of Sciences in 1746, he organized the publication of a second version of his works, which appeared in 1752 in Dresden.[3] In accordance with his new position, he had decided to broaden the range of papers which

K. Chemla (ed.), History of Science, History of Text, 95–103.

were included in the collection. In addition to the works on astronomy and physical geography, the 1752 edition contained philosophical and biological treatises as well as several "academic discourses."[4] A still larger Berlin and Lyon edition in two volumes followed in 1753, featuring additional letters and discourses.[5] This edition finally grew to four volumes in 1756. Under the direction of Maupertuis' friend, the Abbé Nicolas-Charles-Joseph Trublet, the corpus of texts was then completely reorganized and arranged according to subject matter.[6] Maupertuis survived this last edition by another three years. Therefore, to use the strange clause of Giorgio Tonelli (the editor of a reprint of this edition), this last edition could be regarded as conveying an image "which would have been definitive if Maupertuis could have foreseen the imminence of his death."[7] This prospective retrospection, a curious conditional anticipation of a past future, perfectly expresses the unsolvable dilemma of attempting to assemble complete works before the work has been completed, that is, of publishing one's collected oeuvre during one's own lifetime.

Another eighteenth-century scientist editing his collected works is the insect specialist, natural historian, and philosopher Charles Bonnet (1720–1793). A citizen of the Republic of Geneva, he published a "collection complette" of his works on natural history and philosophy at the instigation of a "foreign" publisher, the Royal librarian Samuel Fauche in Hapsburg-ruled Neuchâtel.[8] This collection was quite complete, although it included neither Bonnet's Memoirs as a foreign correspondent to the Royal Academy in Paris nor his contributions to Abbé Rozier's *Journal de Physique*. In the preface, Bonnet advised the reader that the prospect of a complete collection had engaged him in revising his works, and in "making more or less considerable additions, partly in the form of notes, partly as supplements," and he added: "Further writings which I had never published [also] entered into this general revision."[9]

Bonnet divided the collection in two general parts. The first part contained his "Writings on Natural History," and the second, the "Writings on Speculative Philosophy."[10] This "general" division does not, as one might at first assume and as has been suggested at times, reflect a chronological order with Bonnet starting out as a natural historian and ending up as a speculative philosopher. As the historian of science Marino Buscaglia reminds us, we can distinguish four different periods in Bonnet's working life. An early insectological phase until roughly 1740 was followed by a period of experimentation on plants. After this period, Bonnet turned to metaphysical matters, but then he came back to empirical work and conducted regeneration experiments on the heads of snails and the extremities of salamanders between 1765 and 1777.[11] Although, or perhaps because, it took Bonnet three years of effort, from 1775 to 1778, to get the collection ready, he could not avoid the presentiment that, despite his efforts and amendments, part of his work might already be out of date. However, he consoled himself with the wishful thought that this did not prevent his oeuvre from—as he put it—becoming "part of the History of the human Spirit" during his very lifetime. On the contrary, the edition raised it to this level.[12] Bonnet could enjoy this grand feeling, this protracted desire for precocious accomplishment, for another decade of his life after the edition had been published and before he died in 1793, on his estate Genthod at Lake Geneva.

2. WRITING WORKS: GEORGES LOUIS LECLERC, COMTE DE BUFFON

One wonders how far one can take this tendency to anticipate the completion of one's own life's textual production and the codification of its accomplishment. If you embark on editing your own "complete" work, you imply that you have stopped working and will not produce any further scholarly texts. Following this line of thought, at one point in his essay, Michael Cahn concludes: "Sadly enough, however, there has not yet been invented a way to *write* a collected edition. [Books] can be written, but collected works must be *edited*." I know, however, of an example that could be viewed as the realization of this impossibility. The case in point is Georges Louis Leclerc de Buffon's (1707–1788) *Histoire naturelle, générale et particulière*. Buffon's oeuvre is not just an encyclopedic work in the sense of being an exceedingly encompassing natural history; it is more. He crafted it from the very beginning as the successive summation of his scientific life.

Buffon started his academic career as a translator of, among other works, Stephen Hales' "Vegetable Staticks"[13] and Isaac Newton's "The Method of Fluxions and Infinite Series,"[14] each of which he supplemented with an extended preface. In 1739 he succeeded his friend Charles-François de Cisternai Du Fay—famous for his distinction of two kinds of electricity, positive and negative—as director of the Royal Garden and Natural Cabinet. At some point he must have decided that he would not continue working in the usual manner of producing memoirs, which he had been contributing to the *Académie des Sciences* since he had been accepted as "adjoint-mécanicien" in 1733, but rather that he would concentrate all his scientific efforts on the creation of an encompassing collection of works on natural history. Reflecting on this decision, Buffon's later editor Pierre Flourens remarks devoutly: "[He] consecrated ten entire years to deepest meditation before he began to publish his grand oeuvre. The plan he had formed was the largest and most daring a man has ever conceived of."[15] Just as the "desire to distinguish himself"—according to Flourens—had led the young Buffon to the sciences when he came to Paris at the age of 24,[16] this same desire now culminated in the unprecedented endeavor of writing the collected works of natural history—a grand panoply on the formation of the solar system, the history of the globe and the other planets, and the life forms and physical processes upon them.

In contrast to Maupertuis—who had dedicated the last edition of his works to men of commerce and science, including the director of the Indian Company in Paris, Monsieur Duvelaër[17]—Buffon devoted his undertaking to "His Majesty."[18] The first three volumes of the "Histoire naturelle" were published by the Royal Presses in 1749. The last volume came out forty years later, in 1789, one year after Buffon's death. In the end, the whole enterprise comprised 15 volumes of "General and Particular Natural History" (1749–1767) including a "Description of the Royal Cabinet," nine volumes on the "Natural History of Birds" (1770–1783), five volumes on "Minerals" (1783–1788), and seven supplemental volumes. The first of these supplements appeared in 1774, and the last was edited posthumously in 1789 by Bernard Germain Etienne de la Ville, comte de Lacépède. All together, there were thirty-six volumes—parts of which were produced

in collaboration with the anatomist Louis Jean-Marie Daubenton and the ornithologist Guéneau de Montbeillard—a remarkable collection. Buffon's genius worked its way through a "visible filiation of ideas" and a "constant progress," "very capable of development," as Flourens notes, not without a slight hint of irony.[19]

How did Buffon cope with that filiation of ideas and that constant progress? How did he deal with the problem of the development, or more prosaically, the change of his views over time, which would seem to defy any attempt to write one's collected works in a straightforward fashion? Buffon found several solutions to the problem. On the one hand, he revised and rewrote his general considerations from time to time. Volume One (of 1749) contains an opening "Discourse on the Manner of Studying and Treating Natural History", but fifteen years later, Buffon reorganized his thoughts on natural history in a "First View on Nature" in Volume Twelve (1764) and in a "Second View on Nature" in Volume Thirteen (1765). Similarly, the "History and Theory of the Earth" (from 1749) was followed by the "Epochs of Nature" (1778). The 1749 treatise saw the changes on the earth as being engaged in a kind of pendular movement and ultimately in a timeless equilibrium, whereas the 1778 work subjected them to an inexorable gradient of refrigeration. In addition, Buffon made use of supplements. From 1774 onward, he edited a series of volumes in which he assembled an accumulating body of additions and corrections related to his already published materials. Indeed, this strategy of internal referencing on multiple levels imposes a strong incentive and obligation to view the work as a whole. If you want to know what Buffon thinks about a particular aspect of natural history, for example the species concept, you have to consult the complete collection. You cannot just refer to one part of it!

Buffon's "Natural History" sold well from the very beginning. On January 4, 1750, Buffon wrote to his friend and teacher Gabriel Cramer about the publication of the first three volumes: "The edition has sold out in six weeks. Actually, two new impressions are being printed."[20] Not only were the volumes constantly reprinted, but numerous new editions and extensive re-groupings of the "Histoire Naturelle"—with and without Daubenton's anatomical contributions, for example—appeared throughout Buffon's lifetime. One of them, published as early as 1774, carries the premature but revealing title "Oeuvres complètes" fourteen years before Buffon's death.[21] In 1785, the preparation of an edition with colored plates was initiated.[22] Daniel Mornet estimates that almost every second private library during the *Ancien Régime* contained a version of the works of Buffon.[23]

At least as remarkable as this proliferation of editions during Buffon's lifetime is the fate of Buffon's *opus* after his death. First, Bernard Germain Etienne de Lacépède—who edited the last supplement volume—continued Buffon's endeavor and published two volumes on "Egg-laying Quadrupeds" and "Serpents," five volumes on "Fishes," and one volume on "Whales and Dolphins," all between 1788 and 1804.[24] With the publisher Deterville, René-Richard Castel reclassified the contents of Buffon's Histoire following the system of Linnaeus in the year VII of the new calendar (1799).[25] The Deterville edition had a long history of re-publications and additions over almost four decades until 1837—among others conducted by F. Martin Grostête T. de Tigny, E.-F. Guérin, Louis Augustin Guillaume Bosc, and, not least, Jean-Baptiste Lamarck. Between the year VII

(1799) and 1808, the printer François Dufart issued a new Buffon, edited by Charles Nicolas Sigisbert Sonnini de Manoncourt. He increased the corpus to 127 volumes, through additions from Sonnini himself, François Marie Daudin, Pierre Denys-Montfort, Pierre André Latreille and Charles-François Brisseau de Mirbel.[26] Jean-François Bastien published the complete works of Buffon in 1811 and prefaced them with the eulogies of Buffon by Félix Vicq d'Azyr and Marie Jean Antoine Nicolas de Condorcet. The Comte de Lacépède produced a new, reordered edition in 1817–1818, and further reeditions were provided by Hippolyte Romain Duthilloeul in 1822, and by Jean Vincent Félix Lamouroux and Anselme Gaëtan Desmarest between 1824 and 1832.[27] In 1825–1826, Georges Cuvier recast Buffon's "Natural History" to reflect his own taxonomy, a fact that comes as no surprise when one considers this list of predecessors.[28] Another complete edition, including a five-volume supplement on the progress of natural history since Buffon's death also written by Cuvier, was issued by the botanist Achille Richard between 1826–1836.[29] It would be reissued over and over again during the following two decades. The editor Roret started his "Nouvelles Suites à Buffon" in 1834. This endeavor lasted more than half a century, until 1890, and came to include the works of such eminent nineteenth-century biologists as Henri Milne-Edwards, Félix Dujardin, and Alphonse de Candolle.[30] Since Cuvier had decided to graft himself onto Buffon, Etienne Geoffroy Saint-Hilaire could not resist doing the same. His "Oeuvres complètes de Buffon," enhanced by a historical note and some philosophical reflections on the progress of the sciences since Buffon's days, followed Cuvier's edition a decade later, in 1837–1838.[31] More Buffons came from the permanent secretary of the Académie des Sciences, the already mentioned Marie-Jean-Pierre Flourens in 1853–1855, from Ernest Faivre in 1859, and from Jules Pizzetta between 1860–1863.[32] The edition still considered the "best text" from the viewpoint of late nineteenth-century philology is the one published in 1884–1885 by Jean-Louis de Lanessan.[33]

As can be seen from this impressive list, "Buffon" became the synonym for several kinds of authorship "in his name." Among them were editing, reclassifying, rewriting, and writing additions and amendments. These subsequent "authoritative" seizures of an oeuvre could be subtly distinguished from each other and even used to identify and to distinguish their promoters according to the ways and means by which they kept the enterprise going. A century of French naturalists wrote themselves into this procession and gained dignity by associating themselves with a grand predecessor.

The changing cultural meaning of "oeuvres complètes" from the middle of the eighteenth to the late nineteenth century is nicely captured by this prolific expansion—to which most of the leading French naturalists of the nineteenth century contributed—and their subsequent shrinking to the "authentic" Buffon of Lanessan's critical edition. Until the middle of the nineteenth century, the label "complete" meant adding whatever new knowledge was judged to be important and missing from the oeuvre: Linnaean terminology, Cuvierian taxonomy, the supplementation of Buffon's natural history with new groups of animals and plants, and the addition of more details to, and amendments of, Buffon's own descriptions. Even Lanessan, in his emphatic move back to the "sources," that is, to the original Buffon, toward the end of the nineteenth century, did not hesitate to impose an order on the work that does not retain the one established a century earlier in

Buffon's own writing process. If it is a critical edition, it is one that respects the integrity of Buffon's text, not one that renders the work in its chronological intactness.

Buffon editions of all sorts and sizes swamped the French natural history market in the century after Buffon's death. The scientific editions were complemented by popular ones. From 1800 until the 1880s, the catalogue of the *Bibliothèque Nationale* lists various "Abrégés" and "Beautés de Buffon," "Génie de M. de Buffon," "Morceaux choisis de Buffon," even a "Petit Buffon moral et religieux." Different audiences were specifically addressed with "Le Buffon des écoles," "Le Buffon de la jeunesse," "Le Buffon des demoiselles," "Le Buffon des familles," "Le Buffon des enfans," and even a "Buffon des petits enfants" was issued in 1841.[34]

With Buffon, the attempt to write complete works came to its apotheosis. Buffon's *oeuvres* exerted a cultural authority, if not hegemony, in natural history for a long century. With the exception of a few years during the French revolution, Buffon was treated as a national icon. However, his influence also reached beyond France to the entire continent, as is amply demonstrated by translations into German, Spanish, and Italian.[35] The first German edition of the General Natural History of Buffon was prefaced by none other than Albrecht von Haller. Friedrich Heinrich Wilhelm Martini, the founder of the *Gesellschaft naturforschender Freunde zu Berlin*, started a translation of the "Naturgeschichte der vierfüssigen Thiere" in 1772, to which Georg Forster also contributed a volume.[36] "Buffon" became a synonym for natural history for a major part of the nineteenth century. Ironically, Lanessan's critical edition, the restoration of the "best text" after a long succession of "continuations" of the work, also marks the end of its pervasive cultural influence and the transformation of Buffon's writings into an object of purely historical interest.

3. OPERA E PERIODICIS COLLATA: HUGO DE VRIES, CARL CORRENS

The publication of *opera omnia* or, at the limit, their *mise en scène* as a process of writing, does not seem to have survived as a prominent practice in the nineteenth-century life sciences. Let me end, however, with the observation of a different form of *opera* emerging in the early twentieth century. It rests on a transformation in scientific writing during the century before. Increasingly over the course of the nineteenth century, the writing of grand series, handbooks and monographs gave way to the writing of shorter research papers published ever more rapidly and in more and more specialized journals. At the beginning of the twentieth century, writing about research had become almost synonymous with publishing articles in disciplinary journals. As a result, sending, receiving, and collecting offprints became a prominent means of making oneself visible and keeping abreast of what others were doing. As a consequence of this transformation in scientific writing, "opera e periodicis collata" became a way of honoring a productive scientific author in the republic of letters. The papers of the Dutch geneticist Hugo de Vries (1848–1935) nicely testify to this practice. They were collected and published in seven volumes between 1918 and 1927 by an unnamed "aantal vrienden en vereerders." The enterprise started on de Vries' seventieth birthday at the end of World War I.[37] It is clear that the friends and admirers of

the prominent German co-"rediscoverer" of Mendel's rules, Carl Correns (1864–1933), could not leave this challenge unanswered. Only a few years after the takeoff of the Dutch collection, Correns' student and coworker Fritz von Wettstein organized an edition of his teacher's "Gesammelte Abhandlungen zur Vererbungswissenschaft aus periodischen Schriften" on the occasion of Correns' sixtieth birthday in 1924.[38] With that publication, the balance in the claim for national scientific authority in the vigorously burgeoning field of classical genetics was restored.

As Michael Cahn has shown in his beautiful essay, the phenomenon of "opera omnia" is a subject worthy of thorough study in a cultural history of texts. My particular interest in this *addendum* to Cahn's paper has been to comment on the paradox of completion before closure—and the continuation after closure in the case of Buffon. The impossible yet obviously compelling desire of an author to anticipate the fulfillment of his life's literary accomplishment can express itself in many different forms. What I have wanted to highlight, beyond the narrow focus on these examples, is that historians of science could profitably take up this issue in a more general reflection on the historical development of forms and genres of scientific writing.

Max Planck Institute for the History of Science,
Berlin, Germany

NOTES

[1] A German version of this paper appeared as (Rheinberger 2002). I thank Karine Chemla and Laura Otis for good advice.
[2] (Maupertuis 1744).
[3] (Maupertuis 1752).
[4] (Tonelli 1974).
[5] (Maupertuis 1753).
[6] (Maupertuis 1756).
[7] (Tonelli 1974, xiv).
[8] (Bonnet 1779–1781).
[9] (Bonnet 1779, 1:vi).
[10] (Bonnet 1779, 1:viii).
[11] (Buscaglia 1994, 285).
[12] (Bonnet 1779, 1:xii).
[13] (Hales 1735).
[14] (Newton 1740).
[15] (Flourens N. d. [1855], xiv).
[16] (Flourens N. d. [1855], i).
[17] (Maupertuis 1756, 1:1).
[18] (Buffon 1749).
[19] (Flourens N. d. [1855], xix, i).
[20] Letter to Gabriel Cramer (Buffon 1885, 1:61–62).
[21] (Buffon 1774–1778).
[22] (Buffon 1785–1790).
[23] (Mornet 1910).
[24] (Lacépède 1788–1789, 1798–1803, an XII-1804).
[25] (Castel an VII).
[26] (Sonnini an VII-1808).
[27] (Bastien 1811, Lacépède 1817–1818, Duthilloeul 1822, Lamouroux 1824–1832).

[28] (Cuvier 1825–1826).
[29] (Richard 1826–1836).
[30] (Roret 1834–1890).
[31] (Geoffroy Saint-Hilaire 1837–1838).
[32] (Flourens N. d. [1853–1855], Faivre 1859, Pizzetta 1860–1863).
[33] (Lanessan 1884–1885).
[34] See (Lepenies 1976).
[35] (Buffon 1750–1754, Buffon 1772–1804, Buffon 1773–1801, Buffon 1782–1791, Buffon 1785–1805).
[36] (Buffon 1773–1801, 6 [1780]).
[37] (de Vries 1918–1927).
[38] (Correns 1924).

REFERENCES

Bastien, Jean-François. 1811. *Oeuvres complètes de Buffon.* Paris: Jean-François Bastien.

Bonnet, Charles. 1779–1781. *Collection Complette des Oeuvres. Oeuvres d'Histoire naturelle et de Philosophie*. Neuchâtel: Samuel Fauche.

Buffon, Georges Louis Leclerc, Comte de. 1749. "Au Roi." *Histoire naturelle, générale et particulière, avec la description du cabinet du roi, Tome Premier*. Paris: Imprimerie royale.

Buffon, Georges Louis Leclerc, Compte de. 1750–1754. *Allgemeine Historia der Natur, nach allen ihren besondern Theylen abgehandelt (von Buffon); nebst einer Beschreibung der Naturalienkammer Sr. Majestät des Königes von Frankreich (von Daubenton)*, with a preface by Albrecht von Haller. Hamburg and Leipzig: G. C. Grund and A. H. Holle.

Buffon, Georges Louis Leclerc, Compte de. 1772–1804. *Herrn von Buffons Naturgeschichte der Vögel*, translated by Friedrich Heinrich Wilhelm Martini (and Bernhard Christian Otto). Berlin: J. Pauli.

Buffon, Georges Louis Leclerc, Compte de. 1773–1801. *Herrn von Buffons Naturgeschichte der vierfüssigen Thiere*, translated by Friedrich Heinrich Wilhelm Martini, Bernhard Christian Otto (and Georg Forster). Berlin: J. Pauli.

Buffon, Georges Louis Leclerc, Comte de. 1774–1778. *Oeuvres complètes*. Paris: Imprimerie royale.

Buffon, Georges Louis Leclerc, Comte de. 1782–1791. *Storia naturale generale e particolare del Sig. conte di Buffon*. Venezia: A. Zatta.

Buffon, Georges Louis Leclerc, Comte de. 1785–1790. *Histoire naturelle générale et particulière*. Deux-Ponts: Sanson & Compagnie.

Buffon, Georges Louis Leclerc, Comte de. 1785–1805. *Historia natural, general y particular, escrita en francés por el conde de Buffon*, translated by D. Joseph Clavijo y Faxardo. Madrid: D. Joachim Ibarra.

Buffon, Georges Louis Leclerc, Comte de. 1885. *Correspondance générale*, collected and annotated by Henri Nadault de Buffon. Paris: Le Vasseur.

Buscaglia, Marino. 1994. "Bonnet dans l'histoire de la méthode expérimentale." In *Charles Bonnet, savant et philosophe (1720–1793)* (Mémoires de la Société de Physique et d'Histoire Naturelle de Genève, Vol. 47), edited by Marino Buscaglia, René Sigrist, Jacques Trembley, Jean Wüest, 283–313.

Castel, René-Richard. An VII. *Histoire naturelle de Buffon, classée . . . d'après le système de Linné*. Paris: Deterville.

Correns, Carl. 1924. *Gesammelte Abhandlungen zur Vererbungswissenschaft aus periodischen Schriften 1899–1924*, edited by the Deutsche Gesellschaft für Vererbungswissenschaft. Berlin: Julius Springer.

Cuvier, Georges. 1825–1826. *Histoire naturelle de Buffon, mise dans un nouvel ordre, précédée d'une notice sur la vie et les ouvrages de cet auteur*. Paris: Ménard et Desenne.

Duthilloeul, Hippolyte Romain. 1822. *Oeuvres complètes de Buffon. Nouvelle édition*. Douai: Tarlier.

Faivre, Ernest. 1859. *Oeuvres complètes de Buffon, précédées d'une étude historique et d'une introduction sur les progrès des sciences naturelles depuis le commencement du XIX^e siècle. Nouvelle édition*. Paris: J. Poulain.

Flourens, Pierre. N. d. [1853–1855]. *Oeuvres complètes de Buffon, avec la nomenclature Linnéenne et la classification de Cuvier*. Paris: Garnier frères.

Flourens, Pierre. N. d. [1855]. "Notice sur Buffon." In *Oeuvres complètes de Buffon, Tome premier*, edited by Pierre Flourens. Paris: Garnier.

Geoffroy Saint-Hilaire, Etienne. 1837–1838. *Oeuvres complètes de Buffon, précédées d'une notice historique et de considérations générales sur le progrès de l'influence philosophique des sciences naturelles depuis cet auteur jusqu'à nos jours*. Paris: F.-D. Pillot.

Hales, Stephen. 1735. *La statique des végétaux et l'analyse de l'air*, translated by Georges Buffon. Paris: Debure.

Lacépède, Bernard Germain Etienne, Comte de. 1788–1789. *Histoire naturelle des quadrupèdes ovipares et des serpents*. Paris: Hôtel de Thou.

Lacépède, Bernard Germain Etienne, Comte de. 1798–1803. *Histoire naturelle des poissons*. Paris: Plassan.

Lacépède, Bernard Germain Etienne, Comte de. An XII-1804. *Historie naturelle des cétacés*. Paris: Plassan.

Lacépède, Bernard Germain Etienne, Comte de. 1817–1818. *Oeuvres complètes de Buffon. Nouvelle édition*. Paris: Rapet.

Lamouroux, Vincent Félix.1824–1832. *Oeuvres complètes de Buffon. Nouvelle édition*, continued by Anselme Gaëtan Desmarest. Paris: Verdière et Lagrange.

Lanessan, Jean-Louis de. 1884–1885. *Oeuvres complètes de Buffon. Nouvelle édition annotée et précédée d'une introduction par Jean-Louis de Lanessan. Suivie de la correspondance générale de Buffon, recueillie et annotée par Henri Nadault de Buffon*. Paris: A. Le Vasseur.

Lepenies, Wolf. 1976. Das Ende der Naturgeschichte. München: Hanser.

Maupertuis, Pierre Louis Moreau de. 1744. *Ouvrages divers*. Amsterdam: Aux Depens De la Compagnie.

Maupertuis Pierre Louis Moreau de. 1752. *Oeuvres*. Dresden: Georg Conrad Walther.

Maupertuis, Pierre Louis Moreau de. 1753. *Oeuvres* (two volumes). Berlin-Lyon: Etienne de Bourdeaux—frères Bruyset.

Maupertuis, Pierre Louis Moreau de. 1756. *Oeuvres* (four volumes). Lyon: Jean-Marie Bruyset.

Mornet, Daniel. 1910. "L'Enseignement des bibliothèques privées." *Revue d'Histoire Littéraire de la France* 17:449–496.

Newton, Isaac. 1740. *La méthode des fluxions et des suites infinies*, translated by Georges Buffon. Paris: Debure.

Pizzetta, Jules. 1860–1863. *Oeuvres de Buffon*. Paris: Martinon.

Rheinberger, Hans-Jörg. 2002. "Gesammelte Werke." In *Neuzeitliches Denken*, edited by Günter Abel, Hans-Jürgen Engfer, and Christoph Hubig, 13–22. Berlin: De Gruyter.

Richard, Achille. 1826–1836. *Oeuvres complètes de Buffon. Suivies de quatre volumes sur l'histoire des progrès des sciences naturelles depuis 1789 jusqu'à ce jour, par M. le Baron G. Cuvier*. Paris: Baudouin frères (1826–1828).

(Roret). 1834–1890. *Nouvelles Suites à Buffon*. Paris: Roret.

Sonnini, Charles Nicolas Sigisbert. An VII-1808. *Histoire naturelle générale et particulière, par Leclerc de Buffon. Nouvelle édition. Ouvrage formant un cours complet d'histoire naturelle*. Paris: Imprimerie de François Dufart.

Tonelli, Giorgio. 1974. "Introduction: Bibliographie et histoire du texte." In *Pierre Louis Moreau de Maupertuis, Oeuvres*, xi-lxxxiii. Hildesheim: Georg Olms.

Vries, Hugo de. 1918–1927. *Opera e periodicis collata, Vols. 1–7*. Utrecht: A. Oosthoek.

Part III

HOW SCIENTIFIC AND TECHNICAL TEXTS ADHERE TO LOCAL CULTURES

CRAIG CLUNAS

TEXT, REPRESENTATION AND TECHNIQUE IN EARLY MODERN CHINA

ABSTRACT

This paper examines 'number' and numerology as a discursive object among the elite of China in the Ming period (1368–1644). Starting from an anecdote concerning the poet, calligrapher and painter Wen Zhengming (1470–1559), whose refusal to learn these skills from his father led the latter to burn his books, it examines how technical knowledge of this sort was conceived in relation to the humanistic priorities of the Ming elite. It raises the question of how much and what kind of 'numerology', or *shu xue*, (also the modern Chinese word for 'mathematics) learned men of the Ming knew, and in what contexts it was appropriate to admit to knowing it. The ownership and dissemination of the relevant texts is examined, along with the cultural implications of the numerical skills involved in administration and commerce. 'Number' is ultimately seen as problematic for an elite distrustful of 'technique' as a socially compromised form of knowledge.

The term *shu xue* is used in modern Chinese to designate the scientific discourse of mathematics, but to translate it that way in a pre-nineteenth century text is both misleading and anachronistic. As Ho Peng Yoke among others has pointed out, a more satisfactory though still limited translation would be "numerology", incorporating "the art of predicting the future, both in the natural and human world."[1] There is considerable difficulty in gauging the extent to which the specialised knowledge implied by the term was disseminated among the Chinese elite in the Ming period (1368–1644), the era which forms the object of investigation in the present study. However it certainly had one distinguished fifteenth-century practitioner in one elite family of the great Yangtze valley city of Suzhou. This was a man named Wen Lin (1445–1499), father of the famous calligrapher, painter and poet Wen Zhengming (1470–1559).[2] This paper takes as its point of departure an incident in the recorded biography of the latter, as a way of entering into the realm of "number" as a discursive object in at least one social stratum of sixteenth-century Chinese culture. It is an incident which must raise questions concerning the relationship between constructions of knowledge and constructions of the self (often only definable in relation to some Other), and the role of tangible textual artefacts in the creation and sustenance of these constructions. In the context of Ming dynasty China, what one refused to know, what texts one refused to engage with, was to come to be just as important as what one did know, and which books stood on one's shelves.

Wen Lin, according to the official family necrology of his ultimately more famous son, "was expert in numerology" (*shu xue*), and wished to pass the skills and knowledge on to that same favoured second son, Wen Zhengming. Zhengming refused to learn them,

K. Chemla (ed.), History of Science, History of Text, 107–121.

whereupon Wen Lin ordered all his books on the subject to be burned, a command which was immediately executed.[3] Sadly, we have no details of the titles involved, nor of the number of volumes consigned to the flames. The anecdote is recounted immediately after the statement that, though Wen Zhengming was broadly learned and had an extensive library, he refused to read books on the subject of yin-yang cosmology and "magic" (*fang ji*). Clearly the intention of the biographer (Wen Lin's grandson) is to associate *shu xue* with dangerously heterodox areas in which a model Confucian should not dabble. Like many unexamined details in Wen Zhengming's biography, the anecdote when scrutinised closely is more complex than it first appears. Two taboos are violated here, the first being against the wanton destruction of any texts, indeed of any bit of paper with writing on it. There were many powerful social prohibitions on the physical destruction of books, and of the written word in general, in Ming China. The physical destruction of texts, which were not necessarily printed (the wide prevalence of the nearly thousand-year old technology of printing at the time is not equivalent to the extinction of a tradition of transmission of knowledge, especially somewhat arcane knowledge, in manuscript) is the most powerful symbolic act of epistemological rupture possible. It could not fail for an educated person in the Ming to arouse associations of the great "burning of the books" instigated by the tyrant emperor Qin Shihuangdi (r.221-210 BCE), an act which had since antiquity symbolically stood for the potentially fragile nature of culture itself. Loss of the texts (more than loss of people, for example) was the loss of knowledge. Here I wish to argue that the refusal, even the destruction, of texts is also part of the history of those discourses which we now find it convenient to call "science", and that we should not see such acts as purely negative, but as themselves acts which constitute knowledge.

More puzzling still in this tale of texts in flames is the implied lack of filial piety in Wen Zhengming's actions, a challenge to one of the most hallowed norms of the Confucian social order. It is not simply the case that the son represented elite values to which the father did not aspire. Wen Lin was a holder of the highest-level bureaucratic degree, the *jinshi* (achieved in 1472), and he rose successfully to the rank of prefect of Wenzhou, thus doing much better out of the system than did his son, Wen Zhengming, who for all his fame as a writer and artist failed the examination hurdle repeatedly. There is no evidence of imputations of heterodoxy or lack of culture in any contemporary material concerning him, and indeed he was renowned for his active destruction, during his tenure as magistrate of Yongjia in Fujian province, of a range of "lascivious" (*yin*) shrines.[4]

The biographical text in question, "An Account of the Conduct of my Late Father", is by Wen Zhengming's own son (and thus Wen Lin's grandson) Wen Jia (1499–1582), and is intended to show the deceased in a good, not to say a laudatory, light. All biographers go to great lengths to establish Wen's becoming deference, as a model son, to his father's wishes in all other matters, so a point-blank refusal to study a subject of such cherished importance to that father, such that he destroys a library of books on the subject, looks like an act of eccentric unfiliality. To seek to explain it will involve an excursion into the field of the problematic status of geomancy, its texts, and the larger terrain of number in Ming culture.

"Geomancy" is the modern name given to what was in the Ming termed *di li* (literally "the principles of the earth") or *kan yu* ("cover and support" i.e. "heaven and earth").

The modern term *feng shui*, or "wind and water", was at this period something of a colloquialism, rarely used in the written language by the elite. It can even mean "vulgar geomancy", as opposed to "educated cosmology."[5] However the bulk of the evidence suggests that as a practice geomancy was eminently respectable, not a folk superstition but a discourse underpinned for the elite by the most prestigious text in the classical canon, the ancient and arcane "Book of Changes", or *Yi jing*.[6] Why then did Wen Zhengming so vehemently oppose learning the numerology which underlay it? Was it in fact prognosticatory numerology he opposed at all? Perhaps significantly, the explanation given in the biographical literature about him for the repeated failure of this paragon of scholarship to pass the examinations, is that he was too high minded to improve the geomantic prospects of his own dwelling at his neighbours' expense. A patron, Yu Jian is supposed to have built Wen Zhengming a "cottage", and on noticing that the river in front of the house was blocked and dry remarked; "'According to the theories of the geomancers, if this river flows through you will certainly pass the examinations. I will make it flow for you.' Wen refused the offer, saying, 'Opening the river channel will harm the people's cottages and dwellings. I would prefer it if it were not opened.' Yu subsequently regretted this, adding, 'That river ought to have flowed, and if I had not mentioned it to Mr Wen then he would have succeeded long ago.'"[7]

Wen is thus supposed altruistically, and with ultimately unfavourable consequences for himself, to have opposed the re-routing of a watercourse which damaged the geomantic aspect of his own house. There is no sense that he rejected the efficacy of the moves proposed by his patron. Indeed the anecdote (and the excuse) becomes meaningless if Wen was in fact a total sceptic about the subject. The question becomes more complicated when we look at the involvement of the Wen family in geomancy, and at hints of familiarity with the topic in his own writing. That at least one member of the family was prepared in the later sixteenth century to ascribe the family's rise from decent obscurity to geomantic success is suggested by a passage in the "Gazetteer of Tiger Hill" edited by Wen Zhengming's grandson, Wen Zhaozhi. This concerns the family patriarch Wen Hong (1426–1479), whose gaining of the *juren* degree in 1465 set the family on the ladder of bureaucratic success for the first time[8]; "He died and was buried at Huajing in Wuqiu district. He had previously sought for a burial place on Tiger Hill, but a geomancer (*shu zhe*) pointed out an old grave and said, 'If you obtain this you will be noble for generations'. His family thereupon wished to move him, and bought the site. Many people complained that this was pointless."[9]

The potentially transgressive act of moving an ancestor's corpse is presumably justified here by the success which ensued from it, since the Wen family had by the time of writing indeed been "noble for generations". Similarly the account of Wen Zhengming's own burial makes it explicit that divination was used in the choice of a spot and an auspicious day for the interment; "On the 15th day of the 10th month of the year after he died his coffin was placed on the plain of Huajingquao, and after divination of an auspicious time he was interred".[10]

Divination above all implied number. It could be argued that such a phrase, like the use of "he divined and dwelled" (*bu ju*) in the descriptions of the residences of other members of the Wen family contained in the family genealogy, was purely formalistic, but it does at least suggest that in the context of burial some application of the divinatory techniques

derived from numerology was relatively non-controversial, or else the behaviour of Wen Zhengming's family would have looked like a point-blank flouting of his wishes. There is a disjunction here between practice and textual knowledge, which can perhaps be approached through the key Ming discourse of "appropriateness". Just as certain types of picture were indispensable to an elegant interior at certain times of the year, and a vulgar solecism at others, so the seasonal, social, and life-cyclical appropriateness of actions has to be taken into account.[11] To impute inconsistency to Wen Zhengming for refusing *himself* to master numerological techniques, and then acquiescing in their deployment at family funerals or in the purchase of real estate, is to force an inappropriate epistemological category onto the material. Any truly global history of the relationship between texts and knowledge will have to be sensitive to such dimensions, and will have to account for difference at this level, if it is to avoid a naive Euro-centrism.

If geomancy was so respectable, could it therefore in fact be a distaste for what we now classify as "mathematics" which led to the transgressive incident of the book burning? This raises the question of how much mathematics, and what sort of mathematics, did the educated elite of Ming China know? One potential limitation on such study in the Ming lay in the relative rarity of extant editions of the early treatises on number, now accepted as part of the "history of mathematics". These are the books described in some contemporary bibliographies as dealing with *suan shu*, "procedures of calculation", to distinguish them from *shu xue*, or "numerology" (although the extent to which such a distinction is really meaningful in the Ming context seems debatable). This rarity of texts like the *Jiu zhang suan shu* "Nine Chapters on Mathematical Procedures" is asserted by the fifteenth century scholar Wu Jing, who adapted and republished the contents in 1450, and who has been quoted by modern Chinese writers wishing to explain what they see as the lamentable decline of mathematical knowledge in the Ming.[12] This seems an unduly mechanistic explanation, which only produces the further question of *why* were such texts rarely reprinted, at a time when publishing of texts of all sorts expanded dramatically. And Wu Jing's assertion may perhaps be better read as a common strategy of the Ming preface, a claim of rarity being itself part of the marketing of books. A text entitled *Jiu zhang suan fa*, "Nine Chapters on the Methods of Calculation", which may or may not be the same work, does in fact appear in one of the few catalogues we have of a private library of the period, in a section on "miscellaneous arts", which otherwise deals with texts on painting, the playing of chess, and the connoisseurship of embroidery.[13] The owner of this library was no very great scholar himself, but rather someone who aimed at a comprehensive coverage of the knowledge of the day. The existence of the "Nine Chapters" and its contents were certainly familiar to at least one very minor writer of a sixteenth-century *biji*, or "note-form" text, a man named Chen Quanzhi, who displays a passing knowledge of the techniques contained in the "Nine Chapters" in his collection of anecdotes written some time after 1547.[14] And at least one prominent literary figure born in the sixteenth-century did have in his library copies of almost all the early mathematical treatises, including *Jiu zhang suan shu*. This was Qian Qianyi (1582–1664), whose catalogue records at least fourteen titles (significantly some of them Jesuit texts) of this type under the larger category, *Li suan lei*, "Calendars and Calculation."[15] Perhaps mathematical books were not so rare after all (although a wider distribution of texts should not be taken necessarily to imply the presence of lots

of competent readers). The fact is that we are too little-informed about the distribution of books in the Ming to make categorical statements about the rarity or otherwise of a given body of knowledge, whether mathematical or of any other kind.

We are referring here in the main to what were ancient texts, books written many centuries before the Ming, and as such potentially collectable as part of antiquarian studies. Perhaps significantly, Qian Qianyi did not have a copy of what is now considered the major sixteenth-century mathematical treatise, the *Suan fa tong zong*, or "Comprehensive Compendium of Calculative Methods", discussed more fully below. Teleologically oriented accounts of the development of Chinese mathematics view the sixteenth century as pretty much a dark age, devoid of important advances in technique, a stagnant period between the achievements of the Yuan dynasty (1279–1368) and the introduction by Jesuit missionaries of Euclidean geometry and other European mathematical practices at the very beginning of the seventeenth century. It can certainly be shown that techniques laid out in an ancient text like the "Nine Chapters" were either no longer employed, or if transmitted in later texts, no longer fully understood.[16] Positivist writers, particularly those Chinese scholars writing within a Marxist framework, are distressed too by the continued admixture of what they see as "superstitious" elements in those Ming works on number as do exist, such as the *Suan fa tong zong*, of 1592, by Cheng Dawei (1533–1606).[17] Cheng's work opens with illustrations of the most ancient of numerological diagrams, namely the ancient and mysterious *He tu*, "River Chart" and the *Luo shu*, "Luo River Writing", and with a picture of the mythical culture-hero Fuxi inventing the eight trigrams of the *Book of Changes*.[18] The chapters of the main text are, according to modern Chinese scholars, ordered after the cosmological principles of the "Five Elements" (Metal, Wood, Water, Fire, Earth), and the introduction contains the uncompromising assertion that, "Nothing of number is not rooted in the pattern of the Changes".[19] This is material generally dismissed until recently as extraneous to the "real", mathematical content of the book. However such an attitude, and the view of sixteenth-century China in general as a mathematical "dark age", perhaps fails to take account of changes in the social role of the "study of number" (a somewhat literal working translation of *shu xue*), and of important changes too in the application of already existing techniques, particularly in the field of geometry and surveying. These were the cornerstones of Ming mathematics, the areas to which manuals and textbooks devote the greatest amount of space. This had always been the case. The first century AD text, *Jiu zhang suan shu*, "Nine Chapters on Mathematical Procedures", the most influential of all the early mathematical texts, has as its first chapter *Fang tian*, "rectangular fields", and gives the rules for calculating the areas of rectangles, trapezoids, triangles, circles, arcs and annuli. The third century "Sea Island Mathematical Manual" pitches its problems in more practical terms, and perhaps significantly proposes more sophisticated methods of calculating area which are based on a high vantage point, where "looking down on" and "taking control of" are of implicitly military significance (although the extreme rarity of this latter text in Ming times makes it unwise to argue that it was being actively deployed).[20] However these textbook formulae for the measurement of an area of land, which remained in use down to the Ming dynasty (and which were perhaps familiar to the numerologically literate father of Wen Zhengming, whether he learned them from books or not) contain no example of the measurement of an irregular quadrilateral, in actual practice one of the most common forms in which a field could

be found, particularly in south China. According to the modern scholar Kang Chao, the method empirically used between the Han and the Ming for determining the surface area of this shape involved a crude multiplication of half of the products of the addition of opposite pairs of sides. This method of treating a quadrilateral as if it were a rectangle, one "adopted by all ensuing mathematical works and land area conversion tables in China up to the 1578 land survey", always resulted in an overestimate.[21] However in 1578, under the direction of Grand Secretary Zhang Juzheng (1525–1582), orders were given to every magistrate to survey the land in his jurisdiction using a new method, called the *kai fang fa*, "creating a square method". In this method, any quadrilateral to be measured is first surrounded notionally by a rectangle, which can easily be measured accurately. From this total are then subtracted the areas of the right-angled triangles bounded by the notional exterior and the actual field boundary. These are also relatively easy to get right, and so an accurate area for the field can be arrived at. This relatively simple method, which needed no more tools than the chain or the surveyor's measuring rod, was still to prove too cumbersome in practice, and it was never used again after this survey, which lasted from 1578 to 1582.[22]

This has implications for the culture of the elite in general, or at least for that section of it which held or aspired to hold bureaucratic office. It is reasonable to assume that the magistrates who were ordered to undertake the land survey using this new technique had some understanding (to put it no more strongly than that) of what they were being ordered to do. As we have seen, no very high level of numeracy was involved. And there *is* evidence of mathematical interests on the part of individual members of the Ming elite, though it must be remembered that historians of science may be accepting as "mathematics" interests which may well have included the broader cosmological interests better discussed under the heading of "numerology". Tang Shunzhi (1507–1560)[23] and Gu Yingxiang (1483–1565)[24] are two well-connected members of the elite cited by Joseph Needham for their interests in this field.[25] However there was clearly a lot more activity in the period, even if it was not formally innovatory, than has hitherto been acknowledged, and the subject needs further research. For example, although he is hardly of significance to the history of mathematics, the amount of attention given to questions of land surveying by the famous official Hai Rui (1513–1587), especially during his tenures of office as magistrate of Chun'an and as prefect of Suzhou, speaks eloquently of the importance he attached to it, including to the technical aspects which are often deemed to be of little importance by humanistically trained modern sinologists.[26] His essay, "On the principles of the measurement of fields" (*Liang tian ze li*), takes the putative reader, presumably local officials like himself, step by step through all the procedures involved in practical cadastral surveying, beginning with the marking of a blank sheet of paper with the points of the compass, then continuing through the taking of measurements on the ground, their transference to graphic form, and the calculation of the area involved (hence the tax liability). There are also contained in this easy technical diagrams of the types of document which should result from the exercise.[27] Hai Rui's relative success in re-surveying Suzhou, achieved in the teeth of considerable opposition from landholding families, both with and without official status, was instrumental in convincing his superiors that a new cadastral survey of the entire empire would be feasible. Another pioneer of practical survey work was Luo Hongxian (1504–1564), a prominent thinker in the Wang Yangming tradition, as

well as a political associate of Tang Shunzhi, mentioned above.[28] Accounts of his survey of his native area of Jishui, Jiangxi province, undertaken for philanthropic reasons, make it clear that he was prepared to venture out into the fields himself to oversee the practical side of the operation.

A conscientious official was however far more likely to encounter opposition than support from the locally powerful, given that the main aim of the survey was less to increase state revenues from the land tax than it was to curb simple tax evasion and equalise the tax burden. The point was to ensure social harmony by preventing the rich from offloading the entire tax burden for a district onto the poor. But when Tu Long (1542–1605)[29] attempted to carry out the survey as magistrate of Qingbu, east of Suzhou, between 1578 and 1582, he was thwarted by the influence of the retired grand secretary Xu Jie (1503–1583),[30] whose family had huge quantities of land in the area, which they were unwilling to have recorded. Several members of Wen Zhengming's family occupied the kind of posts in which issues of land and the measurement of land were prime concerns; his father Wen Lin was successively both a magistrate and a prefect, his son Wen Peng (1497–1593) was magistrate of Pujiang in Zhejiang province, and his grandson Wen Yuanfa (1529–1602) was (most unusually) magistrate of the same county, during the very decades when the great cadastral survey was carried out. Wen Zhengming himself was associated with the tax-resister Xu Jie as part of his network of clients, providing a laudatory preface for his 50th birthday, and dedicating a picture to him in 1557.[31] Wen Yuanfa alludes in broad terms in his autobiography to the difficulties of collecting taxes in Pujiang, where "the land is barren and the people are devious and addicted to trickery". The local powerful families attempted to bribe him, then to threaten him, finally to impeach him, if he did not relax his rigour. However he survived the ordeal.[32]

For a man like Wen Yuanfa, and equally for his opponents among the wealthy of Pujiang, knowing how to calculate the area of a piece of land was, if not an everyday skill, then at least something not totally alien to their mental universe. It was necessary to the magistrate, for purposes of registration and taxation, but it was necessary too for the private landholder (often one and the same person at a different stage in his career), since land had to be bought and sold and divided equally among a number of heirs. There are some intriguing implications of this for an understanding of the visual culture of the Ming period. The art historian Michael Baxandall has argued that volume and the representation of volume was an issue among the producers and consumers of painting in fourteenth century Italy, men whose education was dominated by mathematical exercises in which gauging, the accurate measurement of volume, played a central role.[33] While it can in no sense be argued that any mathematical skill played an equally prominent part in a Ming landowner's education, the fact remains that if a Ming gentleman knew any mathematics at all it was likely to be a simple planar geometry. How such a way of looking at area, and in particular at a number of interlocking irregular planar fields, might play out in a manner of regarding space and pictorial space is a tantalising subject for further research.[34]

The wide dissemination of geometrical consciousness, to put it no more strongly than that, in the sixteenth century, does not seem to have been matched by any very great measure of prestige. Quite the reverse. Ming calculations were by and large put to practical ends, chiefly land measurement as we have seen but including other types of

sum necessary to a commercial society. This seems likely to have lead to the growth of a cadre of technical specialists, of whom we have but tantalising glimpses in the written record. One Ming land contract dated 1555 is signed by the "surveyor and cartographer" (*zhang liang hua tu ren*) responsible for the map attached to it.[35] Commercial growth and more importantly monetisation of the economy meant that more calculations were now being done more often by more people, and were often now beginning to involve the relatively newly diffused device of the abacus (the date if its invention remains unknown). The abacus is attested by pictorial evidence from 1436, and is mentioned in writings by a member of the elite for the first time in an essay of 1513.[36] It was becoming widespread at the point when Wen Zhengming's refusal to engage with the subject caused his father to burn his books on *shu xue*. The associations of the subject with mercantile operations made it all the more imperative for those who wished to be "pure and lofty" to distance themselves from the very concept of number, as "practical studies" came to be seen as irremediably tainted. There is certainly some sixteenth-century evidence for the patronage of mathematical knowledge within the distinctive merchant culture of Anhui province, and Cheng Dawei was certainly a merchant (which may explain the omission of his work from the library of a scholar like Qian Qianyi).[37] It was to his descendants a matter of pride that Wen Zhengming knew absolutely nothing about the practical details of land management, or of running a household. As his biography records:

> His nature was to detest mundane matters, and all family affairs were entrusted to Mme Wu [his wife], who managed everything to do with funerals and mournings, the marriages of sons and daughters, the building of houses and the purchase of real estate, all without bothering my father one jot. Thus the fact that my father was able to devote himself entirely to literature, and to follow his lofty and sublime resolution, was in truth due to the aid of my mother".[38]

Wen Zhengming's refusal to accept transmission of his father's numerological skills may reflect a change in the perception of what numbers could do, which was all the more vehement in that they were in fact absolutely central to the maintenance of the elite mode of life based on landholding.[39] Numbers now meant economics, and that was what a man like Wen Zhengming could not be seen to dabble in, hence his rejection of both the cosmological and practical implications of the subject.

This rejection of mathematical knowledge, and of the texts which embodied it, thus becomes a historically specific and quite recent event, to do with the maintenance of elite status at a time of growing social mobility and fluidity due to commercial expansion, rather than a timeless characteristic of "the Chinese scholar". This has a bearing on the reception of western pictorial conventions in China in the late Ming, as initially noted by Benjamin March, although he sees the antipathy to geometry as being a constant in Chinese culture. The point may rather be that western conventions were introduced to China precisely at a point when, for other reasons, calculation had uncomfortable resonances.[40] Even in the mid-Ming the wider elite may not have been so absolute. The skilled Ming eye, looking down on the landscape perhaps from an eminence, may well have been able to add an appraisal of area, of the differing proportions of irregularly shaped water and disparate parcels of land to other types of appreciation.

The disdaining of mere technical competence by an elite growing in size, and keen to distance itself from the broader mass of the prosperous, had a possible source in that elite's

constricted access to political power. The number of bureaucratic offices, and the number of examination places which gave access to them, was not expanded during the Ming in line with the growing population, or with the growing segment of the population which was able to turn economic capital into cultural capital through the mechanisms of private tutors and academies. There were more educated men in the Ming than the system allowed jobs for.[41] Arguably, to maintain status this educated elite sought a number of strategies to assert its distinctiveness, one of which might well have been the exaggeratedly fastidious disavowal of subjects (like the study of number) seen as having "practical" connotations, links to trade and commerce especially.

This fastidiousness then had other ramifications. One of these bore upon the whole question or role of pictures in the transmission of technical knowledge. This is a large subject on which work is continuing at present, and conclusions of too firm a nature would be premature. However, some of the broad outlines of the question may be discerned by a brief examination of two texts of the late Ming, one of which is illustrated and one of which is not.

To begin with the unillustrated text first. This is the *Xiu shi lu* ("Records of Lacquering", preface dated 1625) a book entirely without pictures which deals with the manufacture of luxury objects of material culture decorated in various techniques, all of which share a common dependence on the use of the sap of the lacquer tree, *Rhus verniciflua*.[42] This text is obviously of great value to art historians for the unparalleled richness of its material on the techniques and categories of this important Ming craft. However, as well as enlightening us about lacquering *per se*, it can be used to shed some light on how "knowledge" and "technique" were understood in late Ming China, and in particular on the status of the textual transmission of technical knowledge at that time. It is worth remarking here that it shares the characteristic of being unillustrated with the majority of the "handbooks to elegant living", which burgeoned in numbers at the end of the sixteenth-beginning of the seventeenth centuries.

As with the works on *shu xue*, "the study of number", the cosmological schemata of the *Book of Changes* provide the ordering principle for "Records of Lacquering", as well as much of its technical terminology. There should be no need by now to "apologise" for these "non-rational" elements as if they were merely superfluous accretions on what is essentially a work of technical knowledge within a positivist epistemological discourse. Rather it would be widely accepted that it is these very roots in the *Book of Changes* which form a testimony to the work's positioning within norms which were constitutive of the possibility of all sorts of knowledge in the late Ming. The five elements, four phases of time, and the notion of "principle" (*li*), are all parts of the correlative cosmology which underpinned various areas of knowledge at the time. They were as widely accepted, and as necessary to the act of thinking, by the Ming elite, as were macrocosmic ideas, or the Platonic conception of ideal form, to their contemporaries at the other end of Eurasia.[43] Nor are they dismissable as part of "folk belief". We cannot strip them out of the text to reach some notional "real" meaning. Terms which appear everywhere in the text of "Records of Lacquering", like *xiang*, "figure", *fa*, "model" or "pattern", and even *wen*, often translated as "decoration" but which means at the same time "marking" and which has a semantic range right out into concepts like "culture" itself, are essential to educated understanding of the *Book of Changes*, and in particular its claims to provide

an interpretative framework through which all knowledge can be ordered.[44] Any Ming reader with the literacy skills to comprehend *Xiu shi lu* would also have possessed the cultural competencies to read its relationship to a text which possessed unequalled prestige as a foundation for knowledge. These are mainstream ideas, expressed in well-known terminology. They are also ideas which galvanised early western scholars on China, and it is no coincidence that it was the *Book of Changes* which drove Leibniz' fascination with the new accounts of his Jesuit contemporaries, and which he consciously appropriated in his work on the binary system.[45] Only recently, however, has this interest been reaffirmed, and scholarship on the topic of the *Changes* taken to a new level of sophistication.

This text of the "Records of Lacquering" is not an instructional manual. Towards the end of the Ming period, what can be called a "commoditization of culture" coupled with an expansion of printing activity meant it was now possible to buy books purporting to offer accounts of many types of knowledge previously only transmitted orally between actual practitioners.[46] The circulation of texts separated knowledge from practitioners, and inserted that knowledge into a field of market relationships. This phenomenon, tied to the explosion in the quantity of printing in the later sixteenth century, was to become at least as widespread in China as it was in contemporary Europe. The *Xiu shi lu* inhabits a new and still tentative epistemological space, where a gap has opened up between knowing how something is done and actually knowing how to do something. These are two very different kinds of "knowing". It creates knowing subjects who cannot themselves make a lacquer box, subjects who are like those created by the new genre of published merchant route books, which enable you to know the road from A to B without travelling it, or how an elegant garden should be judged, without knowing how to build or irrigate.

Many of these texts have been "rediscovered" in this century, the *Xiu shi lu* itself surviving only in Japan, and have been enthusiastically embraced in twentieth century China as providing material for an alternative and comforting reading of the past in which science and civilisation in China are much more intimately linked than was hitherto realised. It is traditional to ascribe the neglect of these texts, whether about lacquer making or about mathematics, to the humanist culture's disdain for the technical and the practical, the classical pedant's contempt for the artisan. Yet if as I have argued these books in the late Ming embody knowledge itself as a commodity, and are at least as much about consumers as they are about makers (and no-one would suggest that you could use these books unaided to make or do *anything*), then the problem of their existence and flourishing at this precise period assumes a slightly different form. It becomes rather a question of why they exist at all, why humanist disdain at least temporarily allowed a space for them to be brought into being. And why at that moment? Orientalist habits still incline us towards seeing "Chinese thought" as an essentially invariant episteme, when perhaps more attention to cleavages and sites of dissonance over definitions of knowledge would direct attention fruitfully towards historical shifts, moments of contest and appropriation, which we have tended too often to view as mere ripples on an ocean of deep and habitual currents.

To take now an example from among the riches of Ming illustrated books, it is at this same period that we get the first *hua pu*, "pictorial albums", of which the most widely circulated was probably *Gu shi hua pu*, "Master Gu's Pictorial Album" of 1603.[47] This contains reproductive illustrations of one hundred and six paintings by famous artists of

the past and present, with a text commenting on each. The avowed aim of the work is to help the collector avoid forgeries, and to acquaint him with the style of artists whose genuine works are so rare that only the very wealthiest might ever see them. The pictures are referenced back only to other pictures, with the question of content laid to one side. These are not pictures in the sense that the illustrations to a drama or a mathematical manual are pictures, and we can sense here a cleavage opening up, one which was to be fully enforced, at least at the level of aesthetic theory, by the later seventeenth century.

This cleavage between illustrations and pictures was, I would argue, less visible in the fifteenth century, when an elite artist might without discomfort occasionally produce a pictorial scroll depicting coastal defences.[48] Even in the sixteenth century, the evidence of the collected writings of a scholar like Gui Youguang (1507–1571)[49] is that an interest in maps, in visible representations of the perceived world, remained active. The difference between a "map" and a "landscape painting" was at this point a rather fluid and unregulated one. From around 1600 this boundary began to be more forcefully interpreted. It perhaps became visible first in practices like those of book production, where the *absence* of pictures from a text the *Xiu shi lu*, or from one like Wen Zhenheng's, "Treatise on Superfluous Things", with its precise attention to the details of objects in a way which seems to cry out for illustration, is very striking. It was eventually made manifest at the level of explicit aesthetic theory, in the much quoted distinction made by the painter Gong Xian (1599–1689) between *tu*, "pictures/illustrations" and *hua*, "painting". In James Cahill's translation, the relevant passage reads:

> In ancient times there were *pictures (t'u)* but no *paintings (hua)*. Pictures depict objects, portray people, or transcribe events. As for paintings, the same isn't necessarily true of them. [To do a painting], one uses a good brush and antique ink, and executes it on a piece of old paper. As for the things [in a painting], they are cloudy hills and misty groves, precipitous boulders and cold waterfalls, plank bridges and rustic houses. There may be figures [in the painting] or no figures. To insist on a specific subject or the representation of some event is very low class.[50]

Mere "pictures" included maps and diagrams of all sorts, including arrangements of what we would now call "text".[51] They carried meaning and were about something; they were of lower status. Paintings existed in a realm of self-referentiality, within a self-consciously historical tradition, where subject matter takes second place to style as the object of aesthetic contemplation. A distrust of mimesis, of mere transcription of reality, had long existed in Chinese aesthetic theory on visual art, but in the early seventeenth-century it become normative to an even greater degree than hitherto.[52] "Art" in the late Ming sense was far from being an "art" in the sense of a set of transmissible and repeatable techniques (its etymological roots in the European languages, as in e.g. the pseudo-Ciceronian *ars memoriae*). The key word at issue here was *fa*, "method" or "technique", as in *Suan fa tong zong* "Comprehensive Compendium of Calculative Methods", a work written by a man, Cheng Dawei, who came from the merchant background which the landowning elite affected to distance itself from. Techniques were not for gentlemen. Class factors, always encoded in a language of moral worth, now defined in relation to the art of painting who was an artist and who was not, rather than style or subject matter alone. As "pictures" and "painting" came to be two separate things, and as the great apparatus of Ming land registration for taxation purposes, based on a huge collection of pictorial

representations, fell into decline in the later sixteenth century, we can see what is almost an epistemological shift in the way knowledge can be conveyed. This shift is noticeable in the writings, as well as in some of the practices, of the Ming educated elite, and the extent of its hold beyond that narrow group must remain debatable. But broadly speaking, the link between "textuality" and "power", which the critic Margaret Iversen sees as a major theme of Norman Bryson's *Word and Image* would therefore take a substantially different form in Chinese practice from that seen in early modern Europe. In Ming China, "power" adheres more to the "autonomous painterly signifier", the marks of the brush, rather than the thing figured by them.[53]

Wen Zhengming's refusal, in the late 15th century, to accept the transmission of his father's numerological techniques, and the burning of the books to which this refusal gave rise, can thus be read on one level as a straw in the wind, a very early indicator of an early modern Chinese "crisis of representation", which became acute only a hundred years later. To certain Ming thinkers, "number" was diminished by its constant propensity to act as a stable referent, guaranteeing fixed meanings for visible phenomena. It thus claimed to figure the world at a level unworthy of serious philosophical or aesthetic attention. It could not be accepted as "real", any more than the mountain seen out of the window was able to guarantee the value of "realism" in the mountain inscribed by the artist's hand on a surface of paper or silk.

School of Oriental and African Studies, London

NOTES

I am grateful to Karine Chemla, Francesca Bray, Georges Métailié and Michael Lackner for helpful suggestions on an earlier version of this paper.

1 Ho 1991, 506–519; 1985, 6–7.

2 Goodrich and Fang 1976, 2:1471–1474; Clunas 2004.

3 Wen Jia 1987, 1622. A modern collection of biographies of great prognosticators through Chinese history includes Wen Lin, with the statement that, "although he was broadly learned in geomancy and prognostication he was a particular specialist in the *Book of Changes* and in numerology". Yuan Shushan 1948, 24–5. The only source cited is Wen Zhengming's early 18th century *Ming shi* biography.

4 Carlitz, 1997, 631.

5 It is used this way in one mid-sixteenth century text, Chen Quanzhi 1985, *juan* 3:15b. For a fuller account, plus bibliographical references, see Clunas 1996, 177–193.

6 A convenient introduction is Smith 1991, 131–172.

7 Wang Shizhen 1987, 1626.

8 This is the date given for Wen Hong's passing of the examination in both Wen Han, 17th c, *Li shi ke mu zhi*, 19b, and in Zhang Jianhua 1986, 90. However Wen Zhaozhi 1578 gives the date not as Chenghua *yi you* (= 1465), but as Chenghua *yi wei* (= 1475).

9 Wen Zhaozhi 1578, *juan* 1:21a.

10 Wen Jia 1987, 1624.

11 Clunas 1997b, 25–76. An important step in getting beyond a western episteme in the study of a range of Chinese practices linked by the discourse of "propensity" is Jullien 1995.

12 Du Shiran 1989, 10.

13 Gao Ru 1957, 167–168. The edition is said to have been published in Nanjing, with commentary by Yang Rong and Meng Ren, the latter of whom is unrecorded, and the former of whom may be the minor official recorded in Taiwan zhongyang tushuguan 1987, 469 as active in the mid-fifteenth century. The section on 'Calendars and Number' (Gao Ru 1957, 155–156) contains only three entries, all almanac-related. Nor are there any books of number in the 'Minor Studies' (*Xiao xue*) section of the 'Classics' Division.

14 Chen Quanzhi, *juan* 5:10b–11a.
15 Qian Qianyi 1935, *juan* 2:57–58.
16 Chemla 1996, 97–120, also personal communication from Dr Chemla.
17 E.g. Yan Dunjie and Mei Rongzhao 1990, 26–52. I am very grateful to Dr Karine Chemla for supplying a copy of this article.
18 Saso 1977, 399–416 discusses the multiple meanings of this magical diagram, a circular pattern of dots, believed to have been voided out of the Yellow River on the back of a magical beast in remote antiquity. See also Clunas 1997b, 105–108.
19 Yan Dunjie and Mei Rongzhao 1990, 46. Interestingly, Cheng's sequence of the Five Elements accords with neither of the two standard sequences, that of 'mutual production' and 'mutual conquest'. Smith 1991, 371.
20 Needham 1959, 25; Swetz, 1992, 9.
21 Chao 1986, 70–71, cites a land-deed of 81 CE, which shows that peasants already at that time measured a quadrilateral by adding together the lengths of opposite sides, and then multiplying those together.
22 Chao 1986, 71–3. More detail is given by the same author in Zhao 1980, 46–51. Karine Chemla informs me that the term *kai fang fa* is more generally understood within the history of Chinese mathematics to refer to a method for the extraction of roots.
23 Goodrich and Fang, 2:1252–6.
24 Taiwan zhongyang tushuguan 1987, 958.
25 Needham 1959, 51–2.
26 On Hai Rui see Goodrich and Fang 1976, 1:474–9. But see also Huang 1981, 138–141, where a significantly different point is argued.
27 Hai Rui 1962, 1:190–201.
28 Goodrich and Fang 1976 1:980–984. For an inscription by Luo Hongxian on a painting by Wen Zhengming see Wen Zhengming 1987, 1666, but this does not prove they were acquainted.
29 Goodrich and Fang 1976, 2:1324–1327.
30 Goodrich and Fang 1976, 1:570–5766.
31 For the former see Jiang Zhaoshen 1977, 244; for the picture see Liu Jiuan 1997, 219.
32 Du Lianzhe 1977, 7.
33 Baxandall 1988.
34 Some of these issues are tentatively addressed at greater length in Clunas 1996.
35 Zhang Chuanxi 1995, 2:1108.
36 Needham 1959, 75–76.
37 Yan Dunjie and Mei Rongzhao 1990, 28.
38 Wen Jia 1987, 1623.
39 This argument depends on Du Shiran 1989, who also associates the trend with the philosophical ascendancy of Wang Yangming, although there are weaknesses in this, given that Luo Hongxian (mentioned above) was a prominent Wang Yangming disciple.
40 March 1931, 137. For a fuller discussion see Cahill 1982, 70–105.
41 Wakeman 1985, 1:92–94.
42 Wang Shixiang1983 is the first modern edition of this rare text. A fuller analysis than that given here is contained in Clunas 1997a.
43 On the discursive reach of cosmological ideas of this type in the late Ming, see for example Henderson 1984, 132–136. For just one example of recent work tending to undermine older teleological accounts of the "rise of science" in the West, see Grafton 1991.
44 Peterson 1982, 110–111.
45 Kuhn 1973.
46 This forms one of the themes of Clunas 1991.
47 There is a facsimile edition, Gu Bing 1983. For a more extensive discussion, see Clunas 1997b, 138–146.
48 Zhang Jianhua 1986, 265.
49 Goodrich and Fang 1976, 1:759–761.
50 Cahill 1989, 8.
51 Lackner 1992.
52 Ho 1976.
53 Iversen 1990, 213. Bryson 1981.

REFERENCES

Baxandall, Michael. 1988. *Painting and Experience in Fifteenth-Century Italy: A Primer in the Social History of Pictorial Style*, 2nd ed. Oxford and New York: Oxford University Press.

Bryson, Norman. 1981. *Word and Image*. Oxford: Oxford University Press.

Cahill, James. 1982. *The Compelling Image: Nature and Style in Seventeenth-Century Chinese Painting.* Cambridge MA. and London: The Belknap Press of Harvard University Press.

Cahill, James. 1989. "Types of Artist-Patron Transactions in Chinese Painting." In *Artists and Patrons: Some Social and Economic Aspects of Chinese Painting*, edited by Chu-tsing Li. 7–20. Lawrence: Kress Foundation Department of Art History, University of Kansas and the Nelson-Atkins Museum of Art in Association with University of Washington Press.

Carlitz, Katherine. 1997. "Shrines, Governing-Class Identity and the Cult of Widow Fidelity in Mid-Ming Jiangnan." *Journal of Asian Studies* 56: 612–40.

Chao, Kang. 1986. *Man and Land in Chinese History: an Economic Analysis*. Stanford: Stanford University Press.

Chemla, Karine. 1996. "Reflections on the World-Wide History of the Rule of False Double Position. or: How a Loop was Closed." *Centaurus* 16: 97–120

Chen Quanzhi. 1985. *Lian chuang ri lu*, 2 vols. Shanghai: Shanghai shudian.

Clunas, Craig. 1991. *Superfluous Things: Material Culture and Social Status in Early Modern China.* Cambridge: Polity Press.

Clunas, Craig. 1996. *Fruitful Sites: Garden Culture in Ming Dynasty China*. London: Reaktion Books.

Clunas, Craig. 1997a. "Luxury Knowledge: The *Xiushilu* ('Records of Lacquering") of 1625." *Techniques et cultures* 29: 27–40.

Clunas, Craig. 1997b. *Pictures and Visuality in Early Modern China*. London: Reaktion Books.

Clunas, Craig. 2004. *Elegant Debts: the Social art of Wen Zhengming, 1470–1559*. London: Reaktion Books.

Du Lianzhe. 1977. *Mingdai zizhuan wenchao*. Taibei: Yiwen yinshuguan.

Du Shiran. 1989. "Mingdai shuxue ji qi shehui beijing." *Ziran kexueshi yanjiu* 8.1: 9–16.

Du Xinfu. 1983. *Mingdai banke zonglu*, 8 vols. Yangzhou: Jiangsu Guangling guji keyinshe.

Gao Ru. 1957. *Baichuan shu zhi.* Beijing: Gudian wenxue chubanshe.

Goodrich, L. Carrington and Fang Chaoying. 1976. *Dictionary of Ming Biography*, 2 vols. New York and London: Columbia University Press.

Grafton, Anthony. 1991. *Defenders of the Text: The Traditions of Scholarship in an Age of Science, 1450–1800.* Cambridge, MA: Harvard University Press.

Gu Bing. 1985. *Gu shi hua pu*. Beijing: Wenwu chubanshe.

Hai Rui. 1962. *Hai Rui ji*, 2 vols. Beijing: Zhonghua shuju.

Henderson, John B. 1984. *The Development and Decline of Chinese Cosmology.* New York: Columbia University Press.

Ho, Peng Yoke. 1991. "Chinese Science: the Traditional Chinese View." *Bulletin of the School of Oriental and African Studies* 54.3: 506–19.

Ho, Peng Yoke. 1995. *Li, Qi and Shu: An Introduction to Science and Civilization in China.* Seattle and London: University of Washington Press.

Ho, Wai-kam. 1976. "Tung Ch'i-ch'ang's New Orthodoxy and the Southern School Theory." In *Artists and Traditions: Uses of the Past in Chinese Culture*, edited by Christian F. Murck, 113–130. Princeton: Princeton University Press.

Huang, Ray. 1981. *1587, A Year of No Significance: The Ming Dynasty in Decline* (New Haven and London: Yale University Press.

Iversen, Margaret. 1990. "Vicissitudes of the visual sign." *Word & Image* 6.4:212–216.

Jiang Zhaoshen. 1997. *Wen Zhengming yu Suzhou huatan*. Taibei: Guoli gugong bowuyuan.

Jullien, François. 1995. *The Propensity of Things: Towards a History of Efficacy in China*. New York: Zone Books.

Kuhn, Margarete. 1973. "Leibniz und China." In *China und Europa: Chinaverstandnis und Chinamode im 17. und 18. Jahrhundert*, 174–194. Berlin: Verwaltung der Staatlichen Schlösser un Gärten.

Lackner, Michael. 1992. "Argumentation par diagrammes: une architecture à base de mots. Le *Ximing* (l'*Inscription Occidentale*) depuis Zhang Zai jusqu'au *Yanjitu*." *Extrême-Orient—Extrême-Occident* 14; 131–168.

Liu Jiuan. 1997. *Song Yuan Ming Qing shuhuajia chuanshi zuopin nianbiao*. Shanghai: Shanghai shuhua chubanshe.

March, Benjamin. 1931. "Linear Perspective in Chinese Painting." *Eastern Art: An Annual Published for the CAA*, 3: 113–39.

Needham, Joseph, with the collaboration of Wang Ling. 1959. *Science and Civilisation in China. Volume 3: Mathematics and the Sciences of the Heavens and the Earth.* Cambridge: Cambridge University Press.

Peterson, Willard J. 1982. "Making Connections: Commentary on the "Attached Verbalizations" of the *Book of Change.*" *Harvard Journal of Asiatic Studies*: 67–116.

Qian Qianyi. 1935. *Jiang yun lou shu mu*. Shanghai: Congshu jicheng, *ben* 33.

Saso, Michael. 1977. , "What is the *Ho-t"u*?" *History of Religions* 17: 399–416.

Smith, Richard J. 1991. *Fortune-Tellers and Philosophers: Divination in Traditional Chinese Society*. Boulder/San Francisco/Oxford: Westview Press.

Swetz, Frank J. 1992. *The Sea Island Mathematical Manual: Surveying and Mathematics in Ancient China.* University Park, PA: Pennsylvania State University Press.

Taiwan zhongyang tushuguan. 1987. *Mingren zhuanji ziliao suoyin*. Beijing: Zhonghua shu ju.

Wakeman, Jr, Frederic. 1985. *The Great Enterprise: The Manchu Reconstruction of Order in Seventeenth-Century China*, 2 vols. Berkeley/Los Angeles/London: University of California Press.

Wang Shixiang. 1983. *Xiu shi lu jie shuo*. Beijing: Wenwu chubanshe.

Wang Shizhen. 1987. "Wen xian sheng zhuan." In *Wen Zhengming ji*, 2 vols. Edited by Zhou Daozhen, 1624–1629. Shanghai: Shanghai guji chubanshe.

Wen Han. 17th c. *Wen shi zu pu*. Suzhou: n.pub.

Wen Jia. 1987. "Xian jun xing lüe." In *Wen Zhengming ji*, 2 vols. Edited by Zhou Daozhen, 1618–1624. Shanghai: Shanghai guji chubanshe.

Wen Zhaozhi. 1578. *Huqiu shan zhi*. N.p: n.pub.

Wen Zhengming. 1987. *Wen Zhengming ji*, 2 vols. Edited by Zhou Daozhen. Shanghai: Shanghai guji chubanshe.

Yan Dunjie and Mei Rongzhao. 1990. "Cheng Dawei ji qi shuxue zhuzuo." In *Ming Qing shuxue shi lunwen ji*, edited by Mei Rongzhao, 26–52. Nanjing: Jiangsu jiaoyu chubanshe.

Yuan Shushan. 1948. *Zhongguo lidai buren zhuan*. Shanghai: n.pub.

Zhang Chuanxi. 1995. *Zhongguo lidai qieyue huibian kaoyi*, 2 vols. Beijing: Beijing daxue chubanshe.

Zhang Jianhua. 1986. *Ming Qing Jiangsu wenren nianbiao.* Shanghai: Shanghai guji chubanshe.

Zhao Gang. 1980. "Ming Qing diji yanjiu." *Zhongyang yanjiuyuan jindaishi yanjiusuo jikan* 19: 37–59.

GIOVANNA C. CIFOLETTI* [1]

THE ALGEBRAIC ART OF DISCOURSE ALGEBRAIC *DISPOSITIO*, INVENTION AND IMITATION IN SIXTEENTH-CENTURY FRANCE

ABSTRACT

This paper is part of a research project devoted to inquiring into the connections between humanist rhetoric, dialectics and the teaching of the liberal arts on the one hand and the developments occurring in sixteenth-century algebra on the other. In this larger context, we have found that, especially in France, symbolic algebra as we know it grew out of mathematics within humanistic culture, and particularly out the interaction between mathematics and the disciplines of the text. This is what transformed algebra after its importation from Italy (and the German countries), so that it became what we call symbolic algebra.

The paper discusses first the way in which the disciplines of the text modified the way of writing algebra. Secondly, it looks at how one sixteenth-century author theorized mathematical creation in "literary" terms, as invention within imitation. To look at sixteenth-century algebra in this way necessitates our own reflection on the relationship between innovation and tradition or, to use sixteenth-century terms, invention and imitation.

1. MATHEMATICS IN HISTORY AND SIXTEENTH-CENTURY HUMANISM

In the last few years I have been working on the connections between humanist rhetoric, dialectics and the teaching of the liberal arts on the one hand and the developments occurring in sixteenth-century algebra on the other[2].

I have become convinced that many important discoveries of sixteenth-century algebra grew out of other traditions of that era. No matter how translatable these results may be in our mathematics, they remain connected not only to the mathematical tradition but also to other contemporary intellectual traditions and practices. These links are much more than merely stylistic; I am not just suggesting that sixteenth-century mathematics is expressed in a style that is related to sixteenth-century scientific and non-scientific literature. So much is obvious, in any case, and would be admitted even within the field of the history of mathematics. I am suggesting a deeper connection. Mathematicians saw their inventions as contributions to mathematics when they were also consciously transferring to algebra results, meanings and strategies of conceptualization from other disciplines. The perspective of the time was crucial in determining mathematical innovation, i.e. the actual mathematical results, because it promoted explicit "contaminations" with the disciplines of text[3]. These contaminations, typical of the French context, transformed

K. Chemla (ed.), History of Science, History of Text, 123–135.

algebra after its importation from Italy (and the German countries), so that it became what we call symbolic algebra[4]. Thus, it is this wider contemporary context that is the best depositary of the meaning of those results, as opposed to the mathematical, purely disciplinary tradition alone. Furthermore, there has never been a unitary mathematical tradition, but rather a vast field of previous mathematical activity within which the various schools have carved out their own versions of the past[5].

To look at sixteenth-century algebra in this way necessitates a reflection on the relationship between innovation and tradition or, to use sixteenth-century terms, invention and imitation. For sixteenth-century authors, scientific as well as literary, were often very aware of the need for this kind of reflection.

First, I will discuss how symbolic algebra as we know it grows out of mathematics within humanistic culture, and particularly out the interaction between mathematics and the disciplines of the text. Secondly, I will look more closely at the way in which one sixteenth-century author theorized mathematical creation in "literary" terms, as invention within imitation.

2. THE ALGEBRAIC *ORATIO*

For the humanists, any teaching was intended to prepare the future *orator*. This was the very meaning of humanistic education, according to authorities like Cicero and Quintilian. For those who took Horace as their main authority, the stress was not on the *orator*, but on the *poeta*. So, while *oratio* was the general term indicating structured prose, and often meant "text", the *ars poetica* concerned not just poetry but any kind of oral and written communication[6]: the term applying to both prose and poetry was *opus*. In any event, from a humanistic point of view, any work of science, medicine, law, had to meet certain organizational criteria as a text. While the existence of these common criteria is not unique, the role they had is. It is the case that for humanists what had been a crucial feature of science according to Aristotle, its logic, was entirely embedded in these organizational textual criteria, given the vast program of integration of rhetoric and logic started at the end of the fifteenth century[7].

Another level of research was linguistic. At that time more and more people, including scientists, were learning classical Latin and studying classical texts in order to write in neo-Latin as a second language. Another tendency was "vernacular humanism", encouraging the development of literature, including scientific literature, in the vernacular. Both these lines of research heightened the need of conscious choices in style on the writing of science[8].

Within this general and multifarious late humanistic movement, we shall focus on the French algebraic tradition, i.e. that group of French humanists who imported Italian and German algebra and transformed it into a new humanistic discipline: these are Jacques Peletier du Mans, Pierre de la Ramée, Jean Borrel, Guillaume Gosselin. For this group, the *form* and the *matter* of the algebraic text are identified with one another, insofar as algebra is seen not only as a doctrine to be presented, but also as a formal tool to organize reasoning. Algebra not only *used* rhetoric in order to be constituted into a discipline, but *replaced* rhetoric to shape the new kind of scientific *oratio* or text. This later development

would lead us too far, and it is not my intention to develop it here[9]. Rather, I want to look briefly at two of the places in which algebra was connected to texts and to text theories: in the writing of algebraic books and in the theorizing of the evolution of mathematics. The author to be considered will be Jacques Peletier du Mans. He was a well known sixteenth-century scientific writer, particularly in algebra, and also an eloquent author and poet who went so far as committing his theories and practices to a treatise, the *Art poétique*[10].

The two questions I asked earlier take now a more precise form which is the following. In the first part of this paper we shall see how Jacques Peletier du Mans, in his 1554 *L'Algèbre*, develops a new approach to a certain kind of algebraic problem, while insisting that the new solution is a change in the *disposition* of the art, i.e. a change in the rhetorical structure in a sense that we shall see further.

In the second part of this paper, we shall take into consideration Peletier's theory of texts, which is less technical mathematically, but just as technical in poetic theory and dialectics.

3. PELETIER'S ALGEBRA: *INVENTIO*

Peletier published *L'Algèbre* in Lyon in 1554 (Jean de Tournes), fifteen years after the publication of Cardano's *Practica Arithmeticae* (Milano, 1539) and nine years after the publication of the latter's *Ars magna* (Nürnberg, 1545). One might say that Peletier's algebra derives from Cardano and from Stifel, author of *Arithmetica integra*, appeared in Nürnberg already in 1543. For Peletier gives his text the same structure as Stifel's work, although in a much reduced form, while the most important difference is that Peletier's book is entirely devoted to algebra, like Cardano's *Ars Magna*. In fact it is the first printed book on algebra in French and the richest among vernacular books on algebra. Now, we must remember that the selection of topics in any oral or written *oratio* was defined, in dialectics, by the so-called *invention*, whereas *disposition* (or *collocation*) is related to the notion of judgment and order, i.e. the reasoning developed by ordering the "invented" topics. For, all these terms were used in the sixteenth century to indicate specifically the *argumentation* present in the *oratio*: the Orator must build his reasoning starting from the *loci* obtained by the invention, while the construction of the structure of the argument is called disposition. In fact, in the work that we shall examine in the last section, *L'Art poétique*—published a year after *l'Algèbre* and by the same printer—Peletier has different definitions of invention and disposition, more apt to the case of poetry, although poetry is to be taken in its larger sense, as literary work on general topics. "Invention est un dessein provenant de l'imagination de l'entendement, pour parvenir à notre fin. Disposition, est une ordonnance et agencement des choses inventées." (*L'Art poétique*, p. 19)

Going back to *L'Algèbre*, if we describe in some detail the list of topics treated by Peletier, we intend not only to give a sense of the content of Peletier's book, but also to suggest the selection Peletier made among the major algebraic techniques and concepts according to a general plan. In fact, the very specialization of Peletier's book allowed him

to emphasize both the techniques, e.g., calculations of monomials, and the concepts, e.g. what he defines as the two main notions of algebra, equations and extractions of roots of binomials, which correspond, for us, to the solutions of first and second degree equations. The title of the section on equations is, significantly, *De l'Equation, partie essentielle de l'Algèbre*. Peletier writes:

> L'équation et l'Extraction de Racines, sont deux parties de l'Algèbre, en lesquelles consiste toute la consommation de l'Art. (...) équation donc, est une égalité de valeur, entre nombres diversement denommés. Comme quand nous disons 1 Ecu valoir 46 Sous. (*L'Algèbre*, p. 22)[11]

But an equation is built on a problem or, in Peletier's terms, on a question, by identifying unknown numbers through known numbers. To put a problem in the form of an equation is a particular technique:

> Premièrement, il s'entend assez, que les nombres exprimés en Questions sont ceux qui nous guident: et par l'aide desquels nous decouvrons les Nombres inconnus. Il faut donc en cette Question proposée, que par le moyen de 46, Nombre exprimé, se trouve celui que je demande. (*ibidem*)[12]

A little further, Peletier gives "*la grande règle générale de l'algèbre*":

> Au lieu du Nombre inconnu que vous cherchez, mettez 1 R : Avec laquelle faites votre discours selon la formalité de la Question proposée: tant qu'ayez trouvé une Equation convenable, et icelle réduite si besoin est. Puis, par le Nombre du signe majeur Cossique, divisez la partie à lui égalée: ou en tirez la Racine telle que montre le Signe. E le Quotient qui proviendra (si La Division suffit) ou la Racine (si l'extraction est nécessaire) sera le Nombre que vous cherchez. (Peletier J. 1554, p. 46)[13]

We have to assign a symbol to the unknown number, then capture the *form* of the question, i.e. to interpret the problem in function of the unknown and its powers, and set the problem as an equation, then modify the latter, for instance by a reduction, in order to obtain a "good" equation. To interpret this passage in our terms[14], we should remember that a "good" equation, according to Peletier, is for instance, in symbolic algebra, of the form $x = c$ for the first degree and $x^2 = bx + c$ for the second, i.e. it has a coefficient for the highest power equal to 1. Therefore, if one starts with an equation of the form $ax = c$, or $ax^2 = bx + c$, the procedure is to divide the second member by a. In the first case we are done, in the second case we obtain the value of x through an algorithm called by Peletier "extraction of the root of the cossic number"[15]. We can start with any sort of question; the art is to find an equation expressing it, and transform it until it becomes a "good" equation, i.e. an equation whose solution formula is known. For instance, we should first operate algebraic sum and possible divisions. Peletier is the first among his contemporaries to have insisted on this part of algebraic theory. The operation by which a problem is given good form and expressed as an equation, from then on was considered typical of algebraic art of thinking.[16]

Peletier defines algebra as dialectics, i.e. as the art of reasoning. This appears clearly in another passage of the same text, where the goal of algebra is to teach one how to think, *à discourir*:

> L'algèbre est un art de parfaitement et précisément nombrer: et de soudre toutes questions Arithmétiques et Géométriques de possible solution par nombres Rationaux et Irrationaux. La grande singularité d'elle, consiste en l'invention de toutes sortes de lignes et superficies, où l'aide des nombres rationaux nous défaut. Elle apprend à discourir, et à chercher tous les points nécessaires pour résoudre une difficulté: et montre qu'il n'est chose tant ardue, à laquelle l'esprit ne puisse atteindre, avisant bien les moyens qui y adressent. (Peletier, J. 1554, p. 1)[17]

By developing algebra according to the model of dialectics, Peletier intends not only to simplify demonstrations, but also to establish an art of thinking—algebra thus becoming this true art of thinking. We should notice also another use of the word *invention*, here in connection with the finding of algebraic solutions with geometrical meaning, use which we tend to ascribe to Viète. But, more generally, we see here that Peletier's explicit goal is to represent algebra as the art of finding a presentation of problems that can make it possible to solve them all. Peletier is not the first to use the slogan, again usually attributed to Viète, that algebra teaches us how to solve all problems[18].

4. PROBLEMS IN SEVERAL UNKNOWNS. *DISPOSITIO*

If putting a problem into the form of an equation is considered by Peletier to be an operation in itself, and also the strong point of his method, then solving a problem in several unknowns should take his method to task. Peletier innovates with respect to all his predecessors precisely on this point. In fact, he innovates with respect to Stifel, and does so by making use of Cardano's *Ars Magna*, the most recent major work on algebra. Peletier gives the following definition of second unknowns, which he calls, following Stifel, *secondes racines*:

> Les Racines Secondes viennent en usage quand deux nombres ou plusieurs se proposent, entre lesquels ne se fait aucune comparaison expresse par addition, multiplication, division ou proportion, par différence, ni par Racine: qui sont les cinq manières de comparer les nombres ensembles. Desquelles la proportion est la principale, car les autres seules bien souvent n'excusent pas l'usage des Secondes Racines. (*L'Algèbre*, p. 96)[19]

Now, this passage clearly retraces the lines of a similar passage in chapter XI of the *Ars magna*, and shows us that in fact Peletier also adopted Cardano's theoretical point of view with respect to the second unknown:

> We have been using [two] unknowns, and no relation has been assumed between the two numbers at the beginning, either by way of addition or subtraction, multiplication, division, ratio, or root—for numbers may be related in these five ways. But if one [of these relations] exists, there is no necessity for [using] a second unknown, for the problem can be solved by one unknown. (*Ars Magna*[20] ch. X, p. 79)

We may note in passing that we have here a sixteenth-century version of a definition of functional relation, called *comparatio* in the original, insofar as Cardano mentions all determinate operations in a general way. Of course, the use of the *secundae quantitates incognitae* allows one to perform, through the solution of the system, just those operations that would be indefinite.

Now let us look at Peletier's version of the problem that opens the chapter entitled "*De secunda quantitate incognita*" of Cardano's *Ars magna*.

Cardano had written:

> Up to this point we have been treating of new discoveries quite generally. Now something must be said about certain individual types. It frequently happens that we must solve a given problem by using two unknown quantities. There follows an example of this which we could otherwise explain only with difficulty. Three men have some money. The first man with half the others' would have had 32 *aurei*; the second with one-third the others', 28 *aurei*; and the third with one-fourth the others', 31 *aurei*. How much had each? (*Ars Magna* ch. IX[21])

Cardano is not ready to give a general rule. The next sentence gives the procedure of assignment of the unknowns:[22]

> We let the first unknown thing be the first man's share, the second unknown thing the second man's share; thus for the third man there will be left 31 *aurei* minus one fourth of the thing and one fourth of the quantity. (*ibidem*)

Cardano pursues his calculations on this basis for a couple of pages. As we can see the third unknown is defined in terms of the other two. Peletier repeats Cardano's statement and gives a first version of the solution following Cardano's reasoning, adding only a few explanatory comments. At the end he adds some very inspiring remarks:

> En cet Exemple, j'ai suivi de point en point la proposition et la disposition de Cardan. En quoi j'ai été aussi long comme lui, et un peu plus clair. Et n'eut été pour montrer la singularité de l'Algèbre, et comme elle gît en discours, et comme elle exerce les esprits: j'eusse laissé cette explication sienne, laquelle il appelle facile, pour en mettre une autre qui s'ensuit, de notre dessein. (*L'Algèbre*, p. 110)[23]

Cardano's solution gives Peletier the opportunity to state that algebra is explicit reasoning. For our purpose it is particularly important to notice that Peletier uses the word *disposition* for Cardano's reasoning.

In the next sentence Peletier starts his own solution procedure: "Le premier a 1R. The second, 1A. Le tiers, 1B." According to the first hypothesis[24], $1\text{R} + 1/2\,(1\text{A} + 1\text{B}) = 32$. By reduction and "transposition",

I: $2\text{R} + 1\text{A} + 1\text{B} = 64$. Peletier calls this the *first equation*. According to the second hypothesis, $1\text{A} + 1/3\,(1\text{R} + 1\text{B}) = 28$. Thus,

II: $1\text{R} + 1\text{B} + 3\text{A} = 84$, *second equation*. According to the third hypothesis, $1\text{B} + 1/4\,(1\text{R} + 1\text{A}) = 31$, hence

III: $1\text{R} + 1\text{A} + 4\text{B} = 124$, *third equation*. We now **add the third to the second equation**, and we get the *fourth equation*. i.e.

IV: $2\text{R} + 4\text{A} + 5\text{B} = 208$. We now can **subtract the first equation from the fourth**, and we get the *fifth equation*

V: $3\text{A} + 4\text{B} = 144$.

We can now take another direction, and **add the first and the second equation**, we get

VI: $3\text{R} + 4\text{A} + 2\text{B} = 148$, which is the *sixth equation*. By **adding the first and the third equation** we get the *seventh equation*

VII: $3R + 2A + 5B = 188$, whereas **by adding the sixth and the seventh equation** we get $6R + 6A + 7B = 336$, which is the

VIII: *eighth equation*. Let us now multiply the third equation by 6, getting IX: $6R + 6A + 24B = 744$, i.e. the *ninth equation*. Given that the first two terms of equation eight and nine are equal, we can write $17B = 408$, and we obtain the third number, $B = 24$.

Now, let us go back to the fifth equation[25], $3A + 4B = 144$: by subtraction we get $3A = 144 - 96$, we obtain $3A = 48$, i.e. $A = 16$, the second number.

Finally, because of the first equation, $1R + 1/2\,(16 + 24) = 32$, hence $1R = 32 - 20 = 12$, the first number.

Peletier concludes: "*Ce discours est trop plus facile que l'autre. Mais il fait bon voir deux inventions en même intention*." I.e., " This talk is too much easier than the other one. But it is good to see two inventions with the same intention."

This procedure might not look easy or brief to us, but it was remarkably short if compared to Cardano's procedure. What did change? The main innovation is in the introduction of as many symbols as there are unknowns in the problem: here the unknowns in the problem coincide with the unknowns of the equations. Furthermore, Peletier is very systematic in structuring the solution through the various transformations of the first equations, those which, for us, belong to a "system." Then he uses, as Cardano does, the method of addition and subtraction of equations. However, he does this in a different way than Cardano because he does not introduce the arbitrary coefficients which we perceive as artificial and apparently were considered somehow *ad hoc* also at Peletier's time. Peletier's innovations make the solution simpler and shorter or, according to Peletier's stylistic values, "short and clear" and the notation points to the path later taken by Jean Borrel[26], Guillaume Gosselin[27] and then François Viète: the use of a sequence of letters of the alphabet.

What Peletier has done is change a procedure. But he explicitly describes this change as a change in the text. By establishing a specific order for the procedure (immediately represented by ordinal numbers) as well as a general method (attributing different symbols to all the unknowns of the problem) and by transforming the coefficients according to the order established, Peletier intervenes in the typical structuring of algebraic reasoning. He looks at the problem not only as a particular problem but as a pattern, as *suggested by its form*. As it is discussed elsewhere in this volume, this is not specific to sixteenth-century symbolic algebra, but also applies to Mesopotamian and Chinese algebra. What is characteristic here is the form of the solution: not only is the solution procedure given in steps, thereby offering a pattern for the solution of other cases, but also the steps depend on the use of symbols, and are presented in an explicit order denoted by roman numerals. We are therefore in the presence of a text which has been *constructed* as a "second level text," in the same sense as we speak of equations in their general form. Jacob Klein, writing in the first half of last century, has formulated the conjecture that this character of second level text was embedded in the very notion of second order numbers. Klein's point, as I understand it, is that the passage from the "monads" of the Greeks to the more general numbers developed in early modern arithmetic (thanks to the Arabs and to Diophantus) is not a linear development, but a change in perspective. Given that the Renaissance world

rests upon the ancient one, modern numbers would be first of all numbers of numbers, i.e. not numbers about the external world but about human tradition of numbers. But I would argue that general numbers are just the counterpart of (in other words, come with) equations and problems in their general form. In fact, maybe the change in problems is more fundamental that the change in numbers. What we have seen is a way in which problems in their general form come from a theoretical concern about the order and the role of the scientific text.

We shall come back to this peculiarity of algebra in the next section. Here, it is important to notice that Peletier considers his innovation in the treatment of problems in several unknowns as a contribution to the *disposition* of algebra:

> Si la Disposition est celle qui donne dignité aux choses, et si la forme est celle qui fait être une chose celle qu'elle est, je me promets de m'être ici tellement acquitté. (*L'Algèbre*, proème, f. 9)[28]

Peletier does not give us here an explanation, but an example of his notion of disposition, and the example is his entire work, *L'Algèbre*.

The necessity to make explicit the grammar (at least the orthography) and the rhetoric implied in his enterprise shows that to write an algebraic text in French and in accordance with the values of contemporary humanistic culture was, in Peletier's own eyes, an entirely new project. He was in fact one of the main promoters of new scientific writing, among authors who were particularly aware of the requirements and potentialities of printing.

Like the ancient *Orator*, the sixteenth-century writer should not miss out on the opportunities offered by his *oratio*[29]. The first function of the rhetorical art in algebra was, according to Peletier, to provide the tools for rendering a text efficient, and the standards by which to judge it. Thus he used rhetorical technical terms to articulate the organizational criteria for texts that did not belong to the classical mathematical tradition.

5. THE TRUE SENSE OF TEXT AND THE CASE OF ALGEBRA. *IMITATIO*

These examples give us an idea of what the practice of *invention* and *disposition* in algebra meant for Peletier: they allowed him as an algebraic author to write his text, but also were part of the art of reasoning that algebra was supposed to supply.

We can now see more closely Peletier's wider theory of texts. Here appears another quite original contribution of Peletier's thought, which at the same time shows that his humanistic culture gave him an excellent standpoint for a sophisticated view of the matter. Peletier's work *L'Art poétique*, published in Lyon in 1555, a year after *L'Algèbre*, is entirely centered on the notion of *imitatio*. In clarifying his outlook about the imitation of the classics, Peletier makes explicit a theory of the development of scientific disciplines. Already in *L'Algèbre* Peletier gave an account of the history of algebra in terms of a cumulative process, where invention does not start with a person, but with a people. According to his theory, each language is the depositary of the creativity of a people, and is apt to develop the notions which have been the most familiar to that particular people, such as the juridical terms for Latin. In this way, a people is disposed to develop an art further. Authors intervene at this level, and they can be considered inventors because they

play the crucial role of transmitting knowledge. What an author does, says Peletier, is to give newness to old things and authority to new things. As Peletier writes in a long section on imitation,

> L'office d'un Poète est de donner nouveauté aux choses vieilles, autorité aux nouvelles, beauté aux rudes, lumière aux obscures, foi aux douteuses. (*L'Art Poétique*, p. 24)[30]

Peletier is elaborating on Horace's theory of imitation: each author extracts a matter (*materia*: a topic, an idea) from the common knowledge and gives it an original status[31]. For Peletier, this practice of transmission is explicitly connected to the invention of new words[32]. First of all, the stake is the creation of a scientific French. But, more generally, in order to fulfill his task the poet must appropriate "old things" to his language, i.e. to translate them to the new language and to the new time. A people is the creator of an art, and its language its depository, but the plasticity of language makes it possible for a people to introduce new words first to translate from another language then, with luck, practice and new ideas, to create a new art.

This view establishes a chain of authors who are readers of previous authors, where not only imitation is always creation, but any creation is also a translation and an imitation. In our words, a text is a function of the reader, and if the reader is in turn an author, the text will be transformed into another text. There is a use of the text, and this is particularly important not only because it illuminates what scientific creation and invention meant for Peletier, but also because, in the case of algebra, the reader is supposed to be active and creative, is supposed to *use* a procedure applying it to different cases, in fact is supposed to transform a problem into an equation which will yield the solution.

This doctrine by Peletier is meaningful for us in two ways: firstly, it establishes Peletier at the origin of the French algebraic tradition, for after Peletier, Guillaume Gosselin in 1577 and 1583, then Viète in 1591 and Descartes, to mention only the most significant, took the same direction as Peletier, writing texts that are highly sophisticated in terms of language, notation and organization of the material. Algebra became with them an important Latin discipline, whereas, according to them, it had been nothing but an obscure technical calculation in Arabic and vernacular languages. Viète and Descartes neglected to underline their dependence even on their most recent sources.

Secondly, Peletier's lead was followed by later authors insofar as they adopted a similar invention and disposition in their books of algebra, but also stressed the role of algebra as a method, as an art of discovery or of invention, and finally, in Descartes, as the basis for *mathesis universalis*. Algebra as a second level text was born in these writings.

What Peletier did not manage to transmit to his successors, was his concept of transmission, his sense of text. He saw texts as belonging to a chain of texts. For history can be constructed and reconstructed, and this may lead to skepticism or relativism. But the texts exist. Insofar as they are well-built they are a perennial challenge to imitation, the only bridge, though unstable, to the understanding of the past.

I would like to conclude with a passage from Peletier's Latin translation of his *L'Algèbre*[33]:

> Those who have thought a bit more deeply about the beginning of things do not ascribe the invention of disciplines to a single person, but acknowledge that it is for the germs of disciplines as for the virtues located in souls and for the sparks that the eternal spirit

> lights in us, that is to say, that they emerge from a common genius. The destiny of things through time is controversial: destiny, through its vicissitudes, offers such fecundity of things that those who take advantage of present beauty without maintaining any deeper memory not only refer everything they receive to their century but also can not conceive of any transformation in the future. This calamity occurs in an era that we know all too well: in people's minds there is no expectation of betterment. And there where once the arts flourished, people imagine that they are not being reborn but starting from scratch. Because of the perpetual vicissitudes of things, those who do not have a broad outlook consider as unique to the present anything that they did not receive directly from the hands of the ancestors. (Peletier J., 1560, preface).

With this passage, Jacques Peletier stated some of the questions of philosophy of history and history of science that we are now trying to face, such as a philosophy of history which can include a notion of the past and of the development of knowledge. Other historians of the Renaissance have stressed the fact that sixteenth-century people outlined problems of language, interpretation and history that we are still elaborating on: I am thinking of works as different as Donald Kelley's, Richard Waswo's and Ian Maclean's[34]. It has become acceptable, in intellectual history, to state these similarities as an attempt to make explicit the intellectual expectations inspiring our interpretation of ancient texts. However, in the history of science, we are accustomed to oppose innovation and tradition. With Peletier we can see that the invention of a work of art or science does not occur in opposition to the past, but grows out of it, through imitation. His theories make us aware of the necessity to refine our knowledge of the techniques of writing and imitating used in a more or less conscious way by mathematicians of the past.

NOTES

*1 I wish to thank Hannah Davis Taïeb for her precious help in the process of transforming my original paper into an English text.

2 For a more in-depth discussion of these themes, see (Cifoletti G.C. 1992) forthcoming as a book. A global presentation of the French algebraic tradition in the sixteenth-century appeared in (Cifoletti G.C. 1995).

3 By this name I refer to the *artes* known in the Middle Ages as *sermocinales*, the arts of discourse, that is the set of the three arts of the trivium, grammar, rhetoric and dialectic or logic, once they got deeply modified by the humanistic reform. In fact, Lorenzo Valla and other humanists could have collected them under a similar heading for two main reasons: because of the addition of philology and historical studies and because the role of the *orator* had as much to do with writing as with speaking.

4 I call symbolic algebra: 1) a symbolism that allows the treatment of general equations, which is to say different letters for the unknowns on the one hand and the coefficients (or known terms) on the other; 2) the operations with monomials; 3) the determination of solution formulas for equations of the third and fourth degree. And then further, with respect to the theory of equations: 4) the elaboration of techniques for the reduction of equations to some standard cases; 5) the determination of relations between coefficients and roots; 6) the determination of number of roots. All these aspects were not treated in an exhaustive way, but at least partially studied much before the sixteenth-century, in the Arabic texts, and even Descartes and Fermat did not reach exhaustiveness on all the points.

5 This creation of genealogy is crucial to the self-definition of each school with respect to the others. For the foundation of a European tradition of mathematics, see Goldstein, J. Gray, J. Ritter (eds.) 1996.

6 From the rich literature on this topic, I will cite only Gordon, A.L., 1970.

7 Again the choice of works on this topic is great, among them: Jardine, L. 1983, as well as K. Meerhoff, K. 1986. My argument here is somewhat similar to what Lisa Jardine developed with respect to Francis Bacon in Jardine, L. 1974.

[8] For the transformation of scientific writing in the Renaissance, in connection with the development of Neo-Latin, see for instance the essays by A. Blair, I. Pantin and J. Peiffer in: Chartier, R., Corsi, P. (eds.) 1996: 21–42, 43–58 and 79–93. On the question of language in France, see Castor G. and Cave T. 1984.

[9] I have developed this point in (Cifoletti, G.C. 1995), and, in connection with Descartes, in chapter 7 of (Cifoletti G. C. 1992). For, Descartes' *Regulae* are a very accomplished version of the sixteenth-century dream of proposing algebra as the new rhetoric. Algebra, here, is at the same time the method of solution of equations and a form of symbolic calculation concerning equalities. On Descartes' dependence on the French algebraic tradition, see also Cifoletti G.C. 1998.

[10] *L'Art Poétique*, Lyon, Jean de Tournes, 1555.

[11] Peletier was an innovator and a theoretician of texts at yet another level: orthography. I have decided to keep it as much as possible in this first quotation, so that the reader will notice some of the peculiarities of Peletier's orthographic rules. My translation is: "Equation and root extraction are two parts of algebra, in which consists all the refinement of the art. (. . .) Therefore, the equation is an equality of values between two numbers denominated differently. As when we say 1 pound is worth 46 pence."

[12] "First, it is clear that numbers expressed in questions are what guides us, by the help of which we discover unknown numbers. In the question asked here, we must find the number asked by means of 46, number expressed."

[13] My translation is: "Instead of the unknown number you are looking for, put 1 R: by means of this develop your reasoning according to the formality of the question asked, until you have gotten an appropriate equation, reduced if necessary. Then, divide the part put equal to it by the number of the highest cossic sign [a cossic sign being the unknown and its powers, cossic algebra being the four operations and extractions of roots applied to cossic numbers or monomials, my remark], or extract the root according to the sign. And the quotient resulting from this (if division is enough) or its root (if the extraction is necessary) will be the number you are looking for."

[14] Peletier does not use letters for the coefficients, which are taken as positive. Furthermore, he deals almost exclusively with first and second degree equations and positive roots.

[15] In this case, we get the whole second member (a binomial) as a power, of which we extract the square root. This algorithm gives rise to the solution formulas for second degree equations, very similar to the procedures present in the Mesopotamian tradition.

[16] Descartes' *Regulae* is a major later example.

[17] "Algebra is an art of numbering in a perfect and precise way, and of solving all the algebraic and geometrical questions which have a possible solution by rational or irrational numbers. Its great singularity is in the invention [finding] of all sort of lines and surfaces, where the help of rational numbers is missing. It teaches how to reason, and to look for all the points necessary to solve a difficulty: it shows that there is nothing so arduous that the mind cannot reach it, if we decide properly the means useful for it." (My translation.)

[18] It was in fact a *topos* of algebra. Aside from its presence in the Arabic tradition, one should notice that more recently Cardano had made use of it in his *Practica Arithmeticae*. It is therefore particularly interesting to see the same *topos* transformed in the new context.

[19] "Second roots are used when two numbers—or more—are proposed, between which there is no explicit comparison by addition, multiplication, division or proportion, nor by difference or root, which are the five ways of comparing numbers among themselves. The proportion is the most important of these, because the others very often do not justify the use of second roots." (My translation.)

[20] We use here the English translation (Cardano, G. [1545] 1968: 71).

[21] See also Girolamo Cardano [1663] 1966, IV:241.

[22] Here I translate from Latin, because Witmer adopts his own notation directly at this point, and this would obscure matters in our context.

[23] "In this example, I followed point by point Cardano's proposition and disposition. By doing this, I have been as long as him, and a bit clearer. If it had not been for showing the singularity of algebra, and how it consists of reasoning and how it exercises the mind, I would not have mentioned this explication of his, which he calls easy, to replace it by another, the following, which is mine." (My translation).

[24] Here I shall adopt, unlike Peletier, our signs for $+$, $-$ and $=$, where he has p., m. and *égal*. I also introduce the brackets, for typographical convenience.

[25] Peletier writes "3A p 3B étaient égales à 144": the second term is a misprint and should be 4B; the next passage does not carry over the mistake.

[26] Jean Borrel, also known as Buteo, was another Renaissance mathematician devoted to the revival of the classics. He was the author of another algebraic text, the *Logistica* (Buteo 1559), and had a theoretical dispute with Jacques Peletier on the latter's translation of Euclid.

[27] Gosselin published three mathematical works, all related to algebra, the most important of which was his *De Arte magna* (Gosselin 1577). His project, only partially realized, was to recover Diophantus' *Arithmetic* in the terms of the most recent developments in arithmetic, i.e. algebra. Gosselin translates Tartaglia but does not mention Bombelli.

[28] "If Disposition is what gives dignity to things, and if the form is what makes a thing what it is, I consider I have done my duty." (My translation.).

[29] For the discussion of the possibilities of printing by Erasmus, see J. Chomarat 1981, 387–393.

[30] "The duty of a poet is to give novelty to old things, authority to new ones, beauty to the rough, light to the obscure, trust to the uncertain." (My translation).

[31] See Jacques Peletier du Mans 1545, f. 11v.

[32] "Quant à l'innovation des mots, faudra aviser si notre Langue en aura faute: et en tel cas, ne se faut feindre d'en former de nouveaux. Un mot bien déduit du latin aura bonne grâce, en lui donnant la teinture françoise." (*ibidem*, p. 37) i.e., in my translation: "As to innovation in words, it will be necessary to see whether our language will lack of them: in that case, it is not necessary to form new ones. A word well derived from Latin will be fine, if one gives it the French color".

[33] Peletier's Latin version has a few variants, including the entire preface. This is my translation.

[34] See these works also as an illustration of contemporary understandings of sixteenth-century's "disciplines of the text": Donald R. Kelley 1970; Richard Waswo 1987; Ian Maclean 1992. Waswo's can be considered the most radical understanding: one of his "principal theses" is that some very serious doubts about the dualistic model (correspondence between discrete signs and discrete ideas), along with alternatives to it, became articulate and well diffused in the culture of fifteenth and sixteenth centuries.

REFERENCES

Borrel, Jean (also known as Buteo), 1559. *Logistica*. Lyon: Rouillé.

Cardano, Girolamo. 1539. *Practica Arithmeticae*. Milan: B. Calusci.

Cardano, G. 1545. *Ars Magna*. Nürnberg: Petreius.

Cardano, G. [1545] 1968. *The Great Art or The Rules of Algebra*, translated and edited by T. Richard Witmer. Cambridge (Mass.): M.I.T. Press.

Cardano, G. [1663] 1966. *Opera Omnia*. Lyon: I. Huguetan et Ravaud. Reprint Stuttgart-Bad Cannstatt: Frommann.

Castor, G. and Cave, T. (eds.). 1984. *Neo Latin and the Vernacular in Renaissance France*. Oxford: Oxford University Press.

Chartier, R., Corsi, P. (eds.) 1996. *Sciences et Langues en Europe.* Centre A. Koyré (E.H.E.S.S., CNRS, MNHN). Paris: Ecole des Hautes Etudes en Sciences Sociales.

Chomarat, J. 1981. *Grammaire et rhétorique chez Erasme*. Paris: Les Belles Lettres.

Cifoletti, Giovanna C. 1992. *Mathematics and Rhetoric. Jacques Peletier, Guillaume Gosselin and the Making of the French Algebraic Tradition* (Princeton PhD dissertation). Ann Arbor: U.M.I.

Cifoletti, G. C. 1995. "La question de l'algèbre. Mathématiques et rhétorique des hommes de droit dans la France du 16e siècle." *Annales. Histoire, sciences sociales* 6: 1385–1416.

Cifoletti G. C. 1998. "Descartes et la tradition algébrique (XVe–XVIe siècles)." In *Descartes et le Moyen Age*, edited by Joël Biard and Roshdi Rashed: 47–56. Paris: J. Vrin.

Goldstein, C., Gray, J., Ritter, J. (eds.) 1996. *L'Europe mathématique. Mathematical Europe*. Paris: Editions de la Maison des Sciences de l'Homme.

Gordon, A. L. 1970. *Ronsard et la Rhétorique.* Genève: Droz.

Gosselin, Guillaume. 1577. *De Arte magna*. Paris: Beys.

Jardine, Lisa 1974. *Francis Bacon: discovery and the art of discourse.* Cambridge: Cambridge University Press.

Jardine, L. 1983. "Lorenzo Valla: Academic Skepticism and the New Humanistic Dialectic." In *The Skeptical Tradition*, edited by M. Burnyeat: 253–286. Berkeley: University of California Press.

Kelley, Donald R. 1970. *Foundations of modern historical scholarship: language, law and history in the French Renaissance.* New York: Columbia University Press.

Maclean, Ian. 1992. *Interpretation and meaning in the Renaissance. The case of law* Cambridge, New York: Cambridge University Press.

Meerhoff, Kees. 1986. *Rhétorique et poétique au XVIe siècle en France: Du Bellay, Ramus et les autres*. Leiden: Brill.

Peletier du Mans, Jacques. 1545. *L'Art Poëtique d'Horace, traduit en vers françois* par Jacques, recognu par l'auteur depuis sa première impression. Paris: Vascosan.

Peletier, J. 1554. *L'Algèbre.* Lyon: Jean de Tournes.

Peletier, J. 1555. *L'Art Poétique*. Lyon: Jean de Tournes.

Peletier, J. 1560. *De Occulta parte numerorum.* Paris: Cavellat.

Waswo, Richard. 1987. *Language and meaning in the Renaissance*. Princeton, NJ: Princeton University Press.

PIERRE-SYLVAIN FILLIOZAT

ANCIENT SANSKRIT MATHEMATICS: AN ORAL TRADITION AND A WRITTEN LITERATURE

ABSTRACT

The originality of India's mathematical texts is a consequence of the refined culture of the scholars who produced them. A few examples display clearly some salient features of the habits of exposition and the methods of thought of ancient and medieval Indian mathematicians. The attitude of the traditional learned man, called "pandit", is the same, whether he works on literary or technical matter. Propensity to orality, use of memory, brain work are his specific qualities. Composition in verse form, use of synonymous words, metaphorical expression, which are unexpected processes for the exposition of technical matter, have been the rule in all the vast Sanskrit mathematical literature. The present article analyses a technique of memorization of the text of the Vedas, the earliest exposition of geometry rules in the context of Vedic rites of building brick altars, the numeration system, the arithmetical and geometrical concept of square.

India's mathematical texts are highly original in more than one way: not only do they embody original research methods, they also emerge from a refined culture with which every scholar was deeply imbued. When one examines a Sanskrit scientific text, it is proper not to forget the Indian environment of the author, even if the contents of his work consist of positive knowledge existing also in other civilizations at the same time. The purpose of this article is to draw attention to the intellectual background of ancient and medieval mathematicians. A salient feature is that in ancient times they appear to have been working in an environment of pure orality with original intellectual activities, memorization etc. In the period that followed they worked in an environment where the habits of orality were maintained and the tool of writing was used. We will consider briefly these intellectual habits and give a few examples of mathematical expositions.

Indian mathematicians of ancient and early medieval periods whose works have come down to us can be defined as Sanskrit pandits (*paṇḍita* "learned man").[1] What this means is that they underwent an intensive training in handling Sanskrit language, literary composition and in practicing several intellectual disciplines dealing with language, literature and reasoning. Whatever may be his special field, every pandit possesses a common stock of knowledge in grammar (*vyākaraṇa*), exegesis (*mīmāṃsā*) and logic (*nyāya*). Thanks to that, he is endowed with a sharp linguistic awareness, equipped with a rich store of potential resources of expression, and has at his disposal a number of heuristic tools. In addition the pandit's education aims at improving the natural faculties of speech and memory.

K. Chemla (ed.), History of Science, History of Text, 137–157.

In the intellectual history of India there has been a period, namely that of the beginnings of Vedic civilization, during which writing was totally ignored, or was not used, even though its existence was known. No written document pertaining to Vedic or Brahmanic civilization has come down to us from before the 3rd century B.C.[2] The Vedic corpus consists of texts spread out over a very long span of time, perhaps a thousand years, prior to that date. No trace of the use of writing appears in the form of that large corpus of texts. No clue is found to suggest that it may have been committed to writing before the Christian era.

This voluminous mass of texts has been transmitted from generation to generation, without being written. One has to admit that there has been oral transmission and a very efficient means of preservation. The whole corpus, divided in several schools, has survived thanks to the memory of professionals trained for that purpose. We know the techniques of recitation, memorization and conservation, from manuals composed in different periods on that subject and by observing the pandits who still practice this art today.

1. TECHNIQUES OF ORAL TRANSMISSION

From the remotest times the recitation of Vedic hymns was a very important activity. The efficacy of any rite was considered to be dependent on the perfect pronunciation of the accompanying prayer or formula. The Brahmans, who were the repositories of the knowledge of the religion and who had to transmit it to later generations, concentrated their attention and efforts on the conservation of the Vedic texts. Developing techniques of accurate recitation was initially a necessity. In later times, even when writing became a common practice, it was not forgotten, nor neglected. It is still practiced in modern times. It is the profession of a few Brahmans who retain its antic form and still refuse to use the help of writing and other tools offered by modern technology. It is a very elaborate art. It includes eleven modes of recitation of the same text. One purpose of this multiplication is the conservation of the text: if a mistake is committed in one recitation, without being repeated in another, comparison between the two recitations helps to detect and to correct it.

One mode of recitation is taken as basic. It presents the text in the form it takes when words are bound together according to the rules of phonetic euphonic combination (*saṃdhi*) of Sanskrit. This recitation is called *saṃhitā-pāṭha* "continuous recitation". It is followed by a recitation where a pause is marked after each word, even after some grammatical components inside the word. This suppresses the euphonic combinations and restores the original form of the words. It is called *pada-pāṭha* "recitation word by word". In the next recitation, the executant groups words by pairs and proceeds word after word: ab, bc, cd . . . It is called *krama-pāṭha* "recitation step by step". The phonetic combinations are carried out inside each pair. As each word appears twice, successively at the end of a pair and at the beginning of the next one, it comes with its two forms, combined and free. This recitation is the sum of the previous two. The fixation of the recitation word by word is ascribed to Gārgya and Śākalya, the recitation pair by pair to Bābhravya and Gālava. These names are known to Pāṇini. We have also other reasons to think that this fixation was anterior to the famous grammarian, who is tentatively placed in the 5th century B.C.

In their turn these three modes of recitation have been taken as the basis for other modes in which the pairs are repeated with an inversion. For example, the *jaṭā-pāṭha*

"recitation in the form of meshes", which may be pre-Pāṇinian, takes a pair of words from the *krama-pāṭha*, repeats it in reverse order, then in the original order, and then goes on to the next pair, up to the end of a stanza: ab, ba, ab; bc, cb, bc; . . . The *dhvaja-pāṭha* "flag-recitation?" joins the first and last pair of the stanza, then the second and the one before last, and so on, until the last and first pair are reached respectively: ab, yz; bc, xy; . . . ; yz, ab. The most complicated formula is the *ghana-pāṭha* "dense recitation" which takes a pair, reverses it, takes it again with the addition of a third word, reverses the sequence of the three words, repeats them in their original order, goes to the next pair, and so on up to the end of the stanza: ab ba abc cba abc; bc cb bcd dcb bcd; . . .

Example drawn from *Ṛgveda* 8.100.11: *saṃhitā*

devī́ṃ vā́cam ajanayanta devā́s tā́ṃ viśvárūpāḥ paśávo vadanti |
sā́ no mandréṣam ū́rjaṃ dúhānā dhenúr vā́g asmā́n úpa súṣṭutaítu ‖

(Gods engendered the goddess Speech. Creatures of all forms speak her. May this amiable Speech, a cow giving her milk of force and vitality, well-praised, come to us.)

Pada:

devī́m | vā́cam | ajanayanta | devā́ḥ | tā́m | viśvá-rūpāḥ | paśávaḥ | vadanti |
sā́ | naḥ | mandrā́ | íṣam | ū́rjam | dúhānā | dhenúḥ | vā́k | asmā́n | úpa | sú-stutā | ā́ | etu ‖

Krama:

devī́ṃ vā́cam | vā́cam ajanayanta | ajanayanta devā́ḥ | devās tā́m | tā́ṃ viśvárūpāḥ | viśvárūpāḥ paśávaḥ | viśvárūpā iti viśvá-rūpāḥ | paśávo vadanti | vadantī́ti vadanti | . . . úpa súṣṭutā | súṣṭutaítu | súṣṭutéti sú-stutā | aítu | etv ity etu |

Jaṭā:

devī́ṃ vā́caṃ vā́caṃ devī́ṃ devī́ṃ vā́cam | vā́cam ajanayantājanayanta vā́caṃ vā́cam ajanayanta | ajanayanta devā́ devā́ ajanayantājanayanta devā́ḥ | devā́s tā́ṃ tā́ṃ devā́ devā́s tā́m | tā́ṃ viśvárūpā viśvárūpās tā́ṃ tā́ṃ viśvárūpāḥ | viśvárūpāḥ paśávaḥ paśávo viśvárūpā viśvárūpāḥ paśávaḥ | viśvárūpā iti viśvá-rūpāḥ | paśávo vadanti vadanti paśávaḥ paśávo vadanti | vadantī́ti vadanti | . . . úpa súṣṭutā súṣṭutópópa súṣṭutā | súṣṭutaítv etv ā́ súṣṭutā súṣṭutaítu | súṣṭutéti sú-stutā | aítu | etv ity etu | . . .

. . .

Ghana:

devī́ṃ vā́caṃ vā́caṃ devī́ṃ devī́ṃ vā́cam ajanayantājanayanta vā́caṃ devī́ṃ devī́ṃ vā́cam ajanayanta | vā́cam ajanayantājanayanta vā́caṃ vā́cam ajanayanta devā́ devā́ ajanayanta vā́caṃ vā́cam ajanayanta devā́ḥ | . . .

These techniques have been applied to the sacred text of Veda only. One can infer their efficiency from the fact that the *Ṛgveda*, which is the most ancient Indian text, has been preserved without variant readings. These techniques have ensured the conservation of the form of the text.

Similar efforts, perhaps still greater ones, have been made to preserve the knowledge of the meaning. The teacher who makes a disciple memorize a text, explains the text with the purpose of ensuring the transmission of his own way of understanding it. There is a tradition of commentary as old as the tradition of recitation. The teacher's commentary can be memorized in its form. But we observe that less effort has been exerted for preserving the form in the case of commentaries. There is no manipulation of the form of the commentary, no patterns of recitation comparable to those of the commented basic texts. The impression we receive is that the transmission of the ideas here supersedes

the transmission of the form. The emphasis is placed rather upon the acquisition by the disciple of the capacity to comment upon the memorized text and to explain it in the same manner as the teacher.

Thus in the course of development of Vedic culture a technique of commentary was evolved; and it may be as old as the technique of recitation. After the liturgical texts of Vedic religion, the later Vedic corpus consists of a group of commentaries and excursus maintaining a close or loose relationship with the sacred texts. Without being subjected to the same refined techniques of recitation, they have been transmitted through oral teaching and memory for centuries. This character explains some aspects of their form.

In the course of their activities, the intellectuals in charge of conservation of form and meaning of the Vedic texts became more and more aware of the means of their work, started a methodological reflexion and tried to establish the state of knowledge of the subjects dealt with. In India scientific literature started in this line of thought. The main subjects dealt with were Vedic poems, ritualistic formulas, rituals and the calendar. Six disciplines were thus born. Being attached to the use of Vedic texts and ritual performance, they are called *Vedāṅga*-s "Ancillaries of the Veda". Four of them deal with language: grammar (*vyākaraṇa*), etymology (*nirukta*), phonetics (*śikṣā*), metrics (*chandas*). Two others are astronomy (*jyotiṣa*) and ritual (*kalpa*). All are techniques with a practical destination, without any interference of any irrational element, as their sole purpose is the conservation of texts by the knowledge of phonetics, metrics, etc., the conservation of meaning by the knowledge of grammar, lexicon and etymology, the correct performance of rites at the correct time with the aid of knowledge of ritual and astronomy.

The principles and codifications worked out in each of these disciplines have been established and entered in numerous texts, several for a given discipline, because of the multiplicity of Vedic schools. None of these texts is dated with certainty. There is only a general agreement on their relative chronology and on the fact that they form the last layer of Vedic literature (approximately 7th-4th century B.C.). They predate to the use of writing in India and thus pertain entirely to the Vedic oral civilization. Their form is very original: the *sūtra*, the mnemonic form *par excellence*. The masterpiece of this genre is the famous grammar of Pāṇini. The term *sūtra* may be used for the entire work or for each individual formula.

2. THE SŪTRA GENRE

A well-known definition of the *sūtra* is:

अल्पाक्षरमसंदिग्धं सारवद्विश्वतोमुखम् ।
अस्तोभमनवद्यं च सूत्रं सूत्रविदो विदुः ।।

alpākṣaram asaṃdigdhaṃ sāravad viśvatomukham
astobham anavadyaṃ ca sūtraṃ sūtravido viduḥ

(The knowers of the *sūtra* know it as having few phonemes, being devoid of ambiguity, containing the essence, facing everything, being without pause and unobjectionable.)

Concise wording is the first obvious character of the *sūtra*. There are several procedures of abbreviations: use of ellipsis extended beyond the tolerance of natural language;

multiplication of technical names to avoid descriptive expressions; abridged lists through mention of only the first and last items; use of markers; use of variables.[3]

The qualification "facing everything" expresses the fact that the formula which is unique by itself, can be applied to a multiplicity of cases. In the traditional method of teaching a distinction is made between two modes of teaching, each one having a technical name. One is the teaching by express mention, in which each concerned object is mentioned for itself; that is technically called *pratipadokta* "mentioned for each word". There is another mode of teaching in which several objects which have a common character are not individually mentioned, but indirectly referred to by the mere mention of their common feature. This character is called *lakṣaṇa* and the same term is technically used for the general rule based upon it. Technical teaching is mainly done through *lakṣaṇa*-s and the *sūtra* is a *lakṣaṇa* in most cases, *i.e.* one rule "facing" multiple objects.

The expression "being without pause" refers to the fact that often the *sūtra* is not self-sufficient, is linked with one or several others. For instance, several *sūtra*-s are to be used successively to execute an algorithm, to construct a geometrical figure etc. There is also a special feature of composition in the formulary considered as a whole. The *sūtra*-s or individual formulas are organized in groups and they are recited in a fixed sequence. This order of recitation is not without purpose. It does not correspond only to a logical order of the contents. It is also determined by considerations of internal composition of the text, allowing ellipses of words which are tacitly redirected from formula to formula, etc. It corresponds also to an order in the practical use of the formulas. There are precise rules of circulation inside the corpus of formulas to help the user quickly to find in his memory those which are useful for his immediate purpose. The individual character of the *sūtra*, as well as the fact that it is an integrated part of an orderly corpus, is indicated in the usage of pandits who employ the word *sūtra* for the individual formula and for the whole formulary.

One should emphasize the fact that the *sūtra* of the Vedic period is an "oral" text, learnt by heart, recited and mentally used. One who imagines it in a written form does not truly appreciate its character and composition. The most complex text in this *genre* is the *sūtra* of Pāṇini, composed of formulas used to build up words and sentences from the roots, suffixes and diverse elements of Sanskrit language. It is composed in a metalanguage with a very advanced degree of formalization and the arrangement of rules is quite complicated. Using this grammar in the form of a printed book, even equipped with well-prepared *indices*, is a difficult task. On the contrary, it is sufficient to observe traditional pandits who have memorized it and who use it mentally with an extreme facility, to understand that it has been composed in the frame of orality and only for mental use. This can be shown through a simple example. Pāṇini presents the rules of phonetic junction at the frontiers of words (*saṃdhi*) in two parts in two different places in his grammar: the beginning of the sixth section and the end of the eighth and last one. The classification of these rules in two groups and their dispersion have no foundation in the contents of the rules. In each of the two groups the classification does not depend on the subject of the rules. One cannot produce a detailed table of contents of these rules, because their sequence does not correspond to their contents and there is no grammatical subtitle which could be given to a sequence of a subgroup. The reason for this classification and dispersion is that Pāṇini gives a particular significance to the location of a rule in his

formulary: his convention is that a rule presented in the last portion of the eighth section of his book is considered as non-realized at the time of application of a rule located before. Some rules of *saṃdhi* can be applied only after a series, sometimes quite long, of other necessary operations.

For instance, the sequence of two words, *dvau* "two" *atra* "here", being given, the following rules must be applied in a fixed sequence. First, application of the substitute *āv* to the diphthong *au* in presence of the initial vowel *a* of the next word, according to the *sūtra* "eco 'yavāyāvaḥ" 6.1.78: *dvāv atra*. Then elision of the final *v* of *āv* in the presence of the initial *a* of *atra*, according to the *sūtra* "lopaḥ śākalyasya" 8.3.19: *dvā atra*. Thus the final *ā* of the first word is now placed just before the initial *a* of the next word. This is the situation where the *sūtra* "akaḥ savarṇe dīrghaḥ" 6.1.101 prescribing a unique substitute *ā* for the hiatus *āa*, can be applied. But that would produce a wrong form: **dvātra*. In fact, this unwanted application does not occur, because the rule of elision of *v* is placed in the final section of the formulary and, consequently is treated as non-realized at the moment of applying the rule of substitution of *ā* to *āa*. The final correct form is: *dvā atra*.

It appears that the order of the rules in Pāṇini's composition depends more on conventions of circulation in the body of the formulary than on the nature of the subject-matter. The formation of a word or a sentence, which is the target of this practical treatise, requires a long series of rules applied in a definite order. And that compels the user to move about constantly in different parts of the book. For a free and easy circulation it is advisable to bear in mind all the formulas and to know their location. The pandit who has been trained in using this grammar since childhood keeps it in his memory and is able to find quickly any rule relevant for his purpose. Seeking a rule from the non-memorized written text can be done only with the help of an index of incipits of formulas in alphabetical order. The table of contents is of little use. The index is useful only when one has already a good knowledge of the incipits and that amounts to learning by heart the whole of the formulary.

There is no doubt that Pāṇini composed his work for memorization and mental use. The conciseness of the formulas has a mnemonic value. There are a few devices of recitation with a mnemonic purpose, commonly used in Sanskrit traditional schools. For example each chapter is divided in groups of twenty *sūtra*-s. At the end of each chapter the last *sūtra* is repeated. By convention the repetition is a marker of the end of a section. Then one recites the incipit of the first *sūtra* of each group of twenty, then the number of groups and the number of *sūtra*-s in the last group when they are less than twenty, then the total number of *sūtra*-s. Finally there is a colophon at the end of each chapter to specify its location in the formulary. All that is considered as part of the text to memorize. It helps in checking whether a *sūtra* has been forgotten and in finding the location of a *sūtra*, its location being eventually relevant to the mechanism of rule applications. But there is no table of contents, no index.

No ancient manuscript of this text has ever been found. There exist recent manuscripts (17th to 20th century), but these are not very numerous. This paucity of the written form is significant, if we consider that it is a text which has been used by pandits in all times, in all their activities. In a 19th century palm-leaf manuscript we have seen, the scribe has actually declared at the end that he had written the text "out of love" for the work. It implies that there was no necessity for him to put it in written form.

The same features—conciseness, emphasis on the essential point, extent of the field of application, links from formula to formula etc.—appear in all ancient Vedāṅga-s or ancillary disciplines of Vedic culture in varying degrees, but Pāṇini's grammar is the text where they are the most conspicuous. In other disciplines there are less constraints. Less sacrifices are made to conciseness, and refined devices of composition are more scarcely used. Still, the basic features are present. The most remarkable is the emphasis on the essential and on the universality of the formula allowing the multiplicity of practical applications. We will, now, give an example in the ritual called *Kalpa-sūtra*, in a section dealing with geometry. It is an interesting passage in which one sees the quest for the efficient formula in the making, as it contains some observations of particular cases leading to general propositions of geometry.

The *Vedāṅga* of rites contains a section on the construction of brick-altars used for the celebration of solemn sacrifices. Vedic religion gives prominence to the performance of oblations of milk or other substances in fire. The offering requires the construction of brick altars called *agnicayana*, literally "piling up (bricks) for fire", or in a shorter form *agni* "fire". The most common altars had a simple geometrical shape, square, trapezoidal or semi-circular. It was made of five layers and to ensure the tightness of the construction, it was a rule that the bricks should not be assembled in the same manner in two consecutive layers. Therefore the even layers were made of bricks with size, shape or arrangement different from those in the odd layers. The priests who built the altars had to face practical problems of geometry, and thus we see the beginnings of Indian abstract geometry in Sanskrit treatises of ritual practice.

3. THE *ŚULBA-SŪTRA*

In several texts of the ritual literature in the *sūtra* form, there is a section called *Śulba-sūtra* "Formulary of the cord" that deals with such problems of geometry. There exist several *Śulba-sūtra*-s, belonging to different schools. One, traditionally ascribed to an author named Baudhāyana, is presumed to be the oldest in the genre. His date is not known, but a most reasonable estimation is that the manual was composed sometime between 700 and 500 B.C.

Let us take one example from this work. The domestic fire-altar has the form of a square and the area of one sq. *vyāyāma* (a unit of measurement equal to 4 *aratni*-s "cubits"). There are 21 bricks in each layer. One method of arranging them consists in drawing the square, dividing one side into three parts and transversely into seven parts. One obtains 21 rectangles. The bricks are made with this shape and size. On the second layer the bricks will be obtained in the same manner but arranged transversely.

This is prescribed by the following formula.[4]

चतुरश्रं सप्तधा विभज्य तिरश्चीं त्रेधा विभजेत् ।। २.६४ ।।

अपरस्मिन् प्रस्तार उदीचीरुपदधाति ।। २.६५ ।।

catur-aśraṃ saptadhā vibhajya tiraścīṃ tredhā vibhajet | II 64

aparasmin prastāra udīcīr upadadhāti | II 65

(II.64. After dividing a quadri-lateral in seven, one should divide the transverse [cord] in three.

II.65. In another layer one places the [bricks] North-pointing).

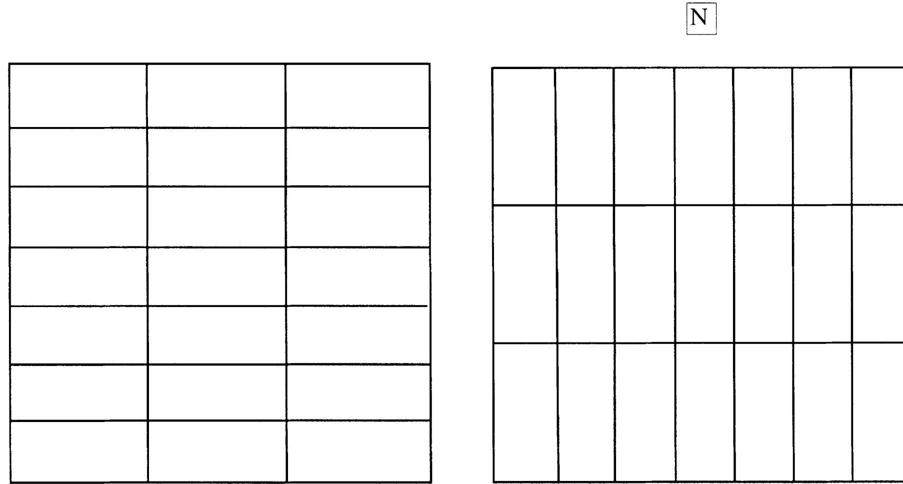


Figure 1. Gārhapatya-citi "altar of the domestic [fire]" II 64–65.

Compared to Pāṇini's *sūtra* this *Śulba-sūtra* is not formalized to the same extent. Still, it is an "oral text" in the same manner. It has been transmitted for many centuries only through memory and has been used on the basis of memory also. It does not contain anything which could indicate that it has been originally written. It is a compendium of formulas composed to be used by performers of the rites and we do not know any other destination for it than the practical one. Its form and composition indicate that the situation in which it is used is the accomplishment of the task of piling up bricks for sacrificial altars. The tools possessed by the officiant in charge of this function were a cord (*rajju*, f.), pegs (*śaṅku*, m.) and clay to make the bricks (*iṣṭakā*, f.). The formulas were in his memory. That was enough for the accomplishment of the rite.

In the example given above, we note several instances of conciseness. The technical vocabulary derives from practical execution. The sides of the oriented rectangle are named by feminine adjectives. The substantive qualified by them is never mentioned and is understood as being the *rajju* "cord". The only hint to this identification is the feminine ending *ī* for the adjective *tiraścī* "transverse" in II.64. In II.65 the isolated feminine plural adjective *udīcīḥ* "North-pointing" is used in the same manner. The understood substantive is *iṣṭakāḥ* "bricks", and this term is indicated only by the feminine plural ending *īḥ*. One understands the cord and the bricks because they are objects designated by feminine words and they are involved in the practical application of the prescription.

It has not been mentioned in the first *sūtra* that the rectangular bricks had their long side placed on an east-west line. The mention of the north direction in the second *sūtra* implies the transverse position for the previous layer, because of the principle that there should be a different disposition of bricks in each layer. This is conciseness, not only in the wording, but also in the meaning. There is ellipsis of an idea, the east-west direction, in II.64, because it can be inferred from the later mention of the north-south direction.

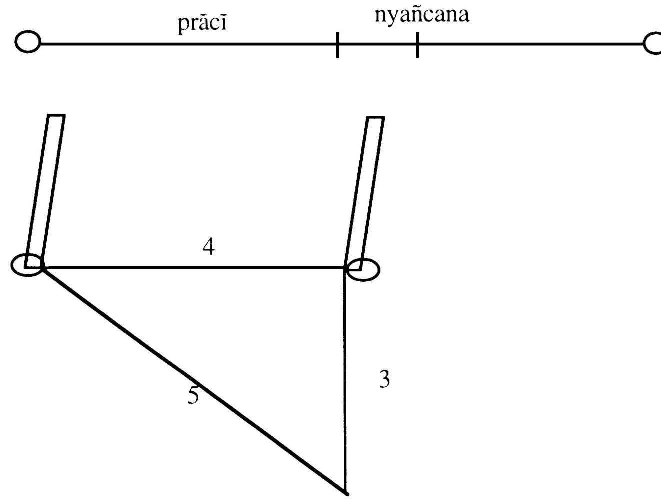


Figure 2. Construction of a square.

Ellipsis is a linguistic device which belongs to natural languages, but to a limited degree. The ellipses found here are clearly beyond the natural limits of usage. This is the result of an effort to formalize the language, and that is explained by the technical destination of the text. The instructions about the "transverse" and the "pointing-to-the-north" can be understood only by the officiant of the sacrifice who has the cord and the bricks in his hands, and whose task it is to arrange the bricks in their respective directions.

The technical vocabulary of the *Śulba-sūtra* is full of imagery. It is always close to practice and derives entirely from the environment in which the officiant works. One does not find here the notions of line or point, but only cords and pegs. Therefore one may raise the question, whether this is a real abstract geometry, different from a collection of pragmatic instructions. Nevertheless, one can recognize here true geometry. In the same text appear formulations of more general geometrical propositions expressed with the same practical vocabulary, as in the following example.

The officiant started his work by molding bricks to a desired shape and size. He was making geometrical constructions with his pegs and cord. In the case of the square, he fixed in the ground two pegs with a distance between them equal to the side of the desired square. He held a cord with a loop at each end and a length equal to two times the distance between both pegs. He put one mark in the middle between the two loops, and another one at a distance from the middle of a quarter of the second half. The second mark had a technical name, *nyañcana*. He attached the loops of the cord on the pegs, then stretched the cord, holding it at the *nyañcana* mark. Thus was obtained a perpendicular to the first side. On that line he marked the length of the side with the help of the middle mark on the cord. By the same process the opposite side was obtained. This procedure is based on the knowledge that the triangle of sides measuring 3, 4, 5 is right-angled.

Baudhāyana's *Śulba-sūtra* prescribes the same procedure of construction for right-angled triangles of sides measuring 5, 12, 13, then 8, 15, 17 etc. Then, after his exposition of such practical procedures for individual cases, he formulates a general rule:

दीर्घचतुरस्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति ॥ १.४८ ॥

dīrgha-catur-asrasya_akṣṇayā-rajjuḥ pārśva-mānī tiryaṅ-mānī ca yat pṛthag-bhūte kurutas tad ubhayaṃ karoti ‖ I.48

(the crosswise-cord of a long-quadri-lateral produces those two which the one-measuring-the-side and the one-measuring-transversely produce).

A cord is said to produce a square, because it is the basic tool to construct a square according to the procedure described above. The long-quadri-lateral is the rectangle. The diagonal is referred to by the word "cord" compounded with an adverb signifying "crosswise". The names of the sides of the rectangle are feminine adjectives qualifying the term "cord", which is understood, and they are compounds of the words *pārśva* "side" and *tiryak* (adverb) "transversely" with the adjective *mānī* "measuring". The word *ubhayam*, literally "both", refers to the two squares which can be constructed on the sides of a right-angled triangle. This proposition amounts to saying that the square constructed on the diagonal of a rectangle is the sum of the squares constructed on the sides. And this is a general proposition, in the sense that there is no more case-wise reference to cords measuring 3, 4, 5 etc., but to cords as sides of a rectangle. The practice-based technical terms are maintained for those cords. But there is also an abstract view of them which leads to the emergence of a general rule from the observed particular cases.

Finally, we wish to emphasize the fact that in this text there are indications only about the gestures and the knowledge of the officiant. The ellipsis, the attribution and choice of technical names are features which belong exclusively to the oral form of language, or to the activity and equipment of the officiant. No writing, no graphical representations are found in it.

4. FROM ORALITY TO WRITING

Vedic civilization gives a remarkable instance of a form of intellectual work operating in the strict limits of orality and still endowed with a striking efficiency. It has been maintained throughout many centuries and some important elements of it, such as recitation and memorization of texts, have been preserved up to the present times. The period around the Christian era witnessed also an evolution of civilization in India in several fields. First, there has been a kind of fixation of the scholarly language. Vedic language had undergone considerable changes and around the Christian era a form of Sanskrit called "classical" became the language of scholars. From that time the grammar has remained practically unchanged until now, even though the vocabulary has been constantly enriched. The name "classical" is often extended to the literature produced in this form of the language. The traditional Sanskrit scholar knows the classical language and often has also memorized one branch of the Vedas. Thus he has the knowledge of both forms of the language and he retains many elements of the old Vedic civilization. It explains the fact that Vedic culture is integrated into the new classical one.

Amongst the innovations of the classical period comes the generalization of the use of writing. With the emergence of writing the already consecrated oral forms, the processes

of work and of expression tested for so many centuries of oral and mental practice, were not forgotten. Classical Sanskrit mathematical literature bears both characters. The typical text of mathematics falls into two parts, one composed for memorization with the methods of orality, another one which is the exposition of the same matters or related matters in written form. The duality of the tools, orality and writing, which are at the disposal of the Sanskrit scholar, explains the originality of the texts composed by him.

First, the Indian mathematician owes to writing a new system of numeration. In the beginning, around the Christian era, numbers were expressed by a system of graphic signs which faithfully reproduces the spoken system of numeration. Sanskrit, like all Indo-European languages, has different words for nine units, for ten and several powers of ten. The written notation initially replaced the basic words by as many basic signs, so that it has more than nine signs, without knowing the zero. Where a number is expressed with a sequence of basic words, numeral notation consisted of a parallel sequence of basic signs. The system of writing numbers was, at that time, only a reflection of the linguistic system.

The principle of the place-value and the use of only nine ciphers with a zero are documented from the 3rd century A.D.[5]. This system appears in its final form in usage in Āryabhaṭa's works in the beginning of the 6th century. With the advent of the place-value system with zero the written expression becomes quite different from the oral one. A new tool is born.

With the new written numeration, the Indian mathematicians and astronomers developed the use of another system of metonymic expression of numbers called *bhūta-saṃkhyā* "object-number". It consists in using, in place of a number-noun, the name of an object regularly attached to that number, for instance in saying "void" to mean zero, "moon" for one, "eye" for two, "fire" for three (because there are three fires used in Vedic ritual), "cardinal point" for four or eight if one counts also intermediary directions, or ten by counting also zenith and nadir, "teeth" for thirty-two, "human life duration" for one hundred etc. All synonyms referring to the object can be used, so that there is a great variety of words for one number. This mode of expressing numbers has its roots in Vedic literature. It became more profusely used, when the place-value system entered in common use. And it was adapted to the model of a number written in this system. Generally it is read from right to left, i. e. the reverse of numeral notation. For example:

nanda-adry-r̥tu-śara

(Nanda-mountain-season-arrow) which means: 5679.

Here Nanda, the name of a dynasty of nine kings, means 9, *adri* refers to seven mountains well-known in Indian mythology, *r̥tu* to the six seasons of Indian climate, and *śara* to the five arrows of the god of Love.

This process of expression of numbers appears in the verse portions of texts. The Sanskrit verse, whose form is determined by the number and rhythm of long and short syllables in a line, cannot contain numbers written in cipher: every number must be mentioned in words. It is clear that the mention of high or numerous numbers cannot enter in a metrical sequence, which may be too short or may impose too many constraints, and that the repetition of the same number-nouns thwarts any quest for literary quality. On the contrary the metonymic procedure of expressing numbers offers a rich lexicon of synonyms. Even if Sanskrit allows some freedom in expressing numbers made of many ciphers, even if it offers several types of periphrases, the basic names are limited and have

no synonyms. With the mere material offered by the natural language, it would often be impossible to place in a verse the name of a high number. The use of "object-numbers" extends considerably the scope of expression.

It has also a mnemonic value. There is more risk of committing mistakes in memorizing a series of number-names, than in memorizing a figurative series of object-names. If that series is cast in the mould of a verse, it will have a stronger hold in memory. One more advantage of this mode of expression is a better textual conservation. It is clear that a text made of various names arranged in verses, which can be verified by scansion, incurs less risk of corruption than a prose text made of many repetitive number-names, or than one written using ciphers, which is still more exposed to the distraction of copyists[6]. This system has been profusely used, wherever numerical information was intended to be learnt by heart or to be used in practical applications in memorized form. That is the case with the enunciation of algorithms and tables of numerical data. When a text is not destined to memorization, as in the case of an illustrative commentary or of the detailed explanation of an algorithm, the written form is more common and numbers are written in cipher.

Another contribution of writing to Indian mathematics has been the written form of operations. The most original feature of Indian practice was the habit of writing operations on sand. This allows one to strike off an element in the course of the procedure. In fact, we have very scanty documentation on this practice. It is no longer in use nowadays and is known only through short descriptions of procedures in medieval texts. The support was probably a small board on which fine sand was spread. It could also have been just the ground. The term *dhūli-gaṇita* "counting on sand" is used to refer to arithmetics in general. There are also a few mentions of chalk used to write on slate, which is equivalent to writing on sand, as chalk can be easily wiped.

Preservable supports for writing in India have been palm-leaf (*Borassus flabelliformis* or *Coryphœa umbraculifera*), generally engraved with a stylus, birch-bark, used in all periods, and paper, starting from the 11th century, for writing upon with ink. In that case the procedure of the operation is not reproduced completely. No data are wiped, there is no re-writing, and only a few steps are represented. Texts describing operations are often illustrated by examples in commentaries, but they fail to give a detailed account of all that could be done on sand but could not be transcribed on other permanent supports of writing. The most ancient document which has come down to us is the Bakhshālī manuscript, called after the name of the place where it was discovered at the end of 19th century. Its date is a subject of controversy. The best palæographical study so far done has led its author to propose the 12th century[7]. It transmits a collection of problems with answers and thus gives us the most ancient known samples of notation of data for mathematical operations. A remarkable feature is the disposition of numbers in tables with frames[8].

5. THE MODEL OF THE MATHEMATICAL TEXT

The origin of a Sanskrit mathematical text, like any text in any other discipline, is the figure of the traditional scholar called "pandit". We have documents about him, consisting not only of data found in the texts produced by him, but also the living figures whom we can meet with nowadays and whose characteristic features can be, for many of them at

least, taken back to former times, because of the perennial quality of intellectual traditions in India. With high probability we can assume the teaching master to have been the most common type among ancient pandits. The typical composition produced for teaching is the *sūtra*, or a composition in the same kind of style, which the master explains orally in his own way. The general rule is that the disciple memorizes the letter of the *sūtra* and remembers the contents, if not the very wording, of the oral explanation. This oral commentary may never have been written but always transmitted orally. Still nowadays, many things told in Sanskrit traditional schools are entrusted only to oral expression and memory. But also the teachings of a master of repute could be couched in writing, by the master himself, by a disciple or by a professional scribe. The last eventuality has probably been the most common, until the recent past. The profession of scribe, active up to the end of 19th century, has disappeared in the 20th century and, now, masters and disciples have to take care of writing themselves, when they accept or wish to write. Even if oral transmission is always appreciated, even if a composition in *sūtra* style and in verse is an aid to memorization, the pandits never refused writing, never neglected the help they could derive from it. And in cases where human memory failed, writing has probably saved a great number of texts.

The standard text of mathematics in the classical period is made of verses in a terse style. It differs from the old *sūtra* only because of its metrical form, which is an additional constraint imposed by the author on himself. Often we are under the impression that it aims at competitiveness in terseness and difficulty. Metrical form and brevity render it all the more easy to memorize. Under this form the mathematical text remains an "oral text", which can be transmitted through memory, used mentally, being well-adapted to such functions. This verse-form, precisely the stanza of four verses in formula-style, has received a technical name, *kārikā*. In some cases the name *sūtra* may be used for the verse-form also.

The difficulty and abstract character of the formulas render explanations necessary. The art of commentary was developed in the same time as the verse-*sūtra*-s were becoming more and more sophisticated. We have mentioned the case of the oral explanation. There also developed a type of commentary composed not only for oral transmission, but chiefly to be couched in writing and so adapted to writing resources. Especially in the field of mathematics the commentator has many occasions to resort to the use of graphic devices, the use of the written place-value numeral system, the graphic arrangement of arithmetical operations, drawing geometrical figures etc. In a general manner we can say that the verses have preserved the style of an oral exposition, and the commentary is an expansion of the memorized knowledge using all the facilities provided by writing.

Its primary purpose is to retell the contents of the *sūtra*-s or verses in expanded form, to state precisely the meaning of each word separately and of the sentence as a whole, to express the implications, to emphasize connotations, linguistic and rhetorical devices, to give illustrations of practical applications and, where necessary, to discuss, criticize, answer to criticism, propose corrections, new formulations etc. The methods of comment are well conceived, have technical designations and are systematically used.

Together with the formulary art, an art of commentary was thus developed, with techniques of interpretation obeying as many rules, constraints and terseness. In this brief article it is not possible to give an idea of this art, because of its technicality and magnitude, otherwise than by giving a sample selected for its simple nature.

6. PLACE-VALUE NUMERAL SYSTEM ACCORDING TO ĀRYABHAṬA

We may draw an example from a masterpiece of Indian scientific literature, the *Āryabhaṭīya* of Āryabhaṭa. Āryabhaṭa, born in 476 A.D., composed two works which bear his name, *Āryabhaṭasiddhānta* and *Āryabhaṭīya*. Only the latter is known to us. It deals with mathematics and astronomy. The first chapter, composed of 13 stanzas, sets forth the basic definitions and astronomical tables. The second, of 33 stanzas, deals with a few subjects of geometry and arithmetic. The third chapter, of 25 stanzas, deals with the various units of time and the determination of the true positions of the Sun, Moon and planets. The fourth one, of 50 stanzas, deals with the motion of the luminaries in the celestial sphere. The first thirteen stanzas are in the metre *gīti*, the other 108 are in the metre *āryā*. They are considered as difficult metres imposing a severe constraint upon the composer. The brevity of expression is carried to its extreme. This is the technique of *sūtra* style. The mnemonic quality of this short verse text for the number of subjects dealt with, is obvious. Like the *sūtra*, it is a text to memorize, and once memorized, to be used for practical purposes, especially for astronomical calculations.

The terseness of expression renders Āryabhaṭa's stanzas particularly difficult. The Indian tradition has itself felt this difficulty and numerous commentaries have been written from medieval to modern times. The earliest of those commentaries, which is available to us, is the *Bhāṣya* of Bhāskara I, composed in 629 A.D. about one century after the composition of the original. The designation of *Bhāṣya* implies that with the explanatory gloss and examples there is an examination of the contents in the form of a debate, with objections and answers.

Āryabhaṭa starts his section of mathematics with a prescription of the place-value system of numeration.

एकं च दश च शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम् ।
कोटयर्बुदं च वृन्दं स्थानात्स्थानं दशगुणं स्यात् ।। २ ।।

ekaṃ ca daśa ca śataṃ ca sahasraṃ tv ayuta-niyute tathā prayutam |
koṭy arbudaṃ ca vr̥ndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt ‖ 2

(One, ten, hundred, thousand, myriad, hundred-thousand, million, ten million, hundred-million, milliard: a place should be ten times a place.) (*Gaṇita* 2)

Sthāna, literally "place", is the technical name of the value signified by a place. Each value has a denomination. Sanskrit has a single and common name for each value up to 10^9, and others, of more limited usage, up to yet larger numbers.

The commentator explains the purpose of using the place value system of numeration. It is the economy of signs in the writing of numbers and the resolution of operations. He states the value of the listed number nouns. He completes the abbreviated prescriptive sentence: "a place is ten times the place one has already created", when one writes a number.

Then he shows the validity of the formula, by questioning and answering possible objections. One could say that the clause "a place should be ten times a place" is applicable to other values than those listed in the stanza. Therefore this short list appears to be useless. The formula has the capacity to make all these values known. It was not necessary to repeat them. One could answer that in the stanza there are two parts: the proposition "a place

should be ten times a place" is the rule (*lakṣaṇa*); the list of values is the example. At this juncture Bhāskara I calls the author Āryabhaṭa "author of *sūtra*-s" and says that it is not the habit of an author of a *sūtra* to formulate examples with the rules. This shows that he considers the verses of Āryabhaṭa as having the status of *sūtra*-s. Then he says that if examples have no role to play in the stanzas, we must understand the list of values as being a presentation of these numerals as technical names of the prescribed places, not as mere examples of values. These numerals, in fact, do not have here their usual meaning of numbers. They are here technical names of the different place-values created by the rule. The Sanskrit words which mean literally "one, "ten", etc. are the names of the first, the second place, etc. in the writing of numbers. They stand in Sanskrit for the designations "units", "tens", etc. Āryabhaṭa's stanza contains a rule to determine the value of a place and to give it a technical name. Bhāskara adds that the rule allows the determination of many more place values than the listed ten, but that only ten names were given, because above that there is no use of particular names. High numbers can be written and understood with this method. Names are not necessary and not in common use.

Finally, after some remarks on the subject, the commentator indicates how to put down the places in writing. This is what is called *nyāsa*, literally "deposit". The *nyāsa* of the places is a row of ten zero-signs written from right to left:

0000000000

The text of Bhāskara's commentary is known to us through five manuscripts, all recent (18th–19th cent.), in South-Indian scripts and probably derived from one and the same source. If we may accept that the manuscript tradition of that text since the 7th century has always faithfully reproduced the original disposition used by the author, this is a document from which one can reconstruct the practice of writing numbers on a plank covered with sand or on a slate. When Bhāskara I represents the "places" by a line of small circles, he prepares the writing of a number: one writes the places with circles, then one writes ciphers, each in its place, by wiping off the circle and substituting the cipher. In the place where no cipher has been entered, the circle remains, and that represents the fact that this place remains empty.

7. THE MATHEMATICAL PROBLEM

When a stanza deals with a longer algorithm involving several steps, the commentator follows a regular model of detailed exposition of the procedure with examples. He starts with an enunciation of the problem, technically called *uddeśaka*. Very often the *uddeśaka* is a stanza. That indicates it was also an item to be learnt by heart, so that the user has in mind models to imitate in the course of his work. This is followed by a *nyāsa*, i.e. writing down the data of the problem. Then comes the execution of the prescribed procedure; that is called *karaṇa*, literally "execution". The result called *labdha* "obtained" is finally stated.

We give as example the procedure of squaring a fraction, given by Bhāskara I in his commentary on the half-stanza composed by Āryabhaṭa to prescribe the procedure of this operation called *varga*:

वर्गः समचतुरश्रः फलं च सदृशद्वयस्य संवर्गः ।

vargaḥ sama-catur-aśraḥ phalaṃ ca sadṛśa-dvayasya saṃvargaḥ (*Gaṇita* 3a)
(A *varga* is of-four-equal-sides; and the area is the multiplication of two equals.)

In common Sanskrit *varga* refers to a group of objects of the same class. Bhāskara I declares that *saṃvarga* is one of many synonyms for multiplication in general. He takes *varga* in a more restricted sense, the multiplication of two equal numbers. He takes that as the primary meaning of the word and interprets the above text as a prescription of an additional technical meaning, namely the meaning of a square. *Phala,* literally "fruit", is the technical name of the area of a figure. Āryabhaṭa's stanza means that one calls *varga* the figure with four equal sides, whose area is the multiplication of two equal numbers, i.e. the measures of two equal sides. One may calculate the area of a square by this operation.

Bhāskara I illustrates that by the following problem:

उद्देशकः
षण्णां सचतुर्थानां रूपस्य च पञ्चभागसहितस्य ।
रूपद्वितयस्य च मे ब्रूहि कृतिं नवमहीनस्य ।।

Uddeśakaḥ:
ṣaṇṇāṃ sa-caturthānāṃ rūpasya ca pañca-bhāga-sahitasya
rūpa-dvitayasya ca me brūhi kṛtiṃ navama-hínasya

(Enunciation [of the problem].

Tell me the square of six with a fourth, of the form with a fifth part, and of a pair of forms minus a ninth.)

The term "form" is a metonymic designation of number "one". The expression "pair of forms" is only a designation of number "two". First, Bhāskara I shows how to write the data of the problem. They are rational numbers which, in Sanskrit, are always expressed by an integer followed by a fraction. When the integer is written on a line, the fraction is placed below it and is itself written on two lines, the numerator called *aṃśa* "part" on the first line, the denominator called *cheda* "divisor" on the second below. If the fraction is written without any particular additional sign, one understands that it is added to the integer above it. If it is marked by a small circle or a cross (the shape of the "plus" sign in the West) placed on its right, one understands that it is subtracted from the integer. Thus Bhāskara I writes:

"न्यासः	६	१	२
	१	१	१ o
	४	५	९
Nyāsaḥ	6	1	2
	1	1	1 o
	4	5	9

(Presentation [of the data] :

6	1	2	
1	1	1 o	
4	5	9	[that is to say 6+1/4; 1+1/5; 2–1/9])

This is followed by the exposition of how the operation is carried out:

करणम् – छेदगुणं सांशमिति २५
४

एतयोः छेदांशयो राश्योः पृथक् पृथग्वर्गराशी १६ ६२५ छेदराशिवर्गेणांशराशिवर्गं हृत्वा लब्धं ३९
१
१६

एवं शेषयोरपि यथासंख्येन १ ३
११ ४६
२५ ८१

karaṇam — cheda-guṇaṃ sa-aṃśam iti 25
4

etayoḥ cheda-aṃśayo rāśyoḥ pr̥thak pr̥thag varga-rāśī 16 625 *cheda-rāśi-vargeṇa_aṃśa-rāśi-vargaṃ hr̥tvā labdhaṃ* 39
1
16

evaṃ śeṣayor api yathā-saṃkhyena 1 3
11 46
25 81

(Realization [of the operation]: [The integer] multiplied by the divisor, with the part, is $[(6 \times 4) + 1]$:

25
4

The squares of these two numbers, divisor and part, are 16 and 625 respectively. After dividing the square of the part by the square of the divisor, one obtains:

39
1
16

In the same way for the other given numbers one obtains:

1 3
11 46
25 81)

In the case of the last operation, one multiplies the integer by the divisor and subtracts the part: $(2 \times 9) - 1 = 17$. The fraction $17/9$ is squared.

At least four commentaries of Āryabhaṭa's work are known. There are many differences of form and interpretation between them. The study of the differences of interpretation goes beyond the limits of this article. It involves many techniques of linguistic and logic analysis of Sanskrit commentators and gives rise to complex problems. Here we will content ourselves with the simple presentation of another example of squaring, given to illustrate the above-quoted formula of Āryabhaṭa by another of his commentators, Sūryadeva Yajvan, born in 1191 in Tamilnadu.

उद्देशकः
बाणार्कसंमिता यस्य चतुरश्रस्य बाहवः ।
त्र्यंशद्वयमिता यस्य तयोः फलमिहोच्यताम् ॥
प्रथमोदाहरणस्य न्यासः

परिलेखः		१२५	
	१२५		१२५
		१२५	

अस्य सदृशद्वयस्य संवर्गफलम् १५६२५
द्वितीयोदाहरणस्य न्यासः–अत्रांशवर्गः ४ छेदवर्गः ९ अनेनांशवर्गे विभक्ते
लब्धं क्षेत्रफलम् ४
९

uddeśakaḥ—
bāṇa-arka-saṃmitā yasya catur-aśrasya bāhavaḥ |
try-aṃśa-dvaya-mitā yasya tayoḥ phalam iha_ucyatām ‖
prathama-udāharaṇasya nyāsaḥ—

parilekhaḥ		125	
	125		125
		125	

asya sadr̥śa-dvayasya saṃvarga-phalam 15 625
dvitīya-udāharaṇasya nyāsaḥ — atra_aṃśa-vargaḥ 4, cheda-vargaḥ 9, anena_aṃśa-varge vibhakte labdhaṃ kṣetra-phalam 4
9

(Enunciation [of the problem].

The fruit of the square whose arms are measured by Suns and arrows[9] and of the [square whose arms] are measured by a pair of thirds, should be told here.

Presentation [of the data] of the first example:

figure		125	
	125		125
		125	

For that [square] the fruit which is the multiplication of two equals, is 15 625.

Presentation [of the data] of the second example: [. . .] In this case, the square of the part is 4, the square of the divisor is 9; when the square of the part has been divided by that [square of the divisor], the area of the square 4/9 is obtained.)

The difference between what we have here and the example of Bhāskara I is the emphasis given to the representation of the geometrical square. Bhāskara I has only written the number to be squared. Sūryadeva has given the figure of the square and written the number as the measure of the side. This detail of presentation is not without significance. In the presentation of Bhāskara I only the arithmetic operation is illustrated. In Sūryadeva's exposition the assimilation to geometrical representation is brought to light. In fact both commentators have underlined the fact that Āryabhaṭa identifies the squaring of a number and the finding of the area of a square. In their mind the word *varga* is primarily the technical name of the arithmetical square and secondarily that of the geometrical square. They consider that, when Āryabhaṭa prescribes it, he does a superimposition of the latter on the former. Superimposition (*upacāra*) is a mode of linguistic expression recognized among Indian grammarians and poeticians. The relation between arithmetics and geometry is translated by them in the form of a linguistic fact. In the sophisticated and compelling poetics of Sanskrit pandits, the rule is that any figurative mode of expression should be justified by an intention of the user. Therefore both commentators specify the intended purpose behind the use of such a device. On that point they have the following difference.

For Bhāskara I the purpose of the superimposition is to exclude the rhombus from the present definition of the *varga*. The term *catur-aśrasya* refers to a figure "of four equal sides". That is ambiguous, being common for the square and the rhombus. The identification with the arithmetical square excludes the rhombus, because the latter has equal sides, but its area is not the multiplication of two equal sides. Thus, Bhāskara I interprets that Āryabhaṭa intended to give a technical name to the geometrical square and selected the name of the multiplication of equal numbers, because that created an identity, excluding the case of the rhombus.

Sūryadeva declares that the purpose of Āryabhaṭa was only to show that the multiplication of equal numbers is the essence not only of the area of the square, but also of the arithmetical square. The superimposition done in the form of a linguistic device of expression is justified by the fact it gives the information that a common property is dealt with here, namely the identity of the operation to be carried out in order to calculate the area of the square and the squaring of a number. And that is a fact of reality. But Bhāskara I has clearly investigated something more and argued for the introduction of an additional piece of information, the exclusion of the rhombus, in Āryabhaṭa's proposition, by exploiting the use of the linguistic device of superimposition. A consequence of this difference in the estimation of the intent of Āryabhaṭa's definition, is that Sūryadeva, who had more interest in assimilating the arithmetical operation and the geometrical conception, has emphasized the latter in his example by drawing the figure of the square and writing down the numbers at the same time.

This pattern of exposition is abundantly represented in Sanskrit mathematical literature. For certain famous texts commentaries have been repeatedly composed in the course of history. They differ in the choice of examples and eventual excursus. There are

cases of self-commentary. There is also a simplified form of mathematical text where the commentary offered by the author of the formulas is reduced to an illustration by a few examples of problems. The *karaṇa* portion is even suppressed in the presentation of the example, because it can be understood easily from the general rule contained in the basic formula. Sometimes the definition itself is called *karaṇa-sūtra* "formula of procedure". It is followed only by the *uddeśaka* "enunciation of the problem" with its *nyāsa* "presentation of the data" and the result (*labdha* "obtained"). In that case the rule of procedure and the *uddeśaka* are in verse, the *nyāsa* is in prose. This is the pattern followed by the *Līlāvatī* of Bhāskara II (12th century A.D.), a collection of arithmetic and geometry which achieved great fame from its high literary style.

Thus Sanskrit mathematics owe their originality not only to the structure of the Sanskrit language, but also to the habits of exposition of Indian pandits. We have highlighted their propensity to orality, their trust in using memory and in mental activity. One could add also their literary inclinations.[10] Together with the metrical form, ubiquitous in all scientific and technical disciplines in India, devices of metaphorical expression, whose natural home is poetry, find their way in their compositions. We have mentioned a metonymic foundation in the terminology. One could also draw attention to the diversity of expression. If, in the West, one notes a tendency to stabilize terminology, definitions, formulations of theorems etc., in the Indian context one has to reckon with a propensity to the perpetual renewal of modes of expression. The Sanskrit mathematical text is a literary text. It imitates the form and spirit of the poetic text. It always tries to renew, even to surpass itself. Sanskrit poetry is definitely situated in the structures of orality. It is a poetry of sound emitted and heard, close to music, an inner object of meditation. The ideal mathematical text aims at being that same kind of mental object which speech transmits.

E.P.H.E., Paris

NOTES

1 The bulk of mathematical literature produced in India is in Sanskrit. It has not been exclusively in this language. Some expositions are found in other languages also, Ardhamāgadhī, Prakrit, Dravidian languages. But behind them there are Sanskrit parallels or sources. A Jaina pandit of ancient times may have had more reverence for the language of the founder of his religion, but has followed the model of the Sanskrit pandit. And in the Middle Ages Jainas have often used Sanskrit directly, especially in non-religious subjects.

2 It is a fact that the Indus civilization (3rd and 2nd millennia B.C.) knew writing and that there may have been a contact between its last period and the beginning of Vedic civilization, which occurred in the second part of the 2nd millennium. But no speculation can be useful on this point as long as the Indus script remains undeciphered. It has so far resisted every attempt at decipherment. We do not even know which language it transcribes. No clue has been found allowing us to suspect that it could be the Vedic language.

3 See Pierre-Sylvain Filliozat 1993a, 1993b.

4 We give the most literal translation. Technical names are rendered by their etymology, and, when several English words are used to translate a single Sanskrit word, they are joined with hyphens. Words in square brackets and figures are added by us. We know only very recent manuscripts of this work. When a figure is found in a manuscript, it is not a part of the ancient text, it belongs to a recent commentary. The ancient text was not written and we do not know any ancient graphic representation.

5 In a very acceptable restoration of the text of a verse from *Yavanajātaka* 79.6d; see text vol. I p. 494 and commentary vol. II p. 406, in Pingree's edition, 1978.

[6] See Billard, R. 1971: 21–22.
[7] Kaye, G. R. 1927, Hayashi, Takao 1995.
[8] See P.-S. Filliozat & G. Mazars 1987.
[9] This is an example of metonymic expressions, namely the mention of something connected to a number in order to express that number, described above. In Indian mythology there are twelve Suns and the Love-god has five arrows (*bāṇa*) in the form of flowers. The compound *bāṇa-arka* "Suns-arrows" is equivalent to 5–12 and is read from right to left: 125. "Fruit" is the technical name for the area of a figure. The problem is: find the area of a square with a side of 125, and of another with a side of 2/3.
[10] See Pierre-Sylvain Filliozat 1997.

REFERENCES

Books:

Billard, R. 1971. *L'astronomie indienne, investigation des textes sanskrits et des données numériques*. Paris: Ecole française d'Extrême-Orient.

Kaye, G. R. 1927. *The Bakhshālī Manuscript, a Study in Mediæval Mathematics*. Calcutta: Archæological Survey of India, New Imperial Series vol. xliii, pts I–II.

Hayashi Takao 1995. *The Bakhshālī Manuscript. An ancient Indian mathematical treatise*. Groningen: Egbert Forsten.

Sanskrit works:

The Śulbasūtras of Baudhāyana, Āpastamba, Kātyāyana and Mānava, with Text, English Translation and Commentary, ed. S.N. Sen and A.K. Bag, New Delhi, Indian National Science Academy, 1983.

Āryabhaṭīya of Āryabhaṭa, critically edited with Introduction, English Translation, Notes, Comments and Indexes by K.S. Shukla in collaboration with K.V. Sarma, New Delhi, Indian National Science Academy, 1976.

Āryabhaṭīya of Āryabhaṭa with the commentary of Bhāskara I and Someśvara, critically edited with Introduction and Appendices by K.S. Shukla, New Delhi, Indian National Science Academy, 1976.

Āryabhaṭīya of Āryabhaṭa with the commentary of Sūryadeva Yajvan, critically edited with Introduction and Appendices by K.V. Sarma, New Delhi, Indian National Science Academy, 1976.

The Yavanajātaka of Sphujidhvaja, edited, translated and commented by David Pingree, Harvard Oriental Series 48, 1978, 2 vols.

Articles:

Filliozat, P.-S. & Mazars, G. 1987. "La terminologie et l'écriture des fractions dans la littérature mathématique sanskrite." *Bulletin d'études indiennes*, n° 5: 91–95. Paris: Association française pour les études sanskrites.

Filliozat, P.-S. 1993a, "Formalisation and orality in Pāṇini's *Aṣṭādhyāyī* ."*Indian Journal of History of Science,* 28(4). 291–301. Delhi: Indian National Science Academy.

Filliozat, P.-S. 1993b. "Ellipsis, Lopa and Anuvr̥tti." *Annals of the Bhandarkar Oriental Research Institute,* vols. LXXII & LXXIII: 675–687. Pune: Bhandarkar Oriental Institute.

Filliozat, P.-S. 1997. "L'utilisation d'outils poétiques dans les mathématiques sanskrites." *Oriens – Occidens*, n° 1. Paris.

Part IV

READING TEXTS

REVIEL NETZ

THE LIMITS OF TEXT IN GREEK MATHEMATICS

ABSTRACT

This article argues for a limited role of the text in Greek mathematics, in two senses of "text": the verbal as opposed to the visual; and the literate as opposed to the "oral" (understood in a wide sense). The Greek mathematical argument proceeds not within the confines of the verbal alone, but essentially relies upon diagrams. On the other hand, it does not use other specific techniques, such as those of the modern cross-reference, relying instead upon verbal echoes. The two, taken together, suggest a model of scientific writing radically different from what we associate with our own mathematics. In methodological terms, the article surveys its evidence in detail, and makes comments concerning the methodology of studying ancient texts through the evidence of those texts alone.

1. INTRODUCTION

The word "text" occurs in the title of this chapter, as presumably in many other titles in this book; yet I doubt if any two chapters share exactly the same definition of "text". Not that it is important to define such a concept. It is the one of the concepts we know too well, which we can only define by artificially limiting our use of it. So I do not intend to define what "text" is; I simply wish to explain what I mean by saying that the text had limits in Greek mathematics. What I mean is that the verbal, written aspect of Greek mathematics had limits. The verbal aspect had limits because of the central importance of the visual aspect, i.e. of diagrams; the written aspect had limits because of the central importance of the verbal, non-written aspect (which we may also call 'oral'—as long as this is not taken as the equivalent of 'illiterate'). So I discuss two limits, the limits of the verbal (measured against the visual), and the limits of the written (measured against the non-written or oral). I start with a relatively detailed discussion concerning Greek diagrams. The discussion of the limits of the written as opposed to the oral will be briefer.

In both cases, my intention is not merely to redress the balance, to reclaim the role of the non-textual elements; my intention is to argue that the textual and non-textual elements cannot be taken apart. This will be clarified in the following.

Before getting started, it is necessary to say something on the nature of the available evidence. An optimist would concentrate on the fact that we have got plenty of material: a few thousand ancient propositions, written by a few dozens of authors. Some of my colleagues in this book can only gasp at such richness of evidence. A pessimist, however,

K. Chemla (ed.), History of Science, History of Text, 161–176.

would stress the fact that practically nothing is available directly, and what we have got are manuscripts written at least a thousand years after the time we are most interested in. The pessimist could also point out that the mathematical works we do have are all first-order texts, none of them is a second-order Greek mathematician talking *about* Greek mathematics. Greek mathematicians are unlike Hilbert, say, in that they speak about triangles and circles but they never speak about mathematics.

Thus, to make a first important clarification, at the current stage of our knowledge it would be rash to offer wholesale conclusions on the precise shape diagrams take in our manuscripts. Secondly, it is a fact that whatever we may know about Greek mathematical attitudes must be deduced from the practice. We interrogate the Greek mathematicians, and the normal methods of interrogation yield nothing, because they keep their silence. We must invent new methods of interrogation to make them speak. In the following, part of the interest lies in what I have learned from Greek mathematicians, another part of the interest, I think, is in the methods of interrogation I have devised.

2. DIAGRAMS

The method of interrogation to be described has to do with what I call the "fixation of reference" or, the one word I will mostly use, "specification"[1]. My meaning is the following. The text of a Greek mathematical proposition always consists of expressions such as

". . . Alpha-Beta is therefore larger than Gamma-Delta"

or

"Since Alpha lies on Beta-Gamma"

and the like. In other words, a Greek mathematical text is, at face value, a discussion of Alpha, Beta, etc. These, of course, refer to other objects. Now, there must be some process of fixation of reference, whereby these letters are related to their objects. The process can be described, complementarily, either as the process in which letters are specified geometrically, or as the process in which geometrical objects are specified by letters. I shall use the two perspectives interchangeably, speaking either of "the specification of letters" or "the specification of objects".

In principle, the process can be verbal or visual; I will show that it is both.

The contrary assumption would take the following shape: that one knew what Alpha stood for by being told. For instance, one may have been told that "Let Alpha be the center of the circle". It should be noted that even here some extra information is called for. For instance, one needs to know that a circle has only one centre. I shall allow for the sake of the following discussion that such assumptions are taken for granted[2]. Even allowing this, I shall still claim that objects in diagrams, as a rule, are specified pragmatically, i.e. through inspection of the diagram.

We now move on to the main table of this article, showing the different ways in which letters are specified in Euclid's *Elements* XIII and Apollonius' *Conics* I. What I ask the

reader to do is to look up the tables as she reads through my explanation of the significance of the different columns (this is my own textual modality, then!).

I shall say, beforehand, why I chose these two works—Apollonius' *Conics* Book I and Euclid's *Elements* Book XIII. The selection is meant to capture a certain middle position of Greek mathematics, between those works of Euclid which are elementary in an absolute sense, and the advanced studies of, say, Archimedes[3]. Furthermore, both works are works of "*Elements*", works which have, apparently, among other things, some pedagogic role; if they exhibit pragmatic specification, then A FORTIORI the same must be true for less self-conscious presentations (though, of course, I do not rely upon this A FORTIORI argument alone[4]).

I also want to add a brief explanation concerning my quantitative methodology. Let me first explain the methodology underlying my use of data such as the following tables. It would be a mistake to think of this methodology as statistical in any sense. First, as representatives of other treatises, the relationship between the two works studied here and the corpus of Greek mathematics taken as a whole is not a matter of statistical sampling but of historical understanding. Sampling would be irrelevant: the field of mathematical treatises is small and uneven; it reflects, more than anything else, the bias of survival whose nature is incompletely understood. Second, as representing the two treatises themselves, the methodology taken here is not statistical, as I survey the *entire* works. My approach, then, is not statistical in the technical sense, but is based on the traditional hermeneutic approach of the historian: I try to understand ancient practices by gaining an empathic understanding of the practice as it is present in a few ideal-type cases which, other things being equal, I take to be representative of the historical reality as a whole. The importance of the quantitative approach of the tables is not that they produce numbers, but rather that, by asking a quantitative question, I force myself to go through the entire treatise in reaching my empathic understanding. The table is an attempt to convey, in succinct form, the experience which underlies this understanding.[5]

Euclid's *Elements,* Book XIII

Prop.	Fully specified	Pre-specified	Under-specified	Un-Specified	Not used in text
1	ΑΒΕΖ	ΓΔ	Η	ΘΚΛΜΝΞΟ	
2	ΑΒΓΖΗ		Δ	ΕΘΜΝΞΚΛ	
3	ΑΒΔΕ	Γ		ΠΣΖΗΘΛΜ ΝΞΟΠ	
4	ΑΒΔΕ	Γ		ΚΘΗΖΛΜΝ	
5	ΑΒΕ	Γ	Δ	ΘΚ	Λ
6	ΑΒ	Γ	Δ		
7	ΑΒΓΔΕ			Ζ	
8	ΑΒΓΔΕΘ				
9	Ε	ΒΓΔΑ			Ζ
10	ΖΘΛ	ΑΒΓΔΕ	ΗΚΜ	Ν	
11	ΖΝ	ΑΒΓΔΕ	ΗΘΚ	ΛΜ	
12	Δ	ΑΒΓ	Ε		

13	ΑΒΓΘ	ΔΕΖΗΚΛ			
13lemma	ΑΒΓΔΕ			Ζ	
14	ΑΒΓΕΖΗΘ	ΔΛΜ	Κ		
15	ΑΒΓΕΖΗΘ	ΔΚΛΜΝ			
16	ΑΒΓΛΜΝΞΟ ΠΡΣΤΨΦΑ′	ΔΕΖΗΘΚΩΨ		Χ	
17	ΑΒΓΔΕΖΗΘ ΚΛΜΝΞ	ΡΣΤΨΦΞΩ	Ψ	ΟΠ	
18	ΑΒΓΔΚ	ΕΗΝ	ΖΛΜ	Θ	
(Final lemma)	ΑΒΓΔΕΖ				
Total	102	55	16	42	2
% (rounded)	47	25	8	19	0

Apollonius' *Conics,* Book I

Prop.	Fully specified	Pre-specified	Under-specified	Un-Specified	Not used in text
1	ΑΒΓΖ	ΔΕ			
2	ΑΒΓΖΗΘΚ	ΔΕ			
3	Α	ΒΓ			
4	ΑΔΕΖΗΘΚ	ΒΓ			
5	ΑΛΖΜ	ΒΓΚΗΘ	ΔΕ		
6	ΑΜΝΔΕΚΖΗ	ΒΓΘ	Λ		
7	ΑΔΕΘΚ	ΒΓ	Ζ	Η	ΛΜ
8	ΑΔΕΗΘΞ	ΒΓ	ΖΚΛΜΝ		ΟΠΡΣ
9	ΑΘ	ΒΓ	ΔΚΕΖΗΛΝΞ	Μ	
10	ΑΗΘ	ΒΓ	ΔΕΖ		
11	ΑΔΕΚ	ΒΓΖΘ	ΗΛΜΝ		
12	ΑΔΕΘΜ	ΒΓΖΛΟΞ	ΗΚΝΠΡΣ		
13	ΑΖΗΛ	ΒΓΔΕΘΝ	ΚΜΞΟΠΡ		
14	Α	ΔΕΖΗΘΚΒΓ ΞΟΡΠ	ΛΥΣΤ	ΜΝ	
15	ΑΒΓΔΕΗΝΦ	ΖΛ	ΘΚΜΞΟΠΥΣΡ ΤΞΨ		
16	ΑΒΓΔΕΖΗΚΛ		ΘΜΝ	Ξ	
17	Γ	ΑΒ			
18	ΓΔΕ		ΑΖΒ		
19	ΓΔ	ΑΒ			
20	ΓΔΕΖΗ	ΑΒ			
21	Γ		ΑΒΔΕΖΗΘΚ		
22	ΓΔΕ	Β	Α	Ζ	
23	ΑΒΓΔΕΖΗ ΘΚΛ				

24	ΓΔΕΖ		ΑΒ		
25	ΑΒΓΔΕΖΗΘΚ				
26	ΔΖΗΘΚΛ	Ε	ΑΒΓ		
27	ΕΜ		ΑΒΓΔΗΖ	Κ	
28	ΑΒΕΖΗΛΝ		ΓΔΘΚΜ		
29	ΑΒΓΔΕΗ		Ζ		
30	ΑΒΓΖΗ		ΔΕ		
31	ΓΔΗΘ	Ε	ΑΒ	Ζ	
32	ΓΕΖ	Δ	ΑΒΘΛΚ	Η	
33	Ζ		ΑΒΓΔΗ	Ε	
34	ΓΔ	ΖΚ	ΑΒΕΗΘΛΜΞ	Ο	
35	ΓΖΔ		ΑΒΕ	Η	
36	ΕΖΘ		ΑΒΓΔΗ		
37	ΕΖ		ΑΒΓΔ		
38	ΓΔΜ	Ζ	ΑΗΒΕΛΘ		
39	ΖΕΗ		ΑΒΓΔ		
40	Κ	ΑΒ	ΖΓΔΕΘΛΗ		
41	ΕΖΗ		ΑΒΓΔΘ		
42	ΓΘ	Η	ΑΒΔΖΕ		
43	ΓΖΗΚΛ		ΑΒΔΕΗΘ	Μ	
44	ΓΟΝΘΞΔ	ΑΒΖ	ΕΗΛΚ	Μ	
45	Δ	ΑΒΓ	ΘΜΛΕΖΚΝ	Η	
46	Λ	ΒΘΜΔ	ΑΓΜΝΖΕΚΗ		
47	ΓΝΛ		ΑΒΔΕΘΟΗΞ ΖΜΚ		
48	ΑΒΓΝ		ΚΛΗΔ	ΟΕ	
49	ΒΗΚΞΜ	Γ	ΜΔΖΝΛΠ	Ε	Ο
50	ΓΚΗΛ	Θ	ΑΒΔΕΖΜΞΠ ΝΠΣΟ		
51	ΑΒΕΗΚΝ		ΓΔΛΜΞ	Ζ	
52	ΓΔΗΘΖΛ	ΑΕΚΞ	ΒΜΝ		
53	ΓΔΒΚ	ΘΑΕΛΜ	ΖΗ		
54	ΑΒΓΕΚΘ	ΔΗΠΡ	ΛΜΞ	ΝΟ	
55	ΑΒΓΔ	ΘΖ	ΗΛΚΜΝΞ	Ο	
56	ΑΒΓΔΕΗΚΜ	Λ	ΞΟΖΘΝ		
57	ΑΒΓΔΕΖΗ				
58	ΒΑΔΕΗΓ	Ζ	ΘΚΛΜΞ	Ν	
59	ΒΕΘΗΑΓΔΖ				
60	ΑΓΔΕΖΗΘΚ ΜΝΟΞ	Γ			
Total	268	100	231	24	7
% (rounded)	42	16	37	4	1

The table can be said to be motivated by the question: is the diagram reproducible from the text? Assume we have only the text (which is the case with a few ancient mathematical works, whose diagrams got lost in the process of transmission: e.g., Archimedes' *Arenarius*). Can we reconstruct the diagram?

The answer is generally positive. But then the question can become more specific: *How* do we reconstruct the diagram then? One way is the global way. In the global way, we read the mathematical proposition from beginning to end, forming a rough impression of what it is trying to say; from the general context, we know the kinds of problems of interest; we have expectations of mathematical relevance; and through the combination of these we gradually may reconstruct a diagram which fits the text and which makes what we see as the "correct mathematical sense". This is the global way, and it is very different from the local way. To reconstruct a diagram locally, what we do is to follow the text as it unfolds. The proposition demands "let a line be drawn"—so we draw that line—and so forth, till the end of the proposition.

Not all the assertions in the text can be used in this way. This is because some assertions clearly are not intended as specifications of objects, but rather assume those specifications. I mean the following:

Consider the following case. Given the following diagram (Fig. 1), the following assertion is made, at some stage of the proof: "and therefore AB is equal to BC". Suppose that nothing in the proposition so far vouchsafed for B being a centre of the circle. Is this assertion then a specification of B as the centre? Of course not, because of the "therefore" in the assertion. The assertion is meant to be a *derivation*, and making it into a specification would make it, effectively, a *definition*, and the derivation would become vacuous. Thus such assertions cannot constitute specifications. Roughly speaking, specifications occur in the imperative, not in the indicative. They are the "let the centre of the circle, B, be taken".

We can thus plot the sequence of moments of specification through a proposition[6]. Each moment of specification sets a locus. Saying "let the centre of the circle, B, be taken" sets a locus consisting of a single point; something like "Let D be taken, so that

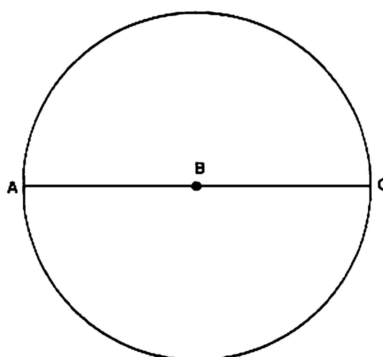


Figure 1.

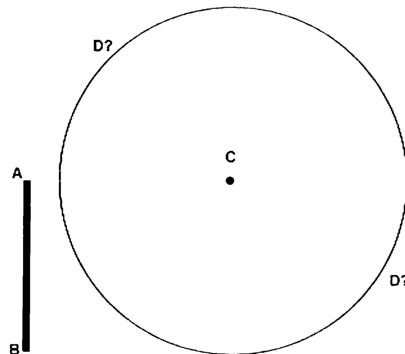


Figure 2.

CD is equal to AB", assuming A, B and C as given, sets up a locus consisting of a circle around C, whose diameter is equal to AB (Fig. 2).

Another locus is the locus which, mathematically speaking, is demanded by the proposition—the locus arrived at "globally". I insist on the concept "locus" here: as a rule, a point may have more than a single legitimate position, as far as the proposition is involved. For instance, there is an important class of true variable points, which, mathematically speaking, should be "any point" within a given domain. Thus, the mathematical locus for such points is the set of points in that domain. On the other hand, the mathematical locus of a point may be more limited. To repeat the example: "Let D be taken, so that CD is equal to AB". This may happen when the mathematical sense demands that D is on a certain given line (Fig. 3). In such a case, the mathematical locus consists of just two points, the intersection of the imaginary circle and the given line. So in such a case the two loci, that defined by the specification, and that demanded by the mathematical situation, diverge. Such divergences are what we are interested in now.

A final general note: it will be seen that the loci involved require, for their definition, letters distinct from the letter which is specified. It may happen that those letters

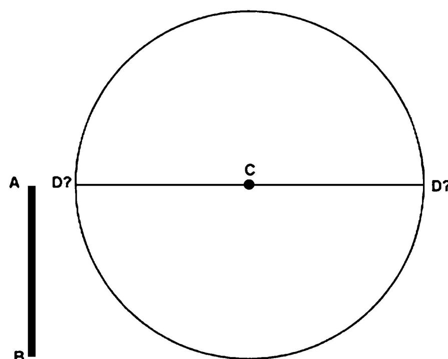


Figure 3.

are underspecified themselves. I have ignored this possibility. I have been like a very lenient teacher, who always gives his pupils a chance to reform. At any given moment, I have assumed that all the letters used in any act of specification were fully specified. I have concentrated on *relative* specification, specification of a letter relative to the preceding letters. This has obvious advantages, mainly in that the statistical results are more interesting: otherwise, practically all letters would turn out to be underspecified in some way.

So let us look at the evidence.

1. The first column, letters which are fully specified, is of course the simplest. The essence of this column is that the locus set up by the specification is identical with the locus demanded by the mathematical situation. A specification setting up a single-point locus, for instance, is bound to be a full specification—the mathematical locus simply cannot be smaller; so, for instance, centers of circles, e.g. E in Euclid's *Elements*, Proposition XIII.9, are indeed ideal. This is a case where specification must be complete because the locus set up by the specification is very small. When the locus demanded by the mathematical situation is very large, full specifications are to be expected, as well; for instance, where a "variable" point is taken, e.g. B in Apollonius' *Conics* I.1: "and let some point on the conical surface, B, be taken[7]" —the locus set up by the specification is the entire conic surface. Nothing more specific is demanded by the mathematical situation, and the point is therefore fully specified. Between these two extremes —the single-point locus, and the "variable" locus— there are many cases, and some of these are fully specified; but the general rule is that full specification is the result of some mathematical necessity — the mathematical locus has such a character which rules out anything short of complete specification.

2. The next column is titled "pre-specified". Pre-specified letters may exist because the same letter may be specified in more than one moment of specification. For instance, Apollonius' *Conics* I.14 (Fig. 4), and the letters B, Γ, Ξ, O. At (Heiberg 1891–1893, 1: 54, lines 7–9) these letters are specified as two pairs of indeterminate points on two respectively given circles. At (Heiberg 1891–1893, 1: 54, line 16) they are re-specified as the result of an intersection of these two circles with a given plane.

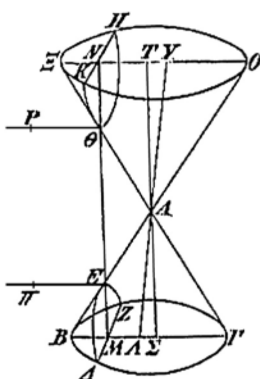


Figure 4. Apollonius' *Conics*, Proposition I.14.

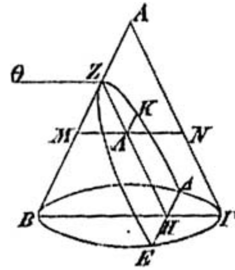


Figure 5. Apollonius' *Conics*, Proposition I.11.

What happened, on the cognitive level, between these two moments? Hypothetically, there are two options. One is the suspension of specification: i.e., the reader did not relate the letters to specific points. The other option is that the reader went beyond what was given in the text, and related the letters to the points as they appear in the configuration of the diagram.

Had this been the only class of letters failing full specification, the choice between these two options would have been real. However, given that there are other, clear cases, where the specification must have been aided by the diagram, I find it hard to believe that, in such cases alone, suspension of specification was exercised[8].

As explained in the table, I do not distinguish, in this column, between two possible cases: one, where the sum of all the moments of specification constitutes a full specification; another, when all the moments of specification taken together still constitute a mere under-specification. I now move on to explain the difference.

3. The third column is titled "under-specified".

Here the confessional mode may help to convert my readers. It took me a long time to realize how ubiquitous this form of specification is. The reason for my obtuseness was the fact that, as soon as one is even slightly acquainted with the Greek approach, one starts to fill the pragmatic gaps automatically. Reading a Greek proposition, one cannot but gaze constantly at the diagram; and visual information compels itself in an unobtrusive, almost unnoticed way. The following example came to me as a shock. It is, in fact, a very typical case.

Look at Λ in Apollonius' *Conics*, Proposition. I.11 (Fig. 5). It is specified at (Heiberg 1891–1893, 1: 38, line 26), where it is asserted to be on a parallel to ΔE, passing through K. This sets up the locus of a line; so how is one to know that Λ is in fact a very specific point on that line, the one intersecting with the line ZH? I know how I knew this: by looking at the diagram.

There is another way by which one could, theoretically, know this: by taking into account the entire proposition. Several later assertions in the proposition make sense only on the assumption that Z is where it is. To repeat: when we discover a partial manuscript, in which the diagrams are absent, we can still reconstruct them. But the process through which a diagram is thus reconstructed from the text is not a true reading of the text, for the following reason, already hinted above. In such a reconstruction, assertions which are meant to *derive*, are taken to *define*. To put this succinctly: un-interpreted by

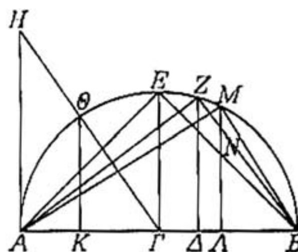


Figure 6. Euclid's *Elements*, Proposition XIII.18.

diagrams, propositions are invalid. Their invalidity consists not in contradictions, but in NON SEQUITURs. The NON SEQUITURs could be rectified, it is true, in principle: but why assume them to begin with?

4. The fourth column is titled "unspecified". This is when letters emerge out of thin air. They occur in assertions, without being anywhere specified. Take for instance Θ in Euclid's *Elements*, Proposition XIII.18. It is first mentioned in the following sentence: "let HΓ be joined, and let a perpendicular be drawn, from Θ to AB, <namely> ΘK" (Fig. 6). Clearly Θ is not specified here at all, it is taken as part of the specification of K. To specify Θ, one would have mentioned the semi-circle; this is not done here. So Θ appears out of thin air. More impressive still is Z in Apollonius' *Conics*, Proposition I.51. At (Heiberg 1891–1893, 1: 156, line 2) the line ΓE is drawn and produced; nothing further is said about it (Fig. 7). Then at (Heiberg 1891–1893, 1: 156, line 9) we are told that a certain property is obvious, "for ΓZ is twice ΓE". That is how Z is introduced into the proposition; very much like "for the Snark was a *Boojum*, you see".

Unspecified letters are relatively speaking rare. They are more common in Euclid's *Elements* Book XIII, but this is only because Euclid uses in propositions 1 to 5 the lazy formula "and let the figure be completed"[9] —an important practice, but not a representative one. Apollonius' *Conics* are more typical, and there unspecified letters occur occasionally, no more: a letter or two, every two or three propositions. This is not much; on the other hand, this is not negligible. It would be extremely rash to suggest that all of these unspecified letters are so many indications of lacunae in the text, though I would agree that some of them may result from such lacunae.

5. The last column is very hard to gauge, and it is included here merely for the sake of completeness. Very rarely, letters—apparently well attested in the manuscripts—occur

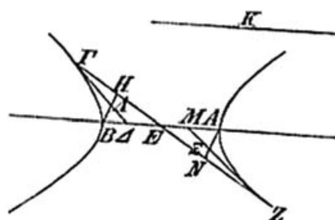


Figure 7. Apollonius' *Conics*, Proposition I.51.

in the diagram but not in the text. Who inserted them and why I do not profess to know: it must be said that most probably these started their lives as accidental marks of ink in some archetype.

The table shows that specification must be to a great extent pragmatic, almost certainly diagrammatic. This is the foundation on which all forms of verbal and visual interdependence are built. I will make two general comments on this, and then move on to the next limit.

A. Why are there so many cases falling short of full specification?

First, it should be quite clear, once again, that the tables have little quantitative significance. It is immediately clear that the way in which letters in Apollonius' *Conics* fail to get full specification is different from that of Euclid; and I suspect that there is a strong variability between works, even by the same author. The *way* in which letters are not fully specified depends upon mathematical situations. Euclid, for instance, may construct a circle, e.g. ABΓΔE, and then constructs a pentagon within the same circle, such that its vertices are the very same ABΓΔE; this is pre-specification, and is demanded by the subject matters dealt with by Euclid's book XIII. In Apollonius' *Conics*, parallel lines and ordinates are the common constructions, and letters on them are often underspecified.

What seems to be more stable is the percentage of fully specified letters: less than half the letters are fully specified —but not much less than a half. It is as if the authors were indifferent to the question whether a letter is specified or not, full specification being left as a random result[10].

This, I would claim, is exactly the case. In other words, I would say that nowhere in Greek mathematics do we find a moment of specification PER SE, a moment whose *purpose* is to make sure that the reference of letters in the text is fixed. Such moments are very common in modern mathematics, at least since Descartes[11]. But specifications in Greek mathematics are done, literally, AMBULANDO. The essence of the moments whose sequence I plot —the moments where the imperative mood is employed, "let a line be drawn . . . "— their essence is to do some job upon the geometric space, to get things moving there. When a line is drawn from one point to another, the letters corresponding to the start and end positions of movement ought to be mentioned; but they need not be carefully differentiated, one need not know precisely which is the start and which is the end, both would yield the same job, the same line (hence under specification); and points traversed through this movement may be left unmentioned (hence unspecification). So, to repeat, there is no specification PER SE, no moment at which the text sets out to take the reins of the proposition in its hands. The text does not try to subdue the diagram, to govern it. The text assumes the diagram. Rather than one of them governing the other, the text and diagram present, let us say, a *cohabitation*. For, indeed, not only is the diagram non-recoverable from text. The following is true as well:

B. The text is not recoverable from the diagram.

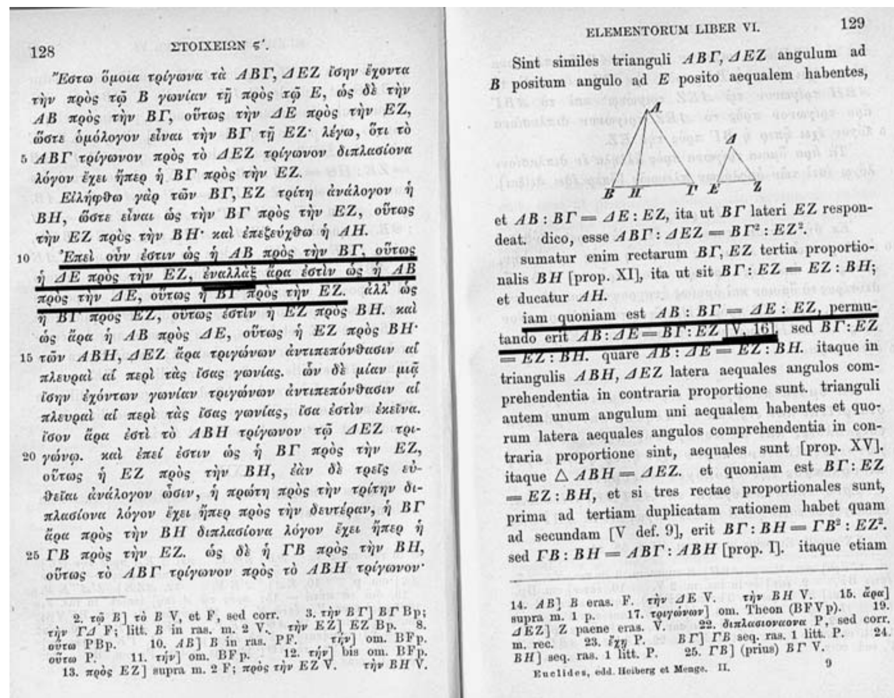
Of course, the diagram does not tell us what the proposition asserts. It could do so, theoretically, by some symbolic apparatus; it does not. Further, the diagram does not specify all the objects on its own. There are many metrical properties —e.g. equality of lengths— that are not expressed diagrammatically in any real sense. When the diagram is "dense", a rich complex of lines and letters, even the attribution of letters to points may not be so obvious from the diagram, and modern readers, at least, using modern diagrams,

use, to some extent, the text to elucidate the diagram. The stress of the discussion so far should be on *inter*-dependence. True to my promise, I did not try merely to offset the traditional balance between text and diagram; I have tried to show that they cannot be taken apart, none makes sense in the absence of the other.

3. CROSS-REFERENCES

I now move to the second limit, the limit of the written as opposed to the verbal (or the oral)[12]. There are various ways in which the oral plays a surprisingly important role in Greek mathematics —e.g., I would say that many arguments are mediated by the *verbal* structure of the assertions manipulated by those arguments. This is an important claim, which however I cannot support in detail here. I have chosen to concentrate here on a simpler practice, that of cross-references, and to show the role of the oral (or the limits of the written) concerning this practice.

The situation is easy to grasp by opening any edition of a Greek mathematical work. Heiberg —who edited practically all of them— had the useful practice of supplying a latin translation. Within this translation (Fig. 8), he also inserted, within square brackets, references. In these references he pointed out that a certain derivation or construction is validated by a certain prior result[13].


128 ΣΤΟΙΧΕΙΩΝ ϛʹ.

Ἔστω ὅμοια τρίγωνα τὰ ΑΒΓ, ΔΕΖ ἴσην ἔχοντα τὴν πρὸς τῷ Β γωνίαν τῇ πρὸς τῷ Ε, ὡς δὲ τὴν ΑΒ πρὸς τὴν ΒΓ, οὕτως τὴν ΔΕ πρὸς τὴν ΕΖ, ὥστε ὁμόλογον εἶναι τὴν ΒΓ τῇ ΕΖ· λέγω, ὅτι τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖ τρίγωνον διπλασίονα λόγον ἔχει ἤπερ ἡ ΒΓ πρὸς τὴν ΕΖ.

Εἰλήφθω γὰρ τῶν ΒΓ, ΕΖ τρίτη ἀνάλογον ἡ ΒΗ, ὥστε εἶναι ὡς τὴν ΒΓ πρὸς τὴν ΕΖ, οὕτως τὴν ΕΖ πρὸς τὴν ΒΗ· καὶ ἐπεζεύχθω ἡ ΑΗ.

Ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΖ, ἐναλλὰξ ἄρα ἐστὶν ὡς ἡ ΑΒ πρὸς τὴν ΔΕ, οὕτως ἡ ΒΓ πρὸς τὴν ΕΖ. ἀλλ' ὡς ἡ ΒΓ πρὸς ΕΖ, οὕτως ἐστὶν ἡ ΕΖ πρὸς ΒΗ. καὶ ὡς ἄρα ἡ ΑΒ πρὸς ΔΕ, οὕτως ἡ ΕΖ πρὸς ΒΗ· τῶν ΑΒΗ, ΔΕΖ ἄρα τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας. ὧν δὲ μίαν μιᾷ ἴσην ἐχόντων γωνίαν τριγώνων ἀντιπεπόνθασιν αἱ πλευραὶ αἱ περὶ τὰς ἴσας γωνίας, ἴσα ἐστὶν ἐκεῖνα. ἴσον ἄρα ἐστὶ τὸ ΑΒΗ τρίγωνον τῷ ΔΕΖ τριγώνῳ. καὶ ἐπεί ἐστιν ὡς ἡ ΒΓ πρὸς τὴν ΕΖ, οὕτως ἡ ΕΖ πρὸς τὴν ΒΗ, ἐὰν δὲ τρεῖς εὐθεῖαι ἀνάλογον ὦσιν, ἡ πρώτη πρὸς τὴν τρίτην διπλασίονα λόγον ἔχει ἤπερ πρὸς τὴν δευτέραν, ἡ ΒΓ ἄρα πρὸς τὴν ΒΗ διπλασίονα λόγον ἔχει ἤπερ ἡ ΓΒ πρὸς τὴν ΕΖ. ὡς δὲ ἡ ΓΒ πρὸς τὴν ΒΗ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΑΒΗ τρίγωνον·

2. τῷ Β] τὸ Β V, et F, sed corr. 3. τὴν ΒΓ] ΒΓ Bp; τὴν ΓΔ F; litt. Β in ras. m. 2 V. τὴν ΕΖ] ΕΖ Bp. 8. οὕτω PBp. 10. ΑΒ] Β in ras. PF. τὴν] om. BFp. οὕτω P. 11. τὴν] om. BFp. 12. τὴν] bis om. BFp. 13. πρὸς ΕΖ] supra m. 2 F; πρὸς τὴν ΕΖ V. τὴν ΒΗ V.

ELEMENTORUM LIBER VI. 129

Sint similes trianguli ΑΒΓ, ΔΕΖ angulum ad Β positum angulo ad Ε posito aequalem habentes,

et ΑΒ : ΒΓ = ΔΕ : ΕΖ, ita ut ΒΓ lateri ΕΖ respondeat. dico, esse ΑΒΓ : ΔΕΖ = $B\Gamma^2 : EZ^2$.

sumatur enim rectarum ΒΓ, ΕΖ tertia proportionalis ΒΗ [prop. XI], ita ut sit ΒΓ : ΕΖ = ΕΖ : ΒΗ; et ducatur ΑΗ.

iam quoniam est ΑΒ : ΒΓ = ΔΕ : ΕΖ, permutando erit ΑΒ : ΔΕ = ΒΓ : ΕΖ [V, 16], sed ΒΓ : ΕΖ = ΕΖ : ΒΗ. quare ΑΒ : ΔΕ = ΕΖ : ΒΗ. itaque in triangulis ΑΒΗ, ΔΕΖ latera aequales angulos comprehendentia in contraria proportione sunt. trianguli autem unum angulum uni aequalem habentes et quorum latera aequales angulos comprehendentia in contraria proportione sint, aequales sunt [prop. XV]. itaque △ ΑΒΗ = ΔΕΖ. et quoniam est ΒΓ : ΕΖ = ΕΖ : ΒΗ, et si tres rectae proportionales sunt, prima ad tertiam duplicatam rationem habet quam ad secundam [V def. 9], erit ΒΓ : ΒΗ = $\Gamma B^2 : EZ^2$. sed ΓΒ : ΒΗ = ΑΒΓ : ΑΒΗ [prop. I]. itaque etiam

14. ΑΒ] Β eras. F. τὴν ΔΕ V. τὴν ΒΗ V. 15. ἄρα] supra m. 1 p. 17. τριγώνων] om. Theon (BFVp). 19. ΔΕΖ] Ζ paene eras. V. 22. διπλασιονασονα P, sed corr. m. rec. 23. ἔχῃ P. ΒΓ] ΓΒ seq. ras. 1 litt. P. 24. ΒΗ] seq. ras. 1 litt. P. 25. ΓΒ] (prius) ΒΓ V.

Euclides, edd. Heiberg et Menge. II. 9

Figure 8. Heiberg's Edition of Euclid's *Elements*, Proposition VI.19.

References have wide applications in science, but in deductive mathematics they are crucial. In the deductive game, things may be asserted without proof only if they are known to have been proved earlier. So the knowledge of what was previously proved is an essential part of doing the actual deductive work. Without references, deduction collapses.

In the Latin side of an edition by Heiberg references are textual in a very precise sense. They are mediated through a written code. The question is, what happens in the Greek side. The answer is, first, that usually nothing happens explicitly.

Still working with the same two books, Apollonius' *Conics*, Book I and Euclid's *Elements*, Book XIII, we may see that Apollonius uses Euclid's *Elements* 195 times; Euclid's *Elements*, Book XIII uses Euclid's *Elements* (book XIII itself excluded) 126 times. For good measure, I add in Archimedes' *Spiral Lines*, where Euclid's *Elements* are used 36 times. So we have got a 357-strong population here; and I believe the reader would agree with me that the results here are not untypical of Greek mathematics as a whole.

What are these results? First, in the great majority of cases, the fact that the author uses a previously established result is not even registered. The derivation or the construction is simply made; it remained for Heiberg to note that some reference to a previous result was necessary (of course, Heiberg was following earlier editors and scholastic readers, perhaps from Late Antiquity onwards). Such completely tacit reference takes place 165 times in Apollonius, 107 times in Euclid and 33 times in Archimedes. The percentages are about 85% for both Apollonius and Euclid, and a (meaningless, given the size of the corpus selected) 92% for Archimedes.

All the remaining cases, except one, make a gesture, a nod, towards Euclid. And this gesture is not made through a written code, similar to Heiberg's. The gesture takes the form of a slight redundancy in the formulation of the construction or the derivation. Consider the following, typical example. There, the structure of the argument is:

since

a:b::c:d[14]

therefore

a:c::b:d

The shorter way to deal with such a derivation is simply to state it, in the abbreviated form above. But, quite often (for such references are especially common with this sort of move), a certain redundancy is made. Instead of saying

a:b::c:d, therefore a:c::b:d

The author will then say

a:b::c:d, therefore, *enallax*, a:c::b:d

The word ENALLAX is unmotivated, it is surprising in the context of the very economic language usually used by Greek mathematicians. What it does is to recall the formulation through which the relevant result was proved by Euclid, in *Elements* V.16[15].

This was an example of a minimum redundancy: a one-word redundancy. Sometimes a fuller evocation of the original formulation is made. This is not done often, but it does occur from time to time. For instance, Apollonius' *Conics* I.15 makes what is practically a quotation of Euclid's *Elements* I.5[16]. There are altogether about 10 such quotations in our sample; here especially it is interesting to note that Euclid's handling of quotations from Euclid is not different from Apollonius' handling of quotations from Euclid. So this is rare, but not absolutely rare.

What is absolutely rare is the single exception to the rule, the reference made by Euclid's *Elements* XIII.17 to Euclid's *Elements* XI.38[17]: "for this was proved in the penultimate theorem of the eleventh book". This is the only Heiberg-like reference to Euclid in our whole sample. And it is typical that this is a reference from Euclid to Euclid; for these rare references do occur elsewhere in Greek mathematics, but almost always in the context of references *within* the same work[18].

So we have seen that the form of reference is usually quite implicit, and when some explicit reference is made, it relies upon verbal formulations, almost never upon citations by books and proposition-numbers.

It should also be remembered that —as I have pointed out above— the sum total of the results required by Greek mathematicians for their proofs is, roughly, that contained in Euclid's *Elements*[19]. There are exceptions to that, to which I will return in a minute; but these exceptions are less important than the rule.

So I repeat:

A. Greek mathematicians refer through verbal echoes, not through reliance upon textual guides.

B. Greek mathematicians refer to Euclid's *Elements* and not to very much else besides.

The two are obviously connected. Euclid's *Elements* represent a pool, a set of results which Greek mathematicians would have internalized, and in a sense the internalization would have been oral. It would have relied upon the verbal formulation of propositions. So there is this verbal, oral pool, which one internalizes once and for all, and then uses as a mathematician. Euclid's *Elements* is the first floor of Greek mathematics, upon which the entire second floor must be built, and this is because the engineering principle which makes each floor rely upon the one beneath it is oral, not textual, and is therefore limited. A few very gifted mathematicians internalized the second floor as well, to some extent. Especially, they became very proficient in the *Conics* and developed, on the basis of the *Conics*, a few advanced theories. But this was the absolute limit of Greek mathematics. Greek mathematics was a three-level building, and you cannot build any taller building on an oral basis. Without (among other things) this emphasis on textual learning, science cannot explode exponentially, recursively, as it did since the invention of printing.

I wish to qualify my picture, however. Greek mathematics is of course not wholly oral. In some aspects, it is very written. The very use of diagrams in which *letters* are inscribed is —need I point out?— a feature of a written culture. While the principle by which Euclid was internalized was oral, the dissemination of Euclid was impossible without writing. Indeed, the numbers of mathematicians were always so small, that writing was a necessary medium of communication. The truth is, I think, one should try not to think of the written and the oral as two polar categories. They are two cognitive processes, which may occur side by side. To put this briefly, one may simultaneously write and

speak. So I am not saying that Greek mathematics is an oral practice, to the exclusion of its written aspect, with the consequence that this written aspect is illusory. The written aspect is real. However, as is true of Hellenistic culture in general, Greek mathematics has a certain dual nature, both very oral and very written and, to repeat our preceding discussion, both very visual and very verbal. Hellenistic culture is shaped by certain dualities, certain tensions—tensions which were extremely fertile. The text had limits in Hellenistic culture. Or is it us who are limited[20]?

Stanford University

NOTES

[1] Not to be confused with the technical term used by Morrow, G.R., in his translation of Proclus' Commentary on the First Book of Euclid's *Elements* (Princeton, 1970), to translate *diorismos* (p. 159).

[2] In other words, I assume that the contents of what we know as Euclid's *Elements* were a background to the very discourse of Greek mathematics; I shall return to this in the second part of this article.

[3] The tripartite scheme which is envisaged here —elementary, advanced, and another option, in the "middle"— will be clarified and defended in the following.

[4] The following is a list of some letters which are not fully specified, collected at haphazard; any Greek mathematical work will do. Archimedes' *Spiral Lines* 6, B; *Balancing Planes* I.13, I; *Sphere and Cylinder* I. 10, EBZ; II.4, ΑΒΓΔ; *Quadrature of Parabola* 16, Θ; *Method* 1, B; Euclid's *Elements* I.47, Z; XI.28, H; *Data* 70, Δ; *Optics* 40, E; Autolycus' *Moving Sphere* 10, ZH; Aristarchus' *On the Sizes and Distances of the Sun and the Moon* 7, Δ. I do not claim at all that these are the only letters in the relevant propositions which are not fully specified.

[5] The above paragraph is my only substantial addition to this chapter since its original writing in the 1990s. I ask the reader to take this paragraph as a note of explanation written as a response to a query I had often heard since publishing Netz (1999).

[6] This is not to say that these moments are *primarily* moments of specification. I will argue below that there are no moments of specification PER SE.

[7] Apollonius' *Conics*, (Heiberg 1891–1893, 1: 8, lines 26–27).

[8] Of course, some suspension of judgment is a feature of Greek—and English—syntax. In expressions such as, say, "and let the point A be taken on the line CD, so that AC is equal to AD", one might have said that there are two separate moments of specification, the "on the line CD" specification, and the "AC = AD" specification, so that we have got here a pre-specification. However, I ignore such bogus pre-specifications, where suspension of judgment is clearly demanded by the syntax. My pre-specifications are all genuine. For bogus pre-specifications, the verb *poiein* is a good guide; see e.g. Apollonius' *Conics*, Proposition I.55 (Heiberg 1891–1893, 1: 172, lines 9–12), the letters ZH.

[9] (Heiberg 1883, 4: 248, lines 12–13; 250, line 27 to 252, line 1; 254, line 6; 258, line 4; 260, line 10). The reference is to a specific type of figure, called by Taisbak 'gnomonic' (Taisbak 2003): the practice is lazy but not arbitrary.

[10] This is a qualitative, not a quantitative comment. The number "fifty percent" is highly suggestive, but it does not directly *prove* randomness: for this, we must have a measure of the "background" (for coins, we assume the "background" is 1/2, for dice, we assume it is 1/6)—and this we do not have here.

[11] In Descartes' *Geometry*, two sign systems are set side by side, the geometric and the algebraic, and a coordination between these two systems is required, hence a moment of specification PER SE.

[12] "Orality"—a concept inevitable in the human sciences, and yet impossibly misleading. So to clarify immediately, by "orality" I do not mean illiteracy. I mean a use of language in which the verbal properties of language (which I take to be essentially predicated upon the spoken and the heard) are seen as the central forms of communication.

[13] Some references, when relevant, point out the existence of a useful comment by Eutocius.

[14] This is a standard and useful way to encode—in a written symbolism, alien to the Greek and more oral approach—the assertion "a is to b as c is to d". Greeks would always say the full phrase—with names fuller than our 'a', 'b' etc.—a verbal formula, the non-written equivalent of our written formula.

[15] I leave aside for the moment an important question, namely the shape and diffusion of Euclid's *Elements* in antiquity. It may well be that Apollonius and Archimedes —and even Euclid!— used "*Elements*" different from those we know. "Euclid's *Elements*" is a label, no more. However, it is possible to reconstruct, as it were, the *Elements* used by Apollonius and Archimedes from their uses of basic theorems; and it becomes clear that, by and large, they had to use something very similar indeed, in content, to what we know as Euclid's *Elements*. The redundancies described here give as a rare glimpse into the *formulations* of the *Elements* they used; on the whole, though not always, these seem to be those of Euclid's *Elements* as we know them.

[16] (Heiberg 1883, 1: 60, lines 22–24). This quotation —and this is true in general of such quotations— is not VERBATIM. One is tempted to deduce that this is yet another manifestation of orality; however, it will be prudent to remember, as mentioned above, that our knowledge of the exact text of *Elements* used by Apollonius is conjectural.

[17] (Heiberg 1883, 4: 322, lines 19–20).

[18] I do not wish to explain away any evidence, but it must be remembered that, in general, such explicit references are the most natural glosses, and while many of them could be original, at least some must be later accretions. This is not a pedantic point; it hints at the important reflection, that the context within which Greek mathematical works were copied, Byzantine scholarship, was already a much more text-oriented context than that in which Greek mathematics originated.

[19] And in general not all of that: almost all of book X, for instance, a good quarter of the *Elements*, is a dead-end for most mathematical purposes.

[20] I wish to thank K. Chemla and B. Vitrac for many useful comments on an earlier version of this article.

REFERENCES

Heath, T.L. 1913. *Aristarchus of Samos, the ancient Copernicus*. Oxford: The Clarendon Press.

Heiberg, J.L. 1883. *Euclidis Elementa*, 5 vols. Leipzig: Teubner.

Heiberg, J.L. 1891–1893. *Apollonii Pergaei quae Graece exstant cum commentariis antiquis*. Leipzig: Teubner.

Heiberg, J.L. 1910–1915. *Archimedis opera omnia cum commentariis Eutocii*, 3 vols. Leipzig: Teubner.

Mogenet, J. 1950. *Autolycus de Pitane: histoire du texte suivie de l'édition critique des traités* De la sphère en mouvement *et* Des levers et couchers. Louvain: Bibliothèque de l'Université.

Morrow, G.R. 1970. *Proclus / A Commentary on the First Book of Euclid's Elements*. Princeton, N.J.: Princeton University Press.

Netz, R. 1999. *The Shaping of Deduction in Greek Mathematics: a Study in Cognitive History*. Cambridge (UK): Cambridge University Press.

Taisbak, C.M. 2003. "Exceeding and Falling Short: Elliptical and Hyperbolical Application of Areas." *Science in Context* 16: 299–318.

JIM RITTER

READING STRASBOURG 368: A THRICE-TOLD TALE

How Does a Poem Mean?
Title of an article by John Ciardi in
Saturday Review of Literature, c. 1959

ABSTRACT

Every reading act takes place within one or more contexts. The choice of a contextualization other than that standardly produced by a given reader can lead to new ways of questioning the text itself. Here three different contexts are constructed in which to view an Old Babylonian mathematical tablet: other contemporaneous Babylonian mathematical texts, Egyptian mathematical texts, Babylonian technical texts of a non-mathematical nature. Each of these leads to a different way of viewing the manner in which the text encodes and structures its information and aids in extending our understanding of it. Finally, on the basis of the foregoing, the nature of "anachronism" in historical studies is queried.

What does a text mean? A question like this poses particularly acute problems for someone who works on ancient texts. This is not to underplay the complexity and plurality of interpretations in, say, contemporary literary texts, but rather that in the case of Antiquity we are often confronted with the opposite situation: the difficulty of finding even a single possible interpretation. The problem may occur at a very concrete level: lacunæ in the texts, hapaxes, a technical vocabulary for which it is difficult or impossible to fix the semantic referents. Moreover, we are often confronted with contemporary constraints, not always immediately perceptible to us, concerning, among other things, the rules of the textual genre, the available concepts or techniques of expression, the aims pursued by the authors, ... Further, in the case of ancient Near Eastern texts no tradition of discourse, no contemporary meta-analyses—either philosophical or even linguistic— exist to provide us with what are seen, in other cases, as a privileged interpretative pattern, allowing us to take up one or another thread in the tapestry of responses already explored or possible.

With respect to this last there is, in fact, a hidden advantage. For the presence of a self-reflexive tradition as one part of a corpus of texts carries with it the risk of causing a relaxation of our critical stance before the rest of the corpus. There is the danger of reading at first degree all the texts in the unique light of this tradition—especially, as in the Greek case for instance, where successive rereadings of this tradition has become so rooted in our own historically determined canonical approaches to intellectual studies.

K. Chemla (ed.), History of Science, History of Text, 177–200.

There is even a tendency to accept the ancient commentary as a sort of primary source for the domain itself—placing Plato or Aristotle on mathematics on the same level (or 'higher') than the texts of mathematical practice themselves.[1] More generally, in the case of the ancient Near East, just because we are less liberally endowed with raw material, the choices we necessarily make in reading any text can more easily be made explicit and visible. In what follows I shall explore the effect of a variety of such choices for a single second-millennium Babylonian text.

No text exists in isolation; the reading of a given text depends, among other things, on the set of texts with which—against which—it is read. Such a set is not uniquely defined; the Western tradition is principally authorally centered and a standard approach in contemporary textual analysis is to assume that the first construction of such a set is to include all or part of the totality of works composed by the same author. Thus *Henry VIII* is normally read, at first at least, in relation to Shakespeare's other histories, *Richard II* or *Henry IV*, and then say in the context of other 'late period' works of the author, such as *The Winter's Tale* and *The Tempest*. Authorship (and the cortege of questions about the preeminence or intention of the author for the determination of the meaning of a text) is not an issue for the ancient Near Eastern texts on which I will focus here, dealing with such 'technical' subjects as computations of areas or treatments of illnesses—they are all anonymous. But their features will allow us to experiment with the contexts in which we read a given text; we can carry out a kind of 'experimental contextualization' in which variations of contexts can reveal to us different aspects of the text under study.[2]

My approach will be to determine the changes made in the understanding of a single text by choosing for an anchorage a series of different *contexts*, each defined and delimited by a specific *corpus*. An important methodological point to note is that this approach does not assume that any of the contexts studied were or were not relevant to the authors or previous readers of the text; it is *neutral* vis-à-vis ontological or epistemological positions. For example, one may hold a pure relativist position (which I do not) and believe such context choices to be all equivalent, or a historicist point of view (which I share), in which some contexts have an historical basis while others serve a heuristic purpose for the modern student. I shall return in a more explicit way to this point at the end of the paper.

As my point of departure I will use Strasbourg 368 (Str 368), a mathematical problem text of the Old Babylonian period (c. 1700 BC).[3] I will choose three different contexts for my text, contexts in which I have worked recently, in order to show how different questions are raised by different choices of corpus—all, I think, interesting—and how different parts of the texts are thrown into relief in these various contexts. If I shall use Str 368 to illustrate my results, the conclusions come, as they must, from a systematic use of the full corpus. The contexts will be selected among:

- contemporary texts in the same field, *viz.* Old Babylonian mathematical problem texts,
- mathematical texts from other civilizations, here Egyptian,
- other contemporary textual genres from Mesopotamia: medical, divinatory and juridical.

Let me begin with a presentation of our central text:[4]



I took a reed. Its length I did not know. I broke off 1 cubit and then I went sixty times along the length (of a field). What I had broken off I restored, and I then went 30 times along the width (of the field). 6 15 was the area (of the field). What was the original length of the reed?

You in your procedure:

Put 1 and 30. For the reed, which you do not know, put 1. You will multiply by 1 the sixty times you went: 1 will be the false length. Multiply 30 by this 1: 30 will be the false width. Multiply 30, the false width, by 1, the false length: 30 will be the false area. Multiply 30 by 6 15, the true area: it will give you 3 7 30. Multiply 5, which was broken off, by the false length: it will give you 5. Multiply 5 by the false width: it will give you 2 30. Fractionize the half of 2 30: 1 15. Square 1 15: 1 33 45. Add to 3 7 30: 3 9 3 45. What is the square root? 13 45 will be the square root. Add the 1 15 that you squared: it will give you 15. Find the inverse of 30: 2. Multiply 2 by 15: 30.

30 was the original length of the reed.

1. STR 368 IN THE CONTEXT OF OLD BABYLONIAN MATHEMATICAL TEXTS

The first context within which we shall view Str 368 is that furnished by the total set of known mathematical problem texts of roughly the same period and cultural area, that of southern Mesopotamia during the Old Babylonian period (roughly the first half of the second millennium BC).[5] I have elsewhere argued that the mathematical texts are principally school texts, divided into the categories of tables and procedural texts with

the latter subdivided between those with and without explicit solutions.[6] Our corpus is the less than one hundred cuneiform texts in which a mathematical problem is posed and a method of solution given immediately afterwards, Str 368 being precisely a typical example of the kind, with its rhetorical and numerical text, its use of standard metrological systems in the posing (and solution) of the problem, coupled with a distinct, positional, base-60 abstract number system for the calculations, etc.[7]

Otto Neugebauer's pioneering work of the 1930s and 1940s sought a unifying mode of reading by using an elementary algebraic interpretation of these texts. I have urged for some years to take seriously the textual form common to all these tablets, namely an algorithmic reading.[8]

From a structural point of view, the identification of each sentence of Str 368 with an algorithmic operation can be seen by simply separating out and numbering the steps:[9]

> I took a reed. Its length I did not know. I broke off 1 cubit and then I went sixty times along the length (of a field). What I had broken off I restored, and then I went 30 times along the width (of the field). 6.15 was the area (of the field). What was the original length of the reed?
>
> You in your procedure:

1	Put 1.00 and 30.
2	For the reed, which you did not know, put 1.
3a	You will multiply by 1 the sixty times you went: 1.00 will be the false length.
3b	Multiply 30 by this 1: 30 will be the false width.
4	Multiply 30, the false width, by 1.00, the false length: 30.00 will be the false area.
5	Multiply 30.00 by 6.15, the true area: it will give you 3.07.30.00.
6	Multiply 0;05, which was broken off, by the false length: it will give you 5.
7	Multiply 5 by the false width: it will give you 2.30.
8	Fractionize the half of 2.30: 1.15.
9	Square 1.15: 1.33.45.
10	Add to 3.07.30.00: 3.09.03.45.
11	What is the square root? 13.45 will be the square root.
12	Add the 1.15, that you squared: it will give you 15.00.
13	Find the inverse of 30.00: 0;00.02.
14	Multiply 0;00.02 by 15.00: 0;30.

> 0;30 was the original length of the reed.

A number of observations can already be made. Each of my numbered lines contains exactly one arithmetical or logical operation. The arithmetical operations present here are addition (**10**, **12**), multiplication (**3–7**, **14**), inverse (**13**), squaring (**9**), square root extraction (**11**). The absence of subtraction is fortuitous, that of division general—our operation of division is carried through in Babylonian texts by a coupled inverse and multiplication (e.g. **13–14**). To these must be added the fractionation of step **8**; not a multiplication by 1/2 (and certainly not a division by 2) but a distinct operation, represented by quite another verb from those used to indicate multiplication.

Steps **1** and **2** are not arithmetical but initialization procedures, signalled by the use of the verb "to put".[10] Step **1** initializes the entire algorithm, using two of the problem's data. This use of two initial values in fact, here and elsewhere, is the introduction of a bifurcation or (non-conditional) branching in the algorithm in which two series of calculations are

carried on "in parallel". Step **2** initializes the value of a 'variable' or 'register' which will ultimately contain the length of the reed. Step **3** carries out two multiplications on these two initial values, before they are folded together in the multiplication of step **4**.[11]

All this is even more transparent if I rewrite the text in modern symbolic terms, with arithmetical operator signs replacing the verbs (**D** represents a datum in the presentation of the problem.):

Data		$\mathbf{D_1} = 1$ cubit (= 0;05 **nindan**)	
		$\mathbf{D_2} = 1.00$	
		$\mathbf{D_3} = 30$	
		$\mathbf{D_4} = 6.15$	
1	1.00	|	30
2		1	
3	$1 \times 1.00 = 1.00$	|	$1 \times 30 = 30$
4		$30 \times 1.00 = 30.00$	
5		$30.00 \times 6.15 = 3.07.30.00$	
6		$0;05 \times 1.00 = 5$	
7		$5 \times 30 = 2.30$	
8		$\frac{1}{2}(2.30) = 1.15$	
9		$(1.15)^2 = 1.33.45$	
10		$1.33.45 + 3.07.30.00 = 3.09.03.45$	
11		$\sqrt{3.09.03.45} = 13.45$	
12		$13.45 + 1.15 = 15.00$	
13		$(30.00)^{-1} = 0;00.02$	
14		$0;00.02 \times 15.00 = 0;30$	

How information flows in our text—that is, the origin of the various numbers used in the algorithm—can be more easily traced if I do one more rewriting, indicating by a bold-face number the number of the step which is at the origin of the numerical values used, without concern for the actual value. Thus step **10** is the sum of the result arising in step **9** with that of step **5** and the operation of **10** will be written **9** + **5**. Using this procedure, with the convention that $L' \rightarrow n$ represents the assignation of a value n to a 'memory' or 'register' L', we have:

Data	$\mathbf{D_1}, \mathbf{D_2}, \mathbf{D_3}, \mathbf{D_4}$,
1	$\mathbf{D_2} \mid \mathbf{D_3}$
2	$L' \longleftarrow 1$
3	$L' \times \mathbf{D_2} \mid L' \times \mathbf{D_3}$
4	$\mathbf{3a} \times \mathbf{3b}$
5	$\mathbf{4} \times \mathbf{D_4}$
6	$\mathbf{D_1} \times \mathbf{1a}$
7	$\mathbf{6} \times \mathbf{1b}$
8	$\frac{1}{2}\mathbf{7}$
9	$\mathbf{8}^2$
10	$\mathbf{9} + \mathbf{5}$
11	$\sqrt{\mathbf{10}}$
12	$\mathbf{11} + \mathbf{8}$
13	$\mathbf{4}^{-1}$
14	$\mathbf{13} \times \mathbf{12}$

As a general rule, one of the values of a given step is to be found as the result of the immediately previous step, that is the flow is linear or sequential. These sequential values are not signalled in any special way by the text; the other values used in a given step are however generally marked either by a name—the datum $\mathbf{D}_4$ used in **5** is glossed as the "true area", its name in the presentation of the problem—or by a reference to its previous functional use in the algorithm—the "1.15 that you squared" in **12**, i.e., the result of **8** used in the squaring operation of **9**.

There are two major exceptions to this: **6** and **13** where only earlier results are used in the calculation; I have indicated them by a line. Though unmarked in our text, such divisions into independent subcalculations are marked in other texts: "Let your head retain n. Return."[12] This marks the end of one calculation—the result of which is to be stored for later use in the algorithm—and the beginning of another, logically distinct, in which the result of the previous calculation will be used at some point (**5** is used in **10**, **12** in **14**).

Up to now, the other texts of the Old Babylonian mathematical corpus have been useful in the elucidation of the calculational and logical dimensions of the algorithm of Str 368—they have indicated that the methods and organization of this text are canonical and, in the case of branching, have shown that an explicit, technical vocabulary existed for this concept. But such a corpus allows us to understand also how Old Babylonian algorithms were articulated *across* problem thematic borders, at the formal and structural level.

To take one example, let us now examine another mathematical procedure text, BM 13901,[13] a collection of pedagogically arranged problems around the determination of the sides of one or more squares whose areas and sides are in various relations. The second problem of this tablet, arranged here in numbered steps, is the following:

I subtracted my side from the surface (of a square): 14.30.

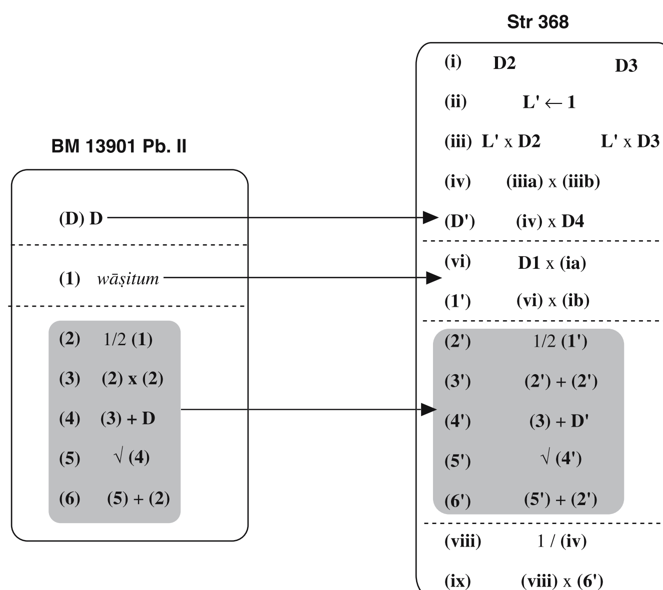
1	You will put 1, the *wāṣitum*
2	You will fractionize the half of 1: (0;30)
3	You will multiply 0;30 and 0;30: (0;15)
4	You will add 0;15 to 14.30: 14.30;15
5	29;30 will be the square root
6	You will add the 0;30 that you multiplied to 29;30: (30)

30 was the side of the square.

Thus, in principal, the subject matter is quite distinct from that of Str 368. Yet if now I rewrite the above algorithm in its symbolic and algorithmic forms, the following is obtained:

	Symbolism	*Algorithm*
Data	$\mathbf{D} = 14.30$	$\mathbf{D}$
1	1	*wāṣitum* $\longleftarrow 1$
2	$\frac{1}{2}(1) = 0;30$	$\frac{1}{2}\mathbf{1}$
3	$0;30 \times 0;30 = 0;15$	$\mathbf{2} \times \mathbf{2}$
4	$0;15 + 14.30 = 14.30;15$	$\mathbf{3} + \mathbf{D}$
5	$\sqrt{14.30;15} = 29;30$	$\sqrt{\mathbf{4}}$
6	$29;30 + 0;30 = 30$	$\mathbf{5} + \mathbf{2}$

Note that steps **2–6** of BM 13901–II are *functionally* fully identical, in terms of information flow, to steps **8–12** of Str 368. That is, after the initialization of the *wāṣitum* in **1**, the sequence of operations on this value is precisely what occurs in the latter text to the value imported from step **7** into the five following steps—with the functional role played by the datum of the problem in step **4** of BM 13901–II replaced by the output of step **5**, the reserved value of the first subcalculation of Str 368. All this can be summed up in the following scheme:



where, for Str 368, I have renumbered the steps I–VII, **2′–6′**, VIII–IX to identify more clearly the identical part of the two algorithms. Seen from this recontextualized perspective, the core of the algorithm proposed by Str 368 is identical to that to be presented by BM 13901–II, the suite of five operations beginning with halving and ending with a multiplication. The role of steps **1–5** (I–V) and **6–7** (VI–VII) is then to transform the problem into one in which the basic algorithm of BM 13901–II can be applied, the first group serving to calculate a quantity which will serve functionally as a datum while that of the second is to create a value to be used as an initialized *wāṣitum*. Finally, steps **13–14** (VIII–IX) are necessary to undo the transformations of the first part, translating the answer found into one adequate for the problem posed. In short the basic algorithm of BM 13901–II finds itself *embedded* as a subalgorithm in Str 368.

The phenomenon is more general. Indeed many of the other 25 problems on the same tablet, BM 13901, are based on this algorithm or some variant; Problem III, for instance, uses a structure very similar to Str 368—transformation of the *wāṣitum*, application of the base algorithm (actually a small modification of it, introduced by Problem I of the

same tablet) and then an inverse transformation of the answer thus produced. If this tablet is particularly rich in such embeddings this is probably due to its systematic, pedagogical purpose. But other mathematical procedure texts operate in very similar ways with this or a reduced number of other basic embedded subalgorithms.

This illuminates the problems of contextualization. Although our choice of corpus here seems rather obvious, its systematic exploitation is not. We have seen how a choice of context throws into relief certain specific aspects of a text. The ordinary modern classification of mathematical texts of this period is by theme or subject of the problem; and in fact there exist a number of mathematical texts where the subject of the exercise is similar, though slightly more complex, i.e., a reed of unknown length is used to measure the various sides of a (trapezoidal) field with bits of the reed being alternately broken off and added on.[14]

The actual value of the field being given, the student is asked to find the original length of the reed. These texts have been treated together—from a modernizing algebraic point of view—by Neugebauer. But if one restricts oneself (implicitly) to this subcorpus, the question of embedding does not appear: all that is seen is a development of the algorithmic procedure. The embedding of algorithms—a witness of the sophistication reached and also of the interest of our interpretation—becomes visible only when our original text finds a corpus which is large enough to show such phenomena, in particular one which does not privilege problem statement and surface subject matter.

2. STR 368 IN THE CONTEXT OF EGYPTIAN MIDDLE KINGDOM MATHEMATICAL TEXTS

I now propose to put Str 368 in quite another context, one that would no doubt have surprised the Akkadian scribe that copied that document: as far as we can tell, the contemporaneous civilization in the Nile Valley had at this time, referred to in modern times in usual Egyptological chronologies as the Middle Kingdom, no significant contact with Mesopotamia. Certainly there is no trace in the mathematical texts of either of influence from the other. Nevertheless, it is precisely out of the Middle Kingdom mathematical texts[15] of this corpus, the twenty-sixth problem of the Rhind papyrus:[16]



Data	**A quantity; its fourth has been added to it.** It has become 15.	$\mathbf{D_1} = 4$ $\mathbf{D_2} = 15$	$\mathbf{D_1}$ $\mathbf{D_2}$
[**0**	Initialization	4	$Q' \longleftarrow \mathbf{D_1}$]
1	Calculate starting with 4. You will make their fourth: 1.	$\frac{1}{4}(4) = 1$	$\overline{Q'}(Q')$
	[1 4]		
	[\ $\frac{1}{4}$ 1]		
2	[Add] Total: 5.	$1 + 4 = 5$	$\mathbf{1} + Q'$
3	Calculate starting with 5 to find 15.	$5 \mid 15 = 3$	$\mathbf{2} \mid \mathbf{D_2}$
	\1 5		
	\2 10		
	3 will result.		
4	Calculate starting with 3, 4 times.	$3 \times 4 = 12$	$\mathbf{3} \times Q'$
	1 3		
	2 6		
	\4 12		
	12 will result.		
	[Verification]		
	1 12 — **The quantity: 12**		
	$\frac{1}{4}$ 3 — Its $\frac{1}{4}$: 3		
	Total: 15 — **Total: 15**		

I have adopted the same series of rewriting as used in the first part of this article but have telescoped the successive stages into a single presentation. In the column at left my numbering of the steps of the algorithm with the name of 'Data' used to indicate the formulation of the problem. The next column to the right contains the English translation of the original hieratic text (roman type indicates the use of black ink in the original, **boldface** indicates the use of red ink), my additions being given in brackets. The third column represents the rewriting in terms of modern symbolism and the final rightmost column the algorithm in general terms.

The only new symbolism adopted is that the inverse of a number Q is represented by $\overline{Q}$ and that $n \mid m$ represents the operation of dividing m by n.

That the text is also algorithmic in my previous sense is clear. The algorithm, like that in Str 368, is given in terms of concrete numbers and in rhetorical form. But there are also a number of striking differences in the manner of organization and of expression of this algorithm. There is of course a great difference in the phrasing—and indeed in the choice—of the various arithmetic operations. The Egyptian term for multiplication of n by m is 'calculate starting with n, m times'. Division, unlike in Mesopotamian practice, exists as a specific Egyptian operation under the expression 'calculate starting from n to find m' for the operation of dividing m by n.

A more striking difference is the fact that almost each step in the Egyptian solution algorithm is accompanied by a small calculation, effected on two columns, with the result indicated following the calculation. Moreover, the whole algorithm is followed by a section marked 'verification' by me, for, though untitled here, one finds just such a heading in other problems in this and other Egyptian procedure texts. And this verification has itself an algorithmic form, paralleled by the actual carrying out of the calculation of each operation.

The existence of a calculational level in the Egyptian texts throws into relief the fact of its 'lack' in the Babylonian texts. By seeing explicitly what choice

of calculational techniques the Egyptian scribe was taught to deploy in the carrying out of each arithmetic operation, we are led to pose the same question of our Mesopotamian sources. In our example, the two explicit calculations of division (step **3**) and multiplication (step **4**) both involve the same choice of technique, successive doubling. This however is far from exhausting the panoply of techniques open to the Egyptian scribe, which involve also halving, tripling and its inverse, decupling and its inverse, inversion, two-thirds, squaring, square root extraction and others besides.[17]

But practically no Babylonian text speaks of calculational techniques. A rare exception is YBC 6295, which discusses interpolation in the calculation of cube roots. The reference in the text to a cube root "which has not been given",[18] together with mentions in other texts of an inverse which "cannot be found",[19] reveal the Mesopotamian manner of effecting the operations in algorithms—the use of tables. Precisely such tables have been found for practically all the operations called for in the various solution algorithms of the Babylonian procedure texts: tables of multiplication, inverses, squares and cubes and square and cube roots, etc.[20]

The explicit two-level structure of the Egyptian algorithm has led to a questioning of the apparent 'flat', single-layer nature of a Babylonian algorithm. In return, the use of a very specific vocabulary in Str 368 involving "false" lengths and surfaces implies that there is yet more to the story, and calls attention to the functional role of the mathematical objects so designated. In Str 368 the false surface, once calculated on the basis of an initialized—and presumably false—value for the length of the reed (step **2**) is used in the determination of the calculation of the true length. This forms part of a standard mathematical strategy in which, given some numerical information about a parameter of a problem, an arbitrary value for the reply is assumed, the same parameter calculated on the basis of this assumed value, their comparative values being then used in the calculation of the true value of the answer.

We are then dealing with yet another level of the algorithm, more general than that of the calculational techniques or that of the arithmetical operations, the level of method of solution, the choice of strategy of resolution. Here it is the turn of the Egyptian corpus to fall silent; no such terminological hints as the 'false length', 'true length' etc. of Str 368 are to be found in Egyptian mathematical texts. An examination however of the method employed in problem 26 of the Rhind papyrus shows that it is a similar method of using arbitrary (but convenient) values for an initial guess at the answer. The unknown quantity is assumed to have the value 4 (so that one-fourth of it becomes simply 1). The conditions of the problem being applied to this false value, the final result is 5, not the 15 demanded (step **2**). A correction factor of 3 is then calculated (step **3**) and applied to the false value 4 (step **4**), yielding the true value 12.

Again we see that a recontextualization of Str 368 situates this text in a quite different manner, creating new questions and highlighting states of the text that are not visible in other contexutalizations. Here, we see methods, operations and techniques operating at distinct levels. Various choices can be—and are—made for each of these levels, choices by civilization, by numerical convenience, by pedagogical aim.

3. STR 368 IN THE CONTEXT OF OB 'RATIONAL PRACTICE' TEXTS

Up to now the contexts we have considered—and the corpora selected by them—have probably appeared to be more or less natural for the modern historian of mathematics. This is in fact due simply to the traditional approach in this field which, as we have already pointed out, tends to classify either by type of problem or by (modern) discipline boundaries projected onto the culture. The third context, to which we now turn, is of quite another sort, akin to more recent trends in discourse and rhetorical analysis.

We shall examine the grammatical structure of Str 368, and specifically its verbal chains. Let us look once more at the translation of the text.

> I **took** a reed. Its length I **did** not **know**. I **broke off** 1 cubit *and then* I **went** sixty times along the length (of a field). What I ***had broken off*** I **restored**, *and then* I **went** 30 times along the width (of the field). 6.15 (was) the area (of the field). What (was) the original length of the reed?
>
> You in your procedure:
>
> **Put** 1.00 and 30. For the reed, which you ***did*** not ***know***, **put** 1. You **will multiply** by 1 the sixty times you ***went***: 1.00 (will be) the false length. **Multiply** 30 by this 1: 30 (will be) the false width. **Multiply** 30, the false width, by 1.00, the false length: 30.00 (will be) the false area. **Multiply** 30.00 by 6.15, the true area: it **will give** you 3.07.30.00. **Multiply** 0;05, which ***was broken off***, by the false length— it **will give** you 5. **Multiply** 5 by the false width: it **will give** you 2.30. **Fractionize** the half of 2.30: 1.15. **Square** 1.15: 1.33.45. **Add** to 3.07.30.00: 3.09.03.45. What (will be) the square root? 13.45 (will be) the square root. **Add** the 1.15, that you ***squared***: it **will give** you 15.00. **Find** the inverse of 30.00: 0;00.02. **Multiply** 0;00.02 by 15.00: 0;30. 0;30 (was) the original length of the reed.

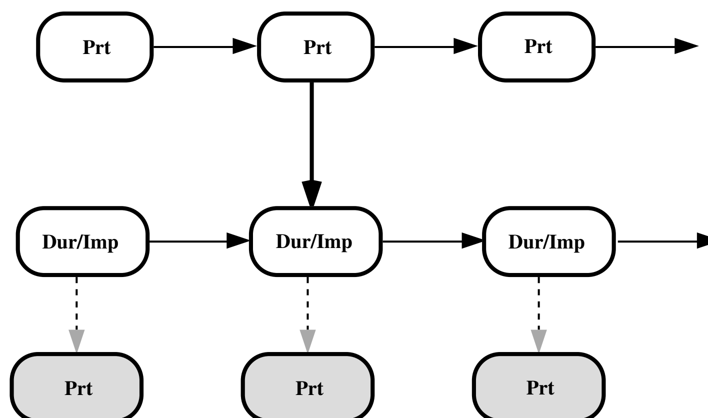
The verbs in our text have been highlighted by a series of typographic markers: **boldface roman** for principal verbs and ***boldface italics*** for verbs in subordinate clauses. In addition, the Akkadian enclitic particle *-ma*, which serves as a verbal suffix, has been translated by '*and (then)*' or '*but*' according to the sense in the presentation of the problem and by ':' in the solution algorithm.

Since they are now of importance, I recall that there exist five finite verbal forms in Akkadian: a block of three which conjugate by means of prefixes and suffixes—durative, preterite, perfect—one which does so by means of suffixes alone—stative, and the imperative/precative. In addition there are two 'moods': indicative, with no marker and subjunctive, marked by the suffix-*u*.[21] In all translations of Akkadian verbs, the following conventions have been used:

- verbs in the imperative are translated by the English imperative,
- verbs in the preterite are translated by the English past tense,
- verbs in the durative are translated by the English future tense,
- since a copula is not expressed in Akkadian nominal sentences, it has been added (in parentheses) in the English translation, without a typographical marker and in a tense which agrees with that of surrounding verbs.

We see clearly that the verbal forms fall into groups which are more or less homogeneous as a function of their logical (as opposed to grammatical) role in the text. That is, the statement of the problem uses the preterite exclusively, while the solution algorithm is primarily composed of either the imperative or the durative. There is an occasional use of the preterite in the second algorithmic part of the problem; in this case, it is invariably in the subjunctive mood, serving in a relative clause modifying the logical subject of a principal clause "the reed, which you ***did*** not ***know***", "0;05, which ***was broken off***", etc. We recall how the numbers used in the algorithm arise: normally, they are the result of the directly preceding step and in this case they are never marked, semantically or morphologically. But in some cases, they are imported from tables or are the results of former steps.[22] The preterite is used in just such cases, as an indicator that the number entering into an operation is not the immediately preceding result and serves to indicate this other origin in a precise manner.

To better seize the grammatical structure of this text I shall represent it in the form of a scheme in which only the verbs appear and then only as a form and a mood. I shall use some conventions: an oval ⬭ indicates a verb with the form inscribed within. A horizontal arrow ⟶ represents the connection, often signalled by *-ma*, between two consecutive verbs. A solid vertical arrow notes an irreversible change in level, here between problem statement and solution algorithm; a dashed vertical arrow indicates a reversible change of level, here between principal clause and possible modifying relative clause in the algorithm proper. Moreover the mood is represented by color: black and white for the indicative, gray for the subjunctive.



To sum up: the first line indicates a potentially unlimited chain of indicative verbal forms in the preterite, connected by the particle *-ma*—the presentation of the problem. The text then shifts to a second, distinct level, that of the 'procedure', in which the verbal forms are in the durative or imperative forms, chained in a sequence of algorithmic steps which can also continue as long as necessary. The alternation imperative/durative in the mathematical texts is not limited to operation = imperative, result = durative as in this text; a glance at BM 13901 quoted in the first section will show that the durative can serve

to indicate arithmetical operations as well. Each operation—or rather its data—can be, if necessary, further specified by a relative subjunctive clause in the preterite referring to previous steps or consultation of a table.

We shall (seemingly) reverse our procedure of contextualization in this third part by *constructing* the corpus on the basis of a criterion—that of the existence of rigid verbal chains—rather than starting from a predefined corpus and picking out salient features. I say "seemingly" because, as will be discussed in the conclusion, I feel that the apparent differences between the two approaches is an (unavoidable) artefact of historically grounded classificatory schema.

We thus pose the question if such rigid verbal chains exist elsewhere in the Akkadian literature. The reply is yes, but only in a few genres of texts. It should be stressed that the issue is not simply one of rigidity of syntax, for accounting texts, among others, exhibit often an even greater degree of this than mathematical texts. It is rather a specific kind of stereotyped, formulaic arrangements of verbal forms, arranged in at least two levels of chains of verbs of the same nature. Using this criterion we can constitute a new corpus of Old Babylonian texts: along with mathematical problem texts we find tablets from exactly three domains:

- divination
- medicine
- jurisprudence

This then is our new context. In what follows I shall give three typical texts drawn from these three domains which will serve both to illustrate their structure and to permit an examination of the concrete points of resemblance and difference with Str 368, that is, simply to work through the chosen contextualisation.

Divination

As our first example I shall take a typical divination text, BM 22446, one involving the prediction of the future from the forms that oil takes when poured into water.[23] Here are the first five entries:

1 If the oil, I **poured** it on water
and the oil **descended** *and then* **rose** *and* **surrounded** its water—
for the military campaign: (it will be) the arrival of calamity;
for the sick man: the hand of the god, the hand **will be heavy**.

2 If the oil **divided** into two parts—
for the military campaign: the two camps **will march** against each other;
I proceed for the sick man: he **will die**.

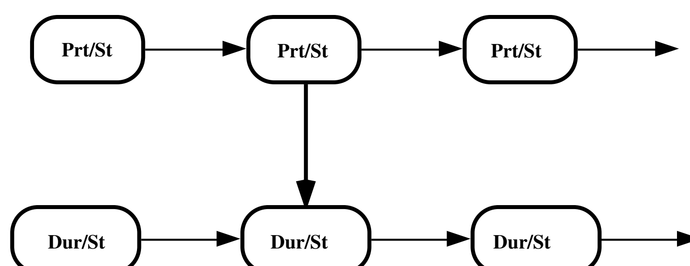
3 If from the middle of the oil a drop **came out** towards the East *and then* **stopped**—
I proceed for the military campaign: I **will carry away** spoils;
for the sick man: he **will recover**.

4 If, from the middle of the oil, two drops **came out**
and one **was large** and the other **was small**—

the man's wife **will give birth** to a boy;
for the sick man: he **will recover**.

5 If the oil **dispersed** *and* **filled** the bowl—
the sick man **will die**;
for the military campaign: the army **will be defeated**.

which gives rise to the following scheme, superimposing the two classes of predictions—for the army and for a sick man—as a single chain of durative clauses:



The structure of the divination texts with their two levels of verbal chains, preterite and durative is familiar. The appearance of the stative as an alternative to the finite verbs in both chains is due to a lexical rather than a structural difference. The expressions 'to be heavy', 'to be large', 'to be small', etc., adjectival predicates in English, are actually stative forms of verbs in Akkadian (the other finite forms have the meanings 'become heavy, large, small, . . .'). These verbs are naturally much more common in the divination texts, with their descriptions of the appearance of oil in water or the organs of sacrificial animals, than in the mathematical texts, though there exist some examples in the latter class too.

Medicine

We have only a handful of medical procedure texts from the Old Babylonian period. The most detailed—and the one I present here—is HS 1883.[24] What follows are the second through the sixth entries on this tablet, followed by the scheme:

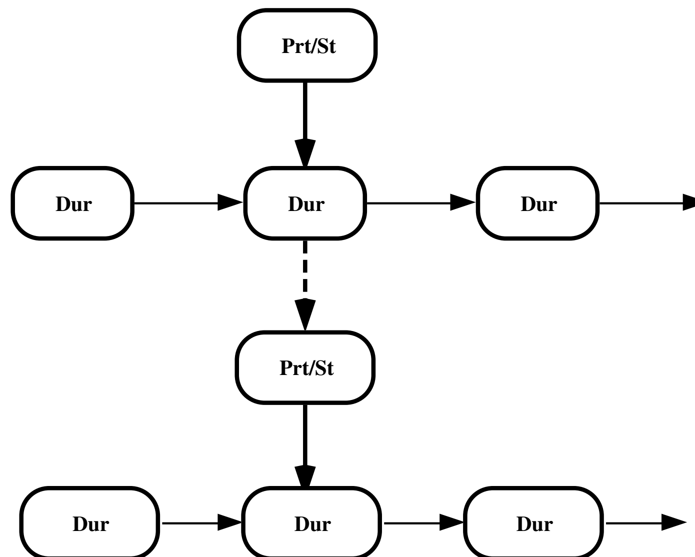
2 If a man **was sick** with jaundice—
you **will soak** licorice root in milk *and* you **will let stand during the night** under the stars
and you **will mix** (it) in oil *and* you **will give** to him to drink *and* he **will recover**.

3 If a man, his tooth (had?) a worm—
you **will grind** (the plant?) "sailor's excrement" in oil *and*
if his right tooth **was sick** *then*
you **will pour** (it) on his left tooth *and* he **will recover**—
if his left tooth **was sick** *then*
you **will pour** (it) on his right tooth *and* he **will recover**.

4 If a man **was covered** with an eruption—
bit by bit, you **will mix** malt flour in oil *and* you **will apply** (it) *and* he **will recover**—
if he **was** not **cured**,
you **will apply** warm *ṣimtum and* he **will recover** —
if he **has** (still) not **been cured**,
you **will apply** the warm residue *and* he **will recover**.

5 If a man, a scorpion **stung** him—
you **will apply** (the plant?) "ox excrement" *and* he **will recover**.

6 If a man, his eyes **were sick**—
you **will crush** the *ḫartītum*-plant *and* you **will apply** (it) *and* he **will recover**.



Once again we have the preterite/stative for the presentation of the (medical) problem—though here only as a single verb—and a durative chain for the algorithm of solution. The novelty here is the occasional reduplication of the whole pair, for example in **3** where there is a branching regarding the method of application, or in **4** where the repetition is a marker for alternative or successive treatments of the same illness. In either case the entire two-chain structure is taken up once again.

Jurisprudence

The final example of the corpus, the well-known 'Code' of Hammurapi, is by far the best preserved of Old Babylonian juridical procedure texts.[25]

Once again, I shall use just the opening articles:

§**1** If a man **accused** a (nother) man *and* he **charged** murder (against him)
but he **has** not **convicted** him—
his accuser **will be killed**.

§**2** If a man **charged** sorcery against a (nother) man *but* he **has** not **convicted** him—
the one against whom sorcery ***was charged*** **will go** to the River *and*
will plunge into the River *and*
if the River **has mastered** him:
his accuser **will carry off** his household;
if that man, the River **has purified** him *and* he **has come out** safe and sound:
the one who ***had charged*** him with sorcery **will be killed**;
the one who ***had plunged*** into the River **will carry off** the household of his accuser.

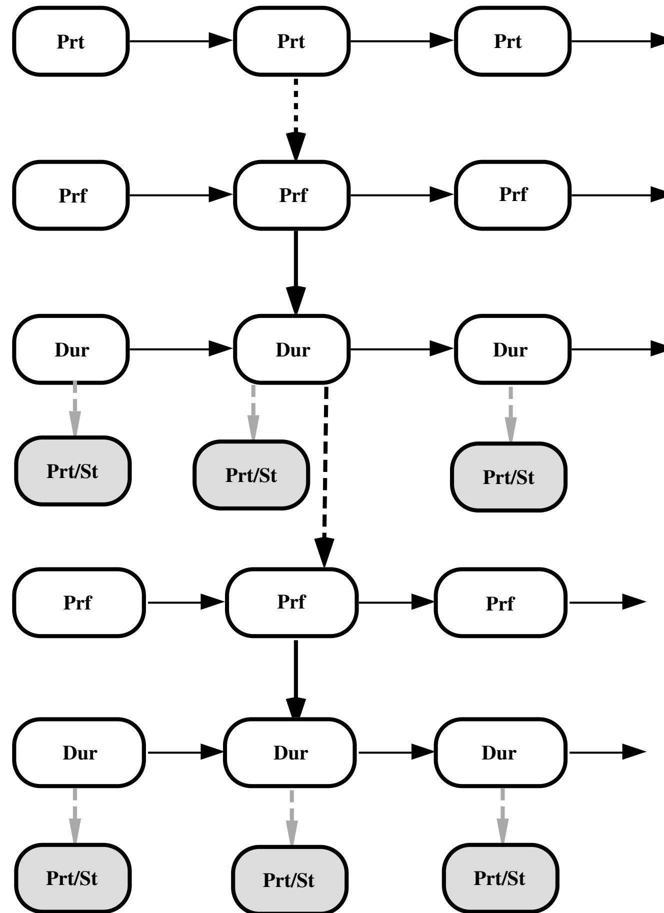
§**3** If a man **appeared** in a trial with false testimony *and* (with) the word that he ***had spoken***
he **has** not **convicted** (the accused)—
if that trial (was) a capital trial:
that man **will be killed**.

§**4** If he **appeared** in order to bear (false) witness (concerning) grain or silver—
he **will bear** the penalty of that trial completely.

§**5** If a judge **judged** a case; **rendered judgement**; **had delivered** the sealed record *but*
later he **has changed** his judgement—
that judge, in the judgement that he ***had rendered***,
one **will convict** him of changing (his judgement) *and*
the fine which ***was involved*** in that trial, he **will deliver** (it) twelvefold *and*
moreover, one **will expel** him from his seat as a magistrate in the Assembly *and*
never again **will** he **sit** with the judges in a trial.
(lit.: he **will** not **return** *and* he **will** not **sit** . . .)

A verbal newcomer in this text is the Akkadian perfect, translated systematically here by the English present perfect: "has convicted", "has mastered" . . . . The appearance of this new verbal form is not at all haphazard but follows a definite pattern as is made clear by the associated scheme which follows:

The juridical literature presents a much more complex picture than the other domains we have seen up to now. We find again the preterite/stative chain and the durative with, as in the mathematical texts, optional use of a subjunctivized preterite (or here also stative) to qualify logical subjects in the durative sequence. Again there is the possibility, as in



the case of the medical texts, of adding reduplicated levels to the procedural level. But the innovation of the jurisprudential domain enters here, since these repetitions involve a totally new level, marked by the use of the perfect verbal form. It is the perfect verbal clauses, referring to the specific circumstance which marks off any particular case from traditional precedents, that are a unique feature of the juridical part of the corpus.

In the scheme I have indicated this level—and its reduplication—as being optional (dashed vertical arrow) in order to cover such cases as **4**. However such short entries are perhaps better understood as abbreviated slight modifications of preceding articles (**4** is a slight variation on the theme of **3**), in which the perfect verbal chain of the last is to be tacitly taken over in its entirety.

What have we accomplished in creating this new contextualization? First of all, the simple fact of the existence of a group of texts coming from a restricted number of Old Babylonian intellectual domains and sharing a common distinctive formal structure,

different from any other in the cuneiform literature of the period such as those pertaining to administrative or mythological texts,[26] signals that we ought to look carefully at what are the similarities—and the differences—among the domains thus singled out. Once such a corpus assembled, we may first of all use our previous analysis of the structures of mathematical texts to pose questions concerning analogous structures in the rest of our corpus.

In the case of mathematical texts we have already seen the important role that tables play in the effecting of the operations called into play by the solution algorithms. Within the context of this new corpus we are encouraged to pose the question of the existence of tables with similar functional roles in the nonmathematical fields. Such tables do in fact exist in the domain of medicine: lists of *materia medica* such as plants or stones, of diseases, and especially, tables which interrelate diseases and the plants useful in combating them. The line between such tables and the procedural texts is not always clear cut; there exists rather a spectrum from the purely tabular verbless tables, through tables involving abbreviated procedures, to the full procedure texts with their structured verbal chains.[27]

This point can be seen in the divination texts with their very simple verbal chains. Are they abbreviated procedure texts or elaborated tables? The lack of fuller documentation does not permit a definite answer.

But we must also pose the inverse question. How does the new context permit us to better understand our starting text, Str 368? One of the ways is the light that the use of a certain technical vocabulary common to our collection of domains sheds on their use in any one of them. Let us take for example the word in Str 368 I have translated by "procedure"; the Akkadian term is *nēpešum*, a nominalizing derivative from the Akkadian verb *epēšum* = "to do". In general, translations of this term in the Assyriological literature remain vague, "action" or the like. Its appearance in very specific contexts, not only in mathematical texts, to introduce or close the solution algorithm, but also in medical texts—at least from the Middle Babylonian period (second half of the second millennium BC) on—as one of the terms for medical treatment, and, in the feminine form *nēpeštum*, for the act of divination, indicates that the term has a quite definite and consistent meaning in this corpus of texts. The fact that it occurs rather rarely outside our corpus is a further indication of its essentially technical nature (Ritter 1990:107–108).

More generally, the similarity in structure can be seen as evidence for a similarity in the functional way in which these domains operated. Those familiar with contemporary legal systems built on precedent, like British and US common law, are not surprised by a juridical code which uses a series of sample cases to establish general rules of jurisprudence. Similarly, even modern medical textbooks make use of typical case studies to teach future practitioners. That Akkadian mathematical texts operate in the same way is not therefore something "recipe-like" or "merely empirical"—these epithets are often traps in that they avoid contact with the need to understand their real cognitive mode of operation—but simply an alternative approach to the development and communication of mathematical practice, one that systematically covers the domain of the possible by a grill of typical examples. Generalization for instance is achieved not by creating englobing "rules" or "laws" but by interpolating any new problem in the mesh of existing known results.[28]

These links can be further traced in the nature of the social groups who learned from and used these texts in their professional practice. Looking at administrative documents and letters yields essentially the same picture—a group of professionals who were seen by their contemporaries as forming a special class—as the close reading we have done above. The social practice of practitioners of medicine, divination and, to a lesser extent, judges can be traced fairly easily in such documents. That of ordinary accounting scribes, those presumably for whom the mathematical corpus was constructed, is much less visible and knowing where to look for analogies in the reconstruction of the practice of the latter can thus be of great utility (Ritter 1989 and 1993b).

A last dividend of this approach is the skepticism it encourages concerning the possibility of creating seemingly natural hierarchies of complexity. From a semantic point of view it would appear that the solution algorithms of the mathematical texts, with their three layers and their embedded subalgorithms, are inherently more complex than the corresponding procedures in the medical or juridical texts. Yet from an organizational point of view, the mathematical subcorpus, like that of divination, is much simpler than the medical texts and even more so compared to the law code. Any attempt to create a hierarchy of complexity must first make explicit the domains constituting the corpus and the ranking values which are being used in the construction of the scale of value.

ENVOI

I have attempted to show in these pages the manner in which the meaning of a text is tightly bound to a series of interconnected constructions. Each reader brings to a text an embedding in a larger corpus of texts, one which shapes his or her understanding. This is inevitably the case but can be done in a more or less conscious manner; I have tried to argue here for the importance of bringing explicitly into play the precise contents of the corpus as well as that of making that corpus as complete as possible. From this point of view ancient mathematics is perhaps privileged since the entirety of known tablets can be realistically surveyed. Moreover I have underlined the advantages of adopting several different explicit contexts in order to explore the maximum of aspects of the text, many of which achieve visibility only under the changing light of multiple recontextualizations.

With regard to the *construction* of the three contextual corpora of Str 368 examined here, there at first glance seems to be a marked difference between the first two and the third. In the former case, the set of texts seem to form a 'natural' set, from which common or contrasted elements could be 'naturally' isolated. In the third case however, the decisive element—verbal chains—determined the choice of corpus. In fact, the difference between the two cases is historical. The criteria for a Mesopotamian text to qualify as 'mathematical' go back to the late nineteenth century—and even then the tools had been well-honed in other contexts for quite some time before; the corpus has thus acquired a consensual status that renders its construction practically invisible to modern eyes. while the corpus of 'rational practice' still shows the marks of its construction.[29] There is then a fundamental symmetry between the criteria for the creation of a corpus and the common traits of a pre-established corpus; one is no more spontaneous or natural than the other.

This modern corpus construction however is only distantly related to the construction of corpora in the initial culture. Each of our three cases had a very different status from this point of view.

The first, the set of all Old Babylonian mathematical texts—and now I refer to both procedure and table texts—was, if not in its totality, at least partially known to the apprentice scribes and their masters who used it for the training of the future accountants and administrative clerks who staffed the bureaucracies, public and private, of the period. They, like the modern historian of mathematics, recognized in it a discipline with a certain autonomy, recognizable by the specificities of its subject and its mode of functioning.

By opposition, my second corpus, that containing Str 368 and the Egyptian Middle Kingdom mathematical papyri, is of modern manufacture. Though the two corpora are often contrasted and compared by modern historians of mathematics, the lack of intellectual contact between Mesopotamia and Egypt during the early second millennium BC, as well as the difference in the historical development of mathematics in the two cultures, makes it virtually certain that no scribe of that era would have been in a position to appreciate—or see the interest in–such a comparison.[30]

Finally, the 'rational practice' texts that were discussed in the third contextualization form a more problematic whole from this point of view. Generally unrecognized by most historians of mathematics—the inclusion of mathematics and divination together forms a particular stumbling block—I have given reasons in the previous section why I believe the grouping would have made sense to the ancient Babylonians.

We have thus three distinct cases: an agreement between moderns and ancients; a modern but not an ancient grouping; and ancient links that have become severed in recent times. But whatever the historical status of the corpus defined, the information that can be gleaned from the use of each contextualization is important, and even the most ahistoric grouping can uncover fruitful dimensions for an appreciation of the ways contemporaries of the texts understood them.

Yet is an "anachronistic" corpus always so ahistoric as all that? For Str 368 makes up as much a part of our environment as it ever did for a Babylonian scribe some four thousand years ago, except that our reading environment includes other things than did his. For us there are, among others, the studies of Babylonian mathematics made two generations ago by Neugebauer, Thureau-Dangin and others; knowledge of Greek mathematics and of modern computer science; knowledge of those parts of mathematics which have entered into school textbooks, university course notes (and, for some, in our research activities); contemporary theories and exemplars of what the history of mathematics is or should be. This is the inescapable contextualization of any reading we might perform and is just as 'historic' as theirs. The explicitation and delimitation of a corpus must include the historical and social situation of its readers. The necessary interaction between our contexts and those of our subject's—both discipline and object—must be dealt with for any history of mathematics (or indeed history *tout court*) that would reach beyond the anecdotal.[31]

The meaning of a text, even one so apparently simple as Str 368, is not to be grasped in some hypostatized 'essence' as an absolute object. It functions, we function, in larger contexts which can be subjected to experiment, modified and changed, to (partially) isolate certain attributes, uncover certain unknown qualities or extrapolate certain previous

results. Perhaps the question that we pose to any text is not so much what, as by what means—*how*—does a text mean?

Université de Paris VIII

NOTES

I would like to thank the Max-Planck-Institut für Wissenschaftsgeschichte in Berlin for its hospitality and assistance during the preparation of a preliminary version of this paper and to have accepted it for diffusion in their preprint series as n° 103 (1998). The present text differs from that version only in the updating of some references.

1 On this issue see (Vitrac 1996).

2 This way of understanding texts owes much to recent work in historiography around 'experimental history', see (Milo and Boureau 1991). The fruitfulness of this approach for the history of mathematics is shown in (Goldstein 1995) which deals explicitly with the question of contexts, see in particular p. 105–12, 180–82.

3 The text is published in (Neugebauer 1935–37):II, pl. 13 (photo) and has been commented a number of times in the literature: (Thureau-Dangin 1933); (Neugebauer 1935–37:I, 311–314); (Friberg 1992:553–554); (Ritter 1993b:152–156, 314–318).

4 My translation. The reader should be aware that it differs in a number of respects from previous translations. These differences will be commented upon in the appropriate parts of the article.

5 The older history of mathematics literature has a tendency to lump together the entire ancient Near East over a period of some 3000 years, the length of time that cuneiform writing was in use in the area. The implicit assumption has been that Mesopotamian culture is an example of a supposed Asiatic 'static' mode, in which all innovative creativity—if such exist at all—was concentrated at some early formative moment, to be simply parroted, with an increasing number of deformations, at all later times. Recent years have seen a more realistic and dynamic picture of this domain being put forward, in which changes in time and space are beginning to receive their full due. See, for example, Høyrup and Damerow 2001.

6 Ritter 1993b: 21–22, 297–318.

7 Editions of these texts can be found in (Neugebauer 1935–37), (Neugebauer and Sachs 1945), (Thureau-Dangin 1938) and in scattered publications mainly in *Revue d'Assyriologie*, 1930–1937, and by various hands in the journal *Sumer* between 1950 and 1962. An invaluable bibliography with commentary is (Friberg 1982).

8 (Ritter 1989a:31–36), (Ritter 1993b:149–158) and see already (Knuth 1972). By algorithmic I do not mean 'merely empirical' or 'recipe-style' collections of isolated solutions, a characterization that was once the standard interpretation before the work of Neugebauer, and remains so in a large number of popularizations. On the contrary, the sophistication, complexity and coherence of these structures will be the main point of this section. This approach is not the only one available, Jens Høyrup for example has proposed a geometric interpretation based on applications of areas. For this approach see (Høyrup 1990) and for a comparison of these three points of view, (Ritter 1993b:152–157 and 304–318).

9 I have silently rewritten the numbers appearing in Str 368, and all further Babylonian texts to be treated here, using the modern transliteration of abstract Babylonian numbers: powers of 60 are separated by periods, with a semicolon used to divide the integer from the (sexagesimal) fractional part and final and medial 00s have been added. All line numberings in this and in all following texts are mine.

10 This activity is indicated by the use of the verb *lapātum*, with the ordinary meanings 'to grasp' and 'to write down'. The reference is to the creation of a space both in the calculation and in the associated mental space of the calculator.

11 I have used these points, along with an apparent algebraic 'error' in the working out of the solution of Str 368 which has no effect whatever on the obtention of the correct result, to argue against the attribution of a 'hidden' algebraic agenda to the Babylonians in (Ritter 1993b:152–157).

12 IM 52301: 5–6, 23–24 (Baqir 1950) and *passim* in the Tell Harmal texts. See also: YBC 10522, YBC 4608, YBC 4186, MLC 1842, (Neugebauer and Sachs 1945: 131, 49, 91, 106) as well as VAT 6546 (Neugebauer 1935–37: I, 268), etc.

[13] This tablet has been the object of a number of treatments since its initial publication in (Thureau-Dangin 1936) (copy). One may consult: (Neugebauer 1935–37:III, 1–14), (Caveing 1994:35–83); (Høyrup 1990:266–281); (Ritter 1993b:316–318). The actual meaning of the Akkadian technical term *wāṣitum* is a subject of debate into which I do not enter here since it has no bearing on its functional use, the only one pertinent in this particular context.

[14] Examples are VAT 7532—one problem (Neugebauer 1935–37:I, 294–303 and II, pl. 21, 46) and VAT 7535—three problems (Neugebauer 1935–37:I, 303–310 and II, pl. 22, 47).

[15] These include the procedural texts that are the two papyri BM10057/8 ('Papyrus Rhind') and Moscow E 4676 ('Moscow Papyrus'), two fragments of a papyrus, Berlin 6619, and three fragmentary papyri from Kahun, as well as the student exercises represented by BM 10250 (leather roll) and Cairo CG 25367/8 (two writing boards).

[16] Publication: (Robins and Shute 1987:pl. 10–11) (photo). Discussion: (Ritter 2000), (Imhausen 2003:36–64).

[17] For a discussion of these various techniques see (Ritter 1989b).

[18] YBC 6295: 3 (Neugebauer and Sachs 1945:42, pl. 22, 46).

[19] Often, see for example BM 85200 + VAT 6599: I 23′ (Neugebauer 1935–37:I, 193–219 and II, pl. 7–8).

[20] These tables were excluded from my first corpus, consisting of procedure texts only. Many are listed and discussed in (Hilprecht 1906), (Neugebauer 1935–37:I, 3–94), (Neugebauer and Sachs 1945:11–36). See also (Friberg 1992:544–552).

[21] The grammar of reference for Akkadian is (von Soden 1969).

[22] See examples in the first part of this article, details in (Ritter 1989b).

[23] (King 1898:pl. 4–6) (copy); translation after (Pettinato 1966:II, 24–27). For Old Babylonian divinatory practices in general, see (*RAI* 1966) and (Bottéro 1974).

[24] (Köcher 1963–:III, n° 393) (copy). My translation. A discussion of our knowledge of Old Babylonian medicine can be found in (Ritter 1993a) and forthcoming.

[25] Publication: (Bergmann 1953); translation after (Finet 1973). On Babylonian 'law codes' in general see (Roth 1995) and on the Old Babylonian period specifically (Driver and Miles 1952–55) and (Bottéro 1987:191–223).

[26] At later periods, and principally during the first millennium, other domains of 'rational practice' such as calculational astronomy and judicial astrology, develop, with a formal structure to their texts similar to that outlined above for mathematics, medicine and divination.

[27] (Goody 1977:74–111), has pioneered the idea of taking lists seriously. However I feel it important to distinguish among the different sorts of lists, to distinguish them carefully from tables and to look more concretely at the way these last may have interacted with the procedure texts. See (Ritter 1989b, 1993a:111–113) for a discussion of this point with respect to mathematics and medicine respectively.

[28] See my discussion of 'rational practice' in (Ritter 1993b).

[29] The unanimity among contemporary historians of mathematics concerning the contents of the Old Babylonian mathematical corpus is to be contrasted with the divergent contours of the Mesopotamian medical corpus as presented by historians of ancient medicine. Though offering no more explicit a discussion than is the case for the former, the classification of texts as "medical", "magico-medical" or simply "magical" by historians depends strongly on their commitment to a 'rational' or an 'irrational' medicine in Antiquity.

[30] On the question of independence of the two traditions see (Ritter 1989b). For a general discussion of the question of borrowing and independent development of 'rational' textual traditions see (Ritter and Vitrac 1998).

[31] In reflecting on and giving flesh to a program squarely facing up to the problem of the construction of the 'long-term' in the history of mathematics, see the work of Catherine Goldstein (Goldstein 1995 and forthcoming).

REFERENCES

Baqir, Taha. 1950. "Another Important Mathematical Text from Tell Harmal." *Sumer* 6: 130–48.

Bergmann, Eugen. 1953. *Codex* Hammurabi. *Textus primigenius*. 3rd edition. Rome: Pontificum Institutum Biblicum.

Bottéro, Jean. 1974. "Symptômes, signes, écritures en Mésopotamie ancienne." In *Divination et rationalité*, edited by J.-P. Vernant et al., 70–197. Paris: Le Seuil.
— 1987. *Mésopotamie. L' Écriture, la raison et les dieux*. Paris: Gallimard.
Caveing, Maurice. 1994. *Essai sur le savoir mathématique dans la Mésopotamie et l'Égypte anciennes*. Lille: Presses Universitaires de Lille.
Driver, G. R. and John C. Miles. 1935–37. *The Babylonian Laws*. 2 vols. Oxford: Clarendon Press.
Finet, André. 1973. *Le Code de Hammurapi* (Littératures anciennes du Proche-Orient 6). Paris: Éditions du Cerf.
Friberg, Jöran. 1982. "A Survey of Publications on Sumero-Akkadian Mathematics, Metrology and Related Matters (1854–1982)." (Preprint n° 1982–17), Göteborg: Chalmers University of Technology and the University of Göteborg.
— 1992. "Mathematik." *Reallexikon der Assyriologie* 7: 531–85.
Goldstein, Catherine. 1995. *Un théorème de Fermat et ses lecteurs*. St. Denis: Presses universitaires de Vincennes.
—- forthcoming. "Long-Term Histories, Contexts and Dynamical Processes in Mathematics." (in preparation).
Goody, Jack. 1977. *The Domestication of the Savage Mind*. Cambridge: Cambridge University Press.
Hilprecht, Hermann V. 1906. *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur* (The Babylonian Expedition of the University of Pennsylvania, A. 20/1). Philadelphia: University of Pennsylvania.
Høyrup, Jens. 1990. "Algebra and Naive Geometry: An Investigation of Some Basic Aspects of Old Babylonian Mathematical Thought." *Altorientalische Forschung* 17: 7–69, 262–354.
Høyrup, Jens and Peter Damerow (eds.). 2001. *Changing Views on Ancient Near Eastern Mathematics*. Berlin: Dietrich Reimer.
Imhausen, Annette. 2003. *Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten* (Ägyptologische Abhandlungen 65). Wiebaden: Harrassowitz.
King, Leonard. 1898. *Cuneiform Texts from Babylonian Tablets in the British Museum* 5. London: British Museum.
Knuth, Donald E. 1972. "Ancient Babylonian Algorithms." *Communications of the ACM* 15: 671–77 [Correction 1976: *Communications of the ACM* 19: 108].
Köcher, Franz. 1963–. *Die babylonisch-assyrische Medizin in Texten und Untersuchungen* 6 vols. Berlin: W. de Gruyter.
Milo, Daniel S. and Alain Boureau (eds.). 1991. *Alter Histoire: Essais d'histoire expérimentale*. Paris: Les Belles Lettres.
Neugebauer, Otto. 1935–37. *Mathematische Keilschrift-Texte* (Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abteilung A: Quellen, 3). 3 vols. Berlin: Springer Verlag [Reprint 1973].
Neugebauer, Otto and Abraham Sachs. 1945. *Mathematical Cuneiform Texts* (American Oriental Series 20) New Haven: American Oriental Society.
Pettinato, Giovanni. 1966. *Die Ölwahrsagung bei den Babyloniern* (Studi Semitici 21–22). 2 vols. Rome: Istituto di studi del Vicino Oriente (Università di Roma).
RAI. 1966. *La Divination en Mésopotamie ancienne* (Rencontre Assyriologique Internationale 14). Paris: Presses universitaires de France.
Ritter, Jim. 1989a. "Babylone –1800." In *Éléments d'Histoire des Sciences*, edited by M. Serres, 17–37.
Paris: Bordas [English translation 1995: "Babylon –1800." In *A History of Scientific Thought*, edited by M. Serres, 17–43. Oxford: Blackwell].
—- 1989b. "Chacun sa vérité." In *Éléments d'histoire des sciences*, edited by M. Serres, 39–61. Paris: Bordas [English translation 1995: "Measure for Measure." In *A History of Scientific Thought*, edited by M. Serres, 44–72. Oxford: Blackwell].
— 1990. "Les Pratiques de la raison en Mésopotamie." In *La Naissance de la raison en Grèce*, edited by J.-F. Mattéi, 99–110. Paris: PUF.
—1993a. "La Médecine en–2000 au Proche-Orient: une profession, une science?" In *Maladies, médecines et sociétés*, edited by F.-O. Touati, II 105–16. Paris: L'Harmattan/Histoire au Présent.

—1993b. "Pratiques de la raison en Mésopotamie et en Égypte aux III[e] et II[e] millénaires", Thesis, Université de Paris XIII, Villetaneuse.

—2000. "Egyptian Mathematics." In *Mathematics across Cultures. The History of Non-Western Mathematics*, edited by H. Selin, 115–36. Dordrecht: Kluwer.

Ritter, Jim and Bernard Vitrac. 1998. "La Pensée grecque et la pensée 'orientale'." In *Encyclopédie philosophique. IV. Le Discours philosophique*, edited by J.-F. Mattéi, 1233–50. Paris: PUF.

Robins, Gay and Charles C. Shute. 1987. *The Rhind Mathematical Papyrus: An Ancient Egyptian Text*. London: British Museum Publications.

Roth, Martha T. 1995. *Law Collections from Mesopotamia and Asia Minor* (Writings from the Ancient World 6). Atlanta: Scholars Press.

von Soden, Wolfram. 1969. *Grundriss der akkadischen Grammatik* (Analecta Orientalia 33/47). Rome: Pontificum Institutum Biblicum.

Thureau-Dangin, François. 1933. "La Tablette de Strasbourg n° 11." *Revue d'Assyriologie* 30: 184–87.

—- 1936. "L' Équation du deuxième degré dans la mathématique babylonienne d'après une tablette inédite du British Museum." *Revue d'Assyriologie et d'Archéologie orientale* 33: 27–48.

—- 1938. *Textes mathématiques babyloniens*. Leyden: E. J. Brill.

Vitrac, Bernard. 1996. "Mythes (et réalités?) dans l'histoire des mathématiques grecques anciennes" In *Mathematical Europe. History, Myth, Identity*, edited by C. Goldstein, J. Gray, J. Ritter, 33–51. Paris: Maison des sciences de l'homme.

KARINE CHEMLA[1]

WHAT IS THE CONTENT OF THIS BOOK? A PLEA FOR DEVELOPING HISTORY OF SCIENCE AND HISTORY OF TEXT CONJOINTLY

ABSTRACT

Based on two examples (one taken from thirteenth-century China and the other from eighteenth-century Europe), this paper discusses why various forms of collaboration between history of science and history of text might prove profitable. Texts are not a historical, transparent forms conveying meanings whose history we would write. Scientific texts as such appear to have taken various forms within space and time, designed as they were through an interaction with local conditions of text production of all kinds. Elaborating a description of these various forms of texts would provide methodological tools to read them, since they can by no means be read without the mediation of a method. Here, the achievements of a history of text would benefit history of science in that it would provide a better grasp of the textual contexts for the production of scientific writing, and it would give a better awareness of the various ways in which texts were meant to mean. On the other hand, the history of scientific text could become a systematic concern in history of science as such: scientists design their texts at the same time as they design concepts and results. This represents a constitutive part of their activity, and the study of the production of texts would bring materials to understand how scientists benefit from the cultural and textual contexts within which they work. It would give us a grasp of how they construct the symbolic tools with which they perform their activities and communicate their results, which in the end are texts. In all these respects, a history of scientific text could then become a specific domain of the history of text.

Why suggest approaching history of science from the perspective of a history of text? In the course of my research, this approach appeared to me a necessity as I found myself repeatedly confronted with the question of how to read the sources I was dealing with. I believe that we would all agree on how not to read them: the limitations of a reading which amounts to a mere reformulation of ancient sources in modern scientific terms have already been denounced frequently enough for me to assume so. Yet, although we all agree on the fact that one should not read in that way, although we are aware that such a reading destroys the networks of concepts in the original, substituting other concepts and later concepts for those expressed in the text, I am of the opinion that we still lack a comprehensive account of what actually happens when one reads that way—what operations our reading performs on the original text, what the reading still tells us about the text.[2] That we should not merely reformulate ancient texts using modern concepts seems to me one of those tenets that everybody grants to be true, even though we do not know precisely how and why they are true.

In this paper, I would like to consider what benefits we might derive from analyzing in a critical way an assumption underlying this mode of reading—an assumption that it indeed shares with other modes of reading: when reading scientific texts in this way,

K. Chemla (ed.), History of Science, History of Text, 201–230.

one assumes that one can apply our own current modes of reading scientific texts to older sources as if texts as such, except for the emergence of modern symbolism, had no history, as if they were invariant in time and space, as if they had always required the same operations from their readers, as if the same elements always meant the same things. Such an assumption now seems to me highly questionable, regarding not only texts in general, as a history of text might teach us, but also scientific texts in particular. I shall start by presenting some of the evidence which led me to question this premise. This will take us to China, in the year 1248, when Li Ye, who had given up the civil service and was living as a hermit, finished his mathematical masterpiece: *Sea-mirror of the circle measurements*.[3]

1. HOW SHALL WE READ *SEA-MIRROR OF THE CIRCLE MEASUREMENTS*?

1.1 *The posing of the problem*

This seems to be the most ordinary book that one might dream of: after the usual preface and table of contents, it opens with a drawing (see figure 1), ordinary as it seems, on which it is entirely based. Right afterwards, the author proceeds to assign names to some of the segments of the drawing, and then goes on to provide numerical values for all the segments taken into consideration and for all the quantities that he will make with them, within the framework of particular dimensions for the drawing. After these preliminaries come the two main parts of the book: first, about 700 formulas are listed stating relationships between the segments of the drawing —one might not have expected that, for such an ordinary drawing, so many relationships could be found; however, here they are, gathered in a compendium which constitutes the last and biggest subchapter of the first chapter of the book. Second, a total of 170 problems concerning the drawing are presented in the next eleven chapters. This last part is famous in the history of mathematics, since we have here the first systematic occurrence, for the solution of most problems, of polynomial computations in order to establish an equation, "the" root of which is the unknown sought for.[4]

As we have described them so far, the elements composing the book appear to be typical: a drawing, names for some segments, numerical values, formulas, and problems. But one has to figure out that *this is the book*: there are almost no remarks by the author commenting on what he is doing. The main second-order statements concerning the content of the book are the titles of the chapters, or, in the case of the first chapter, the titles of its subchapters or their sections: "Map of the circular town" to introduce the drawing, "Set of the names of the *lü*"[5] to introduce the names of the segments considered, and "Correct quantities for the problems to come" to introduce the numerical values of the segments and of the quantities based on them which Li Ye considers.

Then the text reads as follows:[6]

"Set of the names of the *lü*

From C to T, that makes the hypotenuse of [triangle] (13)
From C to Q, that makes the height of [triangle] (13)
From Q to T, that makes the base of [triangle] (13)."

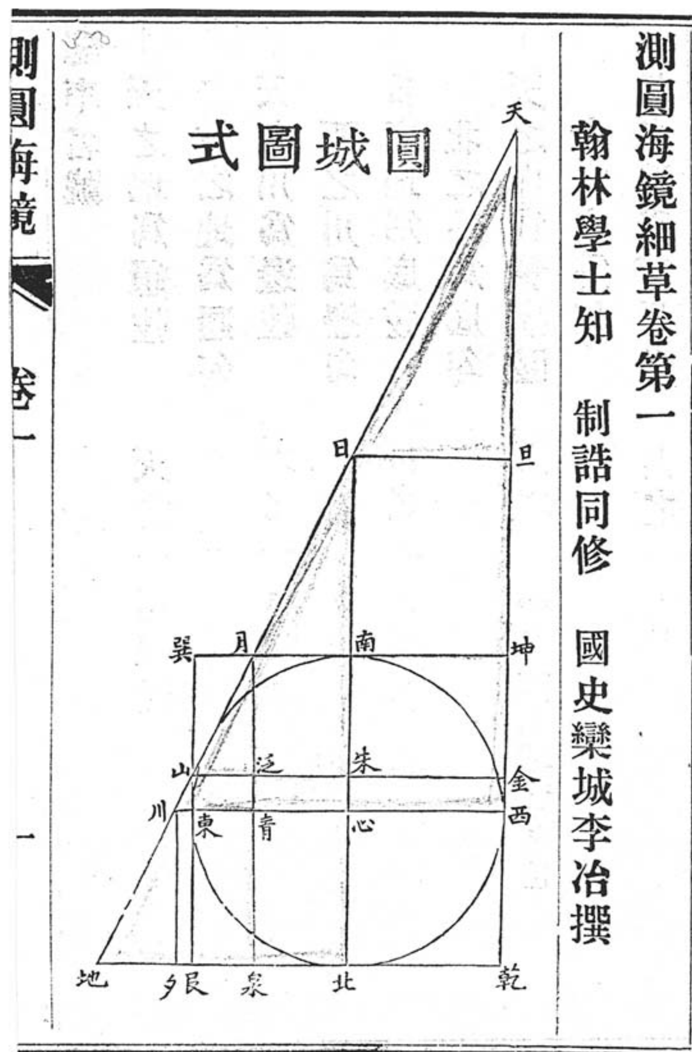


Figure 1. The drawing on which the *Ceyuan haijing* is based, reproduced according to the edition of the Tongwenguan. It is identical to the one contained in the manuscript which is kept in Beijing library (see its attribution in [Mei Rongzhao 1966], p. 111).

In the following, segments are introduced in the same way: each paragraph is devoted to three of them, which form the base, height, and hypotenuse of a triangle. Hence the names refer to a reading of the drawing in terms of triangles: the segments considered are designated according to which function they play in which triangle. As a consequence, the terminology has a tabular structure, the first part of a term in Chinese referring to a triangle, the second to a function.[7]

Figure 2. The letters for points and numbers for triangles used in this paper.

The following subchapter has a comparable structure, and contains only one second-order statement:
"Correct quantities for the problems to come

[Triangle] (13)	c	680	a	320	b	600
	$a+b$	920	$b-a$	280		
	$a+c$	1000	$c-a$	360		
	$b+c$	1280	$c-b$	80		
	$c+b-a$	960	$c+a-b$	400		
	$a+b+c$	1600	$(a+b)-c$	240."		

This is the sober way in which the numerical values used in most of the book are introduced, and it is made clear that thirteen quantities based on its three dimensions are attached to any given triangle.[8]

The simplicity of its style is extreme, and the content of the book seems to have always been clear to its modern commentators: about 700 formulas are displayed so as to function as an appropriate repository of geometrical knowledge needed for the second part of the book—the problems which make use of the relationships between segments to perform polynomial computations. A modern book would organize the presentation of knowledge in a similar way: it would first give a compendium and then rely on it to solve the problems.

Note how much this reading in terms of our own textual categories enhances the practical value of the compendium. Such a qualification (i.e.: to be practically-oriented) has

repeatedly been listed among the "characteristic features" that are sometimes attributed to the so-called "Chinese mathematician." And the usual ways of dealing with the set of numerical values given at the beginning of the book come as a confirmation of the fact that the "Chinese mathematician" is practically oriented. Among the various attitudes towards these numbers adopted by modern commentators, I identify three classes. Some readers choose to forget them—as if they were secondary, or even not present, in the book. Others lament over them: Why did Li Ye give actual values, when he knew that all this was valid with full generality? Still others stress them as, again, an incontrovertible proof of the practical character of the "Chinese mathematician," who would be able to think about things only *in concreto*. Whatever the differences between these attitudes, they all agree in reading the numbers as if they meant the same thing as numbers we would find in a contemporary textbook. This reading turns them into concrete, particular values, and this interpretation is the basis for the three attitudes.

But are we certain that we can read these numbers in that way? Are we certain that we can read the compendium and the problems as we would read their modern counterparts? These are the issues that I would like to address. Let us start with the numbers.

1.2 *The numerical values*

Li Ye's drawing is constituted in such a way that all its triangles are similar to each other. With the numerical values which Li Ye attached to them at the beginning of his book, they appear to be dilatations of the right-angled triangle whose smaller side, a, measures 8, whose height, b, measures 15, and whose hypotenuse, c, is 17. For example, the largest triangle of the figure, (13), whose dimensions we listed above, can be obtained as the result of a dilatation with a factor 40 on the basis of this small triangle with sides 8, 15, and 17.

Now, the dimensions 8, 15, and 17 immediately raise an echo. The resonance comes from a mathematical classic of the Chinese tradition *The nine chapters on mathematical procedures*, a text compiled around the beginning of the common era and which became an official book, or even "the" classic, in mathematics —hence, the first book which would be known to any scholar learned in this topic in China until the Song and Yuan dynasties at least.[9] The ninth chapter of this book, which Li Ye knew,[10] is devoted to the right-angled triangle. The only problem in this chapter that involves a triangle with the dimensions (8, 15, 17) is precisely —is this surprising?— the problem devoted to finding the diameter of a circle inscribed in a right-angled triangle. *The nine chapters* provides a procedure for finding the diameter, and in Liu Hui's commentary the reader can find —Li Ye could find— proofs establishing its correctness. One of these proofs happens to have an intimate connection with Li Ye's drawing. Let us sketch it (see figure 3 and the text in [Qian Baocong 1963], p. 252). Liu Hui draws a parallel to the hypotenuse of the triangle going through the center of the circle and makes the square whose sides are equal to the radius of the circle appear in the right angle of the triangle.[11] Then he relies on two related properties concerning the triangles which, as a consequence, appear within the original

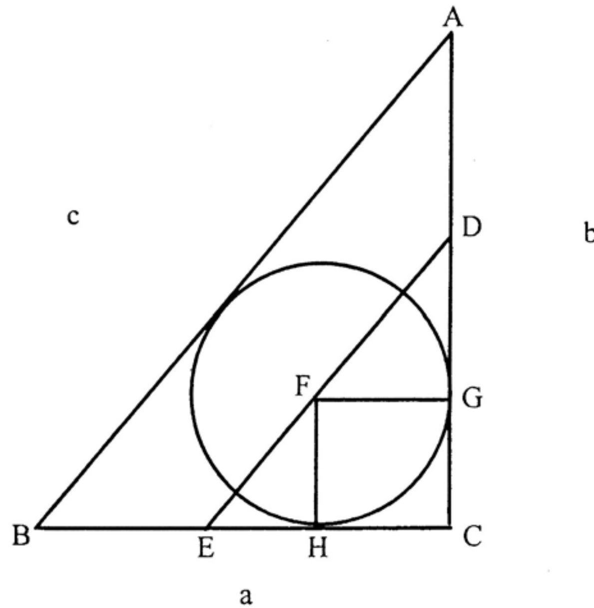


Figure 3. The drawing to which Liu Hui alludes in his commentary on the procedure to find the diameter of the circle inscribed in a right-angled triangle.

triangle: the sum of the lengths of the three sides of triangle DFG is b, whereas the sum of the lengths of the three sides of triangle EFH is a. Therefore, one can compute the coefficients expressing the similarity between the original triangle, on the one hand, and, respectively, triangles DFG and EFH, on the other hand, with respect to the dimensions of the original triangle, which gives, respectively:

$$\frac{b}{a+b+c} \quad \textit{and} \quad \frac{a}{a+b+c}$$

Now, taking into account the fact that the sought-for radius is, respectively, the base of triangle DFG and the height of triangle EFH, one obtains its value by using the previous coefficients of similarity:

$$\frac{b}{a+b+c} \cdot a \quad \textit{or} \quad \frac{a}{a+b+c} \cdot b$$

If, following this incitation, we look back into Li Ye's drawing and consider the sum of the three sides of each of the triangles introduced, as we saw above, by the terminology, we discover a striking fact: the sum of the three sides of each triangle is one of the 13 quantities attached to the largest triangle of the drawing (13). The following table displays the relationship between the triangles and quantities thereby established:

1	b
2	a
3	$b + c - a$
4	$a + c - b$
5	$2b$
6	$2a$
7	$c + b$
8	$a + c$
9	$c - a$
10	$c - b$
11	$a + b - c$
12	c
13	$c + b + a$

Hence, each triangle appearing on Li Ye's drawing has a property similar to the one Liu Hui brought into play in his proof. And we have an almost complete system of such triangles: but for two exceptions, the thirteen triangles prove to be connected precisely with the thirteen quantities attached to a right-angled triangle.[12] A correlation between the triangles and the quantities considered appears: the drawing might have been the fundamental figure attached to the right-angled triangle. Li Ye's reading of the drawing in terms of triangles echoes its relevance for linking triangles and quantities. This stresses the singular nature of the figure, when compared to the drawings considered in Greek geometrical texts such as Euclid's *Elements of geometry*.[13] Drawings seem to have been worked out in different ways in distinct traditions, which can also cause problems in their reading, whether one projects a modern conception of a mathematical drawing onto an ancient one, or a conception elaborated in one tradition onto another. Drawings raise problems of interpretation too.

As a consequence of the property exhibited, each numerical value given by Li Ye, being an integer, can be decomposed into two multiplicative factors: one indicates the triangle on which it is computed, the other the kind of quantity it represents for this triangle.[14] The values Li Ye provides constitute the smallest possible set based on the triangle (8, 15, 17) for which all quantities computed are integers. In addition, they have the property of being the unique set of dimensions for which the multiplicative factors referring each value to a triangle and a quantity correspond exactly to the values computed with respect to (8, 15, 17). For example, the values for the largest triangle, (13), which we gave above, 680, 320, and 600, can be decomposed as 17.(8 + 15 + 17), 8.(8 + 15 + 17), and 15.(8 + 15 + 17), where (8 + 15 + 17) constitutes precisely the one among the thirteen quantities that corresponds to the largest triangle.[15] Note that, on the one hand, numbers have a structure parallel to that of the terminology, and, on the other hand, both manifest the mathematical structure of the situation.[16]

Let us recapitulate the correlations we have obtained. We have correlations of three types between the situation within which Li Ye operates and the problem from *The nine chapters* that it evokes:

— the basic numerical values (8, 15, 17).
— the situation of the circle inscribed in a triangle.

— the property of having triangles inserted in the largest triangle, where the sum of the three sides of the smaller triangles is expressible as a function of the dimensions of the largest triangle.

This accumulation seems to me to rule out the possibility of a mere coincidence.

And, therefore, if we recapitulate in terms of readings, we get the following hypothesis:

— The numbers given by Li Ye at the beginning of his book might not be related in the least to "concrete numbers" or to "particular values," as a projection of contemporary readings would suggest.

— On the contrary, they might function as a quotation from *The nine chapters*, thus placing Li Ye's book in a tradition of research that stems from the classic. They might also indirectly express a property of the drawing, which turns out to be fundamental for the compendium.[17] This property would be stated as such nowhere in the book, but rather disseminated in the compendium.

With this very simple first example, we have arrived in my view at the heart of the problem. Quotations of classics and, thereby, indirect expressions are frequent in ancient Chinese texts.[18] We would just have here a modality of their functioning within mathematics. An echo being raised, its meaning gets unfolded by the reader. When Li Ye writes, he might be expecting such operations from his readers, as they can be expected from any reader in general in this context.

Therefore and more generally, in order to be read by us, mathematical texts need to be conceived as embedded in textual cultures, since they are written in ways showing that they require specific operations of reading —even though these operations might take particular forms in a mathematical context.[19] Only a confrontation between mathematical sources on the one hand, and between them and other kinds of texts on the other hand, can secure the modes of reading to be used. So far, they are just hypothetical. This is a point where a history of text, a history of reading, would be of importance for history of science. But, conversely, scientific sources could in this respect contribute to elaborating the description of the ways in which texts made sense, and in this way prove to be precious material for a history of text.

What is at stake here is clear. If we proceed by projecting our modes of reading onto ancient documents, as if texts had been used everywhere and constantly in the same way, we face the danger of creating, on the basis of our own artifacts, a "practical Chinese mathematician." In response to this danger, our method suggests a remedy.

Note that such a reading lets us see that , in what could appear to be as a tedious table, there is a structure the meaning of which has to be unfolded, though we would probably choose to express this meaning discursively. The same phenomenon repeats itself with the compendium, to which we shall now turn briefly.[20]

1.3 *The compendium*

How is the set of 700 formulas that follows the table of numerical values expected to be read? What were the motivations for laying down these formulas? What mathematical information is there made available? Before I deal with these questions, a much more elementary issue needs to be addressed: Why should such questions be raised? The simple

explanation, which would see the compendium merely as the repository of geometrical knowledge designed to serve as a foundation for the second main part of the book, and the simple reading of this piece of text as a directory of formulas, have some validity. Indeed, to solve the problems given thereafter, one needs to use relationships between the various segments of the drawing such as those given in the compendium, and the solutions of the problems regularly refer to it explicitly when they use one of its formulas. Why then not keep to this "natural" hypothesis to account for the composition of the compendium and its inclusion in the book?

My reason for finding this explanation insufficient, though it is certainly partly valid, is that this hypothesis cannot explain certain facts related to the compendium. Indeed, if it was composed only with a view to the second part, one would expect that Li Ye would incorporate in the compendium all the formulas he knows concerning the drawing, or at least all the formulas he needs to solve the problems in the second part. However, the formulas used in the problems do not all come from the compendium. This is easy to prove, by comparing the incorrect formulas recorded in the two parts.[21] Some of those used in the course of the solution of one of the problems cannot be found among the incorrect formulas in the compendium. For example, problem 17 of chapter III contains the incorrect formula

The difference between hypotenuse and height (of triangle (13)) added to the height of triangle (7) gives twice the difference between base and height (of triangle (13)),

which is not found in the compendium.[22]

This means that Li Ye has not considered the compendium the only source of formulas for the problems. Nor has he felt it necessary to record there all the formulas that might have to be used in the second part. In any case, once he had finished solving his problems, he did not enter all the formulas he used in the compendium. These details indicate that Li Ye did not conceive the compendium as a place for gathering all relevant geometrical knowledge concerning the drawing.

Moreover, a second point also remains unexplained by the simple hypothesis. The formulas presented in the compendium are not all used in the second part of the book. On the contrary, it is surprising how few formulas actually occur in the problems, compared with the multiplicity of formulas registered in the first part of the book. So there seems to be an interest in formulas themselves that exceeds their being useful for problems and, correlatively, an interest in the compendium that exceeds its practical use. As a consequence, the compendium must also be considered as a piece of text *per se,* devoted to formulas, and not as a directory used for reference only when one needs a specific formula to solve a problem, as the form of the text would incline us to believe. So we again encounter the same warning related to the recognition of a textual unit familiar to the contemporary reader, who is hence tempted to project his accustomed modes of reading onto the text.

How, then, should we read the compendium? And how can we argue that this reading is valid? These questions are not easy to answer, since Li Ye leaves us with a mere juxtaposition of groups of formulas, organized in sections, sub-sections and paragraphs, with no commentary. I shall not develop a full answer to this problem here, but just indicate the direction that, I believe, must be followed, inasmuch as it will again give us interesting material for our purpose.

If something is said in the compendium, if it is to be read as a text on its own, what is said ought to have some connection with the activity that produced it, or at least with the motivations for this activity. And the best entry point for describing the activity that produced the compendium seems to be the incorrect formulas it contains. If a correct formula tends to be mute concerning the reasoning which yielded it, an incorrect formula by contrast is full of information for the historian of science. In that respect, a couple of the incorrect formulas are extremely interesting, since besides the fact that each is incorrect, they follow each other in the compendium and are phrased as grammatically parallel statements:[23]

> "Twice the height of triangle (1) and the difference between the base and height of triangle (11) taken together generate once the difference between the height (of triangle (13)) and the diameter of the circle.
>
> Twice the base of triangle (2) and the difference between the base and height of triangle (11) taken together generate once the difference between the base (of triangle (13)) and the diameter of the circle."

To make clear what I mean by parallel statements here, let me illustrate this textual structure on the basis of two lines of a poem written by the famous Tang writer Wang Wei:[24]

> "The bright moon shines among the pines
> The clear source flows on the rocks"

The two verses have the same syntactical structure and correspond character by character to each other —an adjective to an adjective, a noun to a noun, light to flowing water— building a relationship between the two facts as well as between the elements composing them. Such parallel sentences are extremely frequent not only in Chinese poetry, but also in Chinese philosophical, strategic, political texts, and so on, as well as in mathematical texts[25] like our compendium. And one can take for granted that the Chinese reader reads not only each of the two sentences, but the relationship between them as well—that is to say the correspondence that it establishes between the two facts.

Now, if our reader reads our pair of incorrect formulas as parallel, he is likely to note a mathematical transformation that enables one to obtain one formula from the other. And from this fact, we can deduce much information.

1. Here the parallelism between sentences has a mathematical meaning, and even though each formula is incorrect, the transformation itself is valid. It refers to a symmetry in the drawing, which is invariant with respect to the exchange between base and height.

2. This meaning is related to Li Ye's mathematical activity: since both formulas are incorrect in the same way, Li Ye did not obtain them independently, but used this mathematical transformation. This is the only way to account for such a pair of mistakes. Hence the parallelism —the production of a textual phenomenon— is correlated with a transformation the author used —a mathematical act. Note that the terminology has been selected in such a way that the result of the transformation can be obtained by a mere translation, character by character, applied to the statement of a formula: linguistic operations parallel mathematical operations.

3. Therefore some of Li Ye's mathematical work is expressed by the parallelism: Li Ye uses mathematical transformations to produce new formulas, and he records all the formulas so produced. They become parallel formulas that Li Ye might expect his reader to read as such. His mathematical activity is correlated with what, we can assume, he has given us to read. Hence the conclusion: Li Ye expresses the transformation *via* the parallel formulas that it links.

In fact, this kind of parallelism between formulas pervades and structures the compendium. Hence we are led to the following hypothesis, the consequences of which will not be developed here: the compendium deals with the transformations that produce formulas and their properties, and the results are expressed by the structure of the text. This content would be invisible if we were reading the text as a mere juxtaposition of formulas. In terms of the text, beyond the statements of particular formulas, the groups within which they are organized—subsections, paragraphs and subparagraphs—would as such be meaningful. We again come to the conclusion drawn above that the structures of the text convey meanings to be unfolded by the reader. This refers also to a kind of reading which we can consider as usual for communities of Chinese scholars.

The same way of reading holds true for the problems organized in chapters parallel to each other. But, as regards the problems, we are now confronted with a much more perplexing problem of reading.

1.4 *The problems*

After the compendium, entirely devoted to formulas, *Sea-mirror of the circle measurements* presents 170 problems, grouped in eleven chapters. Within the framework of the particular dimensions given for the drawing at the beginning of the book, the problems generally aim at determining the length of the diameter of the circle when given two or sometimes three pieces of data: the lengths of some segments of the diagram or simple combinations of such lengths. For this purpose, the so-called "procedure of the celestial element" is used.[26] Let us first describe the structure of most of the solutions given to a problem, using the example of problem 13 in chapter III. Its terms read as follows (see figure 4):

"One asks: it appears that the height of triangle (7) is 480 and the hypotenuse of triangle (9) is 153. The question and the answer are the same as before."[27]

As for most of the problems presented by Li Ye, the unknown is found as "the" root of an equation whose computation is described twice, in two distinct parts of the solution. First comes the part called "method" (*fa*), where Li Ye describes, one after the other, how to compute each coefficient of the equation. To give a flavor of what the "methods" look like, let us translate the description of the computation of the first coefficient, the constant term, replacing the names of the data by the letters A and B:

"Method: The two given quantities being subtracted from each other, again double this and subtract from it A. Again multiply this by A and put above. Moreover, B square being multiplied by the position above, that makes the constant term."

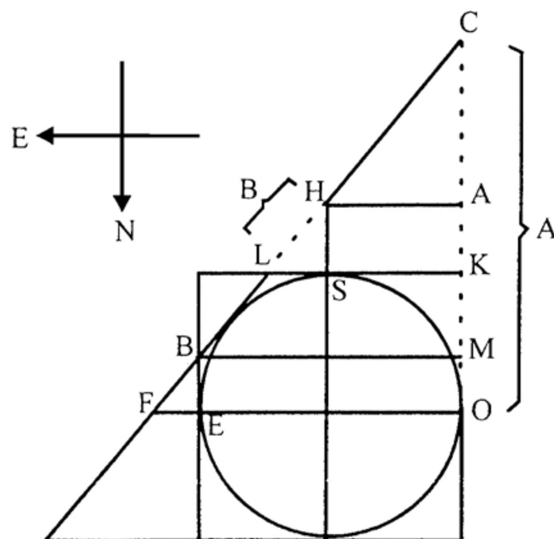


Figure 4. The data of problem III.13.

A procedure is hence provided to compute this term, followed by as many similar procedures as the equation has coefficients. Its description makes use of the names of the data as they are given in the outline of the problem as well as of intermediate quantities designated by the name of the position where they are provisorily placed during the computation. No numerical computation is appended there. We can translate the successive stages in the computation of the constant term as follows:

$A - B$
$2(A - B) - A$
$A[2(A - B) - A]$
$B^2.A[2(A - B) - A]$

Then comes the second part of the solution, called "sketch" (*cao*), where the same equation is produced in an entirely different way. First an unknown is chosen that is linked with the desired diameter in the course of the text, when it happens to be different from this diameter, as is the case here. A geometrical reasoning and polynomial numerical computations follow. The geometrical reasoning makes use only of the unknown, of the two data, and of the quantities that can be obtained on this basis, through reasonings where formulas show their efficiency. Along with these sequences of formulas, polynomials that represent the intermediate quantities are computed numerically, and in the end, the equation is obtained numerically. Let us translate the beginning of the "sketch", again to give a flavor of what the text looks like:

"Sketch: Take one celestial element[28] as the base of triangle (9). If one adds the hypotenuse of triangle (9), one gets

1
153•

as the height of triangle (1). If one subtracts the height of triangle (1) from A, it remains

−1
327•

as hypotenuse of triangle (1)."

Such is the way in which the solutions to the problems are presented in general. And this method of presentation immediately raises three questions:

1. Why does Li Ye systematically give two descriptions of each equation in solving a problem?

2. Where does the description in the "method" come from? How does Li Ye know that the procedures he gives actually compute the coefficients of the equation? How did he find them? The means of obtaining them are nowhere stated explicitly. Their description is, in the "method," rhetorical, whereas in the "sketch," the coefficients are produced only numerically.

3. Our translation into modern symbolism underscores that Li Ye is not using the most efficient algorithm to compute the constant term. Why does he first compute $A - B$ and then $2(A - B) - A$, instead of computing directly $A - 2B$? This question is even more intriguing if we consider the second solution that he gives to the same problem III.13, as he regularly does. As above, it is constituted of two parts. Its "method" reads as follows:

"Additional method: From A, subtract twice B. Again multiply this by A. Again multiplying this by B square makes the constant term of an equation of the fourth degree. All the other coefficients are as before."

Thus, in addition to the constant term, the equation given for this other solution is said to be the same as the one obtained in the first solution. But again, if we use modern symbolism to transcribe the successive quantities this new algorithm prescribes to compute the constant term supposed to be different from the previous one, we get:

$A - 2B$
$A(A - 2B)$
$B^2.A(A - 2B)$

Doesn't Li Ye know that this algorithm and the previous are equivalent in yielding the constant terms? Why does he give a cumbersome description first and then this simpler one? Why again does he give all coefficients except the constant term as the same, while giving a new description of this constant term, when it is obvious that even the constant terms are the same in both cases?

These are the questions to which the form of the text seems to lead the reader. We shall now show that they *can all receive answers simultaneously*. To this end, let us perform

an experiment on the text. Here is the procedure: let us take the sequences of geometrical formulas presented in the "sketch" and, using the tools of modern algebra, compute symbolically, with respect to the data given in the outline, the successive polynomial expressions that correspond to those Li Ye computes numerically. In the case of the first solution of problem III-13, the following sequence of polynomials occurs in the "sketch":[29]

(1)	$1\,x$ B	(2)	$-1\,x$ $A-B$	(3)	$-2\,x$ $2(A-B)$	(4)	$-2\,x$ $2(A-B)-A$
(5)	$-2A\,x$ $A[2(A-B)-A]$		(here the text suggests to put this intermediate result "above")			(6)	$-2B^2.A\,x$ $B^2.A[2(A-B)-A]$

It is striking that there is a strong correlation between the sequence of intermediaries Li Ye used in the description of the "method" and the sequence of states of the constant terms appearing now in the successive polynomials, which, in the end, gives the constant term of the equation. Hence the description of the constant term of the equation given in the "method" follows the steps through which this coefficient is progressively computed in the "sketch." We can even notice that the intermediary term said to be put "above" in the "method" corresponds exactly to the polynomial said to be put "above" in the "sketch".

The correlation between the two parts of the solution, which then proves to be very strong, is actually even stronger. Let us indeed observe the second "sketch". It differs from the first one only at the beginning, and if, again, we compute symbolically the successive polynomials that Li Ye computed numerically there, we get the following sequence:

(1)	$A-B$	(2)	$-1\,x$ $A-B$	(3)	$1\,x$ B	(4)	$-1\,x$ $A-B-B$
(5)	$-2\,x$ $A-2B$						

After this stage, this polynomial that now represents a given geometrical entity will be used in the computations exactly as polynomial (4) above was used in the first solution. From this point onwards the two "sketches" are the same. Hence, first, we again find that the way of describing how to compute the constant term in the second "method" is the same as the way this constant term is actually progressively computed numerically in the second "sketch." But, second, we can see that the difference between the *descriptions* the two "methods" give to obtain the constant term is *correlated with a difference in the ways through which they are computed in the two "sketches,"* whereas the statement that the other coefficients are "the same" corresponds to the fact that they are, in both "sketches," computed in exactly the same way. Therefore the correlation between the "sketch" and the "method" appears once more in this second solution, and the comparison of the cases indicates how strong this correlation is: the reason why the constant term is given a new description in the second "method" might be that it is computed in another way, through the second "sketch." This indicates the direction to follow to solve our third question, since this shows both why the constant term is first given an

"awkward" description from our point of view, and why it is then given a second description in the second "method." Moreover this reveals that the lists of computations given in the "method" are symbolical expressions, and have a meaning as such, and do not refer to an actual numerical computation.

I repeated this experiment for each problem where polynomials occur in the course of the solutions, and the same correlation appeared: the same transposition of the numerical computations performed with polynomials in the "sketch" provides the successions of states of the coefficients corresponding to the intermediaries in the description of the equation as given in the "method"; this kind of relation between the two parts of the solutions holds throughout the book.[30] Much can be concluded from this.

First, in terms of history of mathematics, we can tackle the second question we raised, concerning the origin of the "method." It seems to be an inconvertible conclusion that Li Ye deduced the "methods" on the basis of the computations he recorded in the "sketch." But how? To do so, he must have performed the computations not only numerically, as he recorded them, but also, in a way, symbolically. Should we conclude that he has been computing not only with polynomials that have numbers as their coefficients, but also with polynomials having sequences of characters as their coefficients?[31] That would be another perspective from which to consider how unimportant the actual numbers are in Li Ye's book. But that would also assign to the book a content which has until now not been recognized in it and would modify its position within the history of mathematics. Indeed, within the framework of such a conclusion, we come to realize that what we are given in the "methods" appears to be precisely *equations with general coefficients*. In the Chinese mathematical tradition, as opposed to other traditions, an algebraic equation has the symbolic identity of an operation, given by its list of coefficients.[32] Such is the object Li Ye described in the "methods." Moreover, again in the Chinese tradition, what we would now write as formulas or symbolic expressions was expressed by algorithms.[33] Here, in the case of our equations, the coefficients that we would write as symbolic expressions, as I did above, are expressed through algorithms, but this should conceal neither their identity as symbolic expressions of another type, nor their generality.

From this, two consequences can be drawn. First, our approach leads us to recognize, under another form of expression, an object again so far not identified as such in this text: the general expression of an equation the coefficients of which are described through algorithms.[34] Second, the object that we recognize presents interesting correlations with the mathematical work that we are tempted to assign to Li Ye as bridging the gap between the "method" and the "sketch" of each solution, namely computations with polynomials with symbolic coefficients.

This last conclusion has to be taken into account when discussing the methodological problem which our approach raises: we used mathematical knowledge which is not contained explicitly in the text, which is, at least in some aspects, posterior or exterior to the text, to perform an experiment on it and to bring to light some of its underlying structures. The properties of the text so discovered enabled us to solve the second and the third of the problems its reading raises, by providing a hypothesis for the origin of the "method." And I know of no other way to solve this problem and account for the "method" as well as for the descriptions of the constant terms above. Should we then assign to Li Ye precisely

this knowledge which we brought into play to work on his book, a knowledge which bridges the gap between the two parts of each solution, and conclude that, computing with polynomials with characters as coefficients, he also obtained general equations as a result, that he conceived of the equation and the polynomial in a general way? To answer this question involves not only methodological problems in the history of mathematics regarding the status of the use of mathematics exterior to a given text to deal with this very text;[35] it also raises issues related to the history of text. And this brings me back to questions 1 and 3 which I formulated above.

In terms of the text, two textual phenomena caught my attention. The first was Li Ye's careful and artificial distinction between the descriptions of the two equal constant terms, while he gives the other equal coefficients as being the same. The second was the division of each solution into two parts describing, in different ways, the same equation. Following these hints, I did an experiment and discovered a mathematical relationship between the two parts of each solution, which explains the origin of the "method" (question 2) and accounts for the strange form it sometimes receives as well (question 3). But, are these mere traces which the historian of mathematics can seize to reconstruct the mathematical work which goes along with the text, as we treated them so far? Or are these conscious indications, modes of expression: shouldn't we form the hypothesis that these textual devices, the structure of the text as well as the structure of the sentences, are precisely the means of expression Li Ye chose to call his reader's attention to a mathematical content which he did not express discursively in his book? So far, I can see only this way of accounting for the textual acts Li Ye performs.

What kind of research program can we set for ourselves to assess the validity of such a hypothesis? A better knowledge of how texts were written in Li Ye's time might enable us to confirm our conclusion as well as to understand why he used such a device. This is another point where history of science and history of text can collaborate. Until now, I know of no other contemporary Chinese text presenting the same phenomena. Li Ye's text constitutes precious evidence, since its mathematical content enables us to describe its structure rather precisely. It constitutes an invitation to look for other similar writings. In addition to yielding a textual context for the production of such writings, a history of text could help us to determine the sociological context within which they were produced. In particular, we could obtain information to determine whether Li Ye expected that any reader would read such a structure or whether it was meant only for initiates. In any case, we know that some Chinese readers of the *Sea-mirror of the circle measurements* read this structure as such in the text.[36]

At this point, let us recapitulate our findings. We started with some of the most elementary pieces of text that can be found in a scientific book: a set of numerical values, a set of formulas and a set of problems. In each case, it appeared that the kind of reading which would turn them into their modern counterparts might completely miss the way in which they make sense, excising them from the textual context to which they pertain. In consequence, one would fail to grasp what is at stake mathematically. Hence the problem of how to read sources appears crucial for the historian of mathematics and difficult to solve with certainty: elaborating our description of the reading of the *Sea-mirror of the circle measurements* constitutes a fundamental prerequisite for interpreting it, but what arguments can we offer to prove the validity of our answer?

Before we consider this question and related ones with full generality, another point naturally arises: Do we have here a unique case or do such problems recur constantly? We saw that, in many ways, Li Ye made use of the structure of a text to convey mathematical meanings:

- The parallelisms between the formulas, between the paragraphs of his compendium point out, it seems to me, the mathematical transformations that enable one to move from one formula to another, from a group of formulas to another, and from a member of a formula to the other.
- The systematic relationships between the two parts of the solution to any problem, indicated by textual hints, might refer to a computation on polynomials of a nature different from those recorded "discursively" in the text.

If in both cases to overlook the necessity of reading these structures would lead us to miss part of the content, the interpretation of these structures is at least as open as the interpretation of discourse. Now, should we conclude that "the Chinese" or "the Ancients" distinguished themselves by using, for such a distinct topic as mathematics, strangely indirect modes of expression, without commenting on them, a sin which would have been eradicated in modern science? Certainly not. It seems rather that an interest in certain kinds of phenomena regularly led groups of individuals to elaborate specific kinds of texts for the purpose of their research and that these texts, whatever their origin, raise the same problems of reading and of interpretation.

Even though such examples are manifold, I shall deal with only one in what follows. It will lead us to Potsdam, Saint-Petersburg, and Paris during the second half of the eighteenth century and the beginning of the nineteenth century.

2. THE HISTORY OF DUALITY AND THE DESIGN OF TEXTS[37]

2.1 *A similar problem of reading*

In 1753, the *Mémoires de l'Académie des Sciences de Berlin* published a *Mémoire* by Euler, a reasonable product of eighteenth-century Europe, entitled "Principes de la trigonométrie sphérique tirés de la méthode des plus grands et des plus petits." Euler's intention in this *Mémoire* is to derive the whole body of spherical trigonometry from an analytical method that he was largely responsible for developing and that he had placed at the center of a newly established branch of mathematics: the calculus of variations.

Why do so? One perfectly knew at the time how to solve any problem that could be raised in spherical trigonometry: whatever triplet of sides and angles of a triangle drawn on a sphere was given, one knew formulas to find the three other sides or angles. One of Euler's explicit motivations came from geodesy: when dealing in such a way with triangles drawn on the surface of the sphere, one could extend the treatment to triangles drawn on a surface resembling more closely the surface of the earth, a problem which he tackled in a subsequent memoir. But let us keep to the sphere.

Euler starts from the hypothesis that the sides of such triangles are the shortest paths that one can draw between its three vertices, and on this basis, embarks on pages of computations. He has already produced dozens of formulas when suddenly, at a point

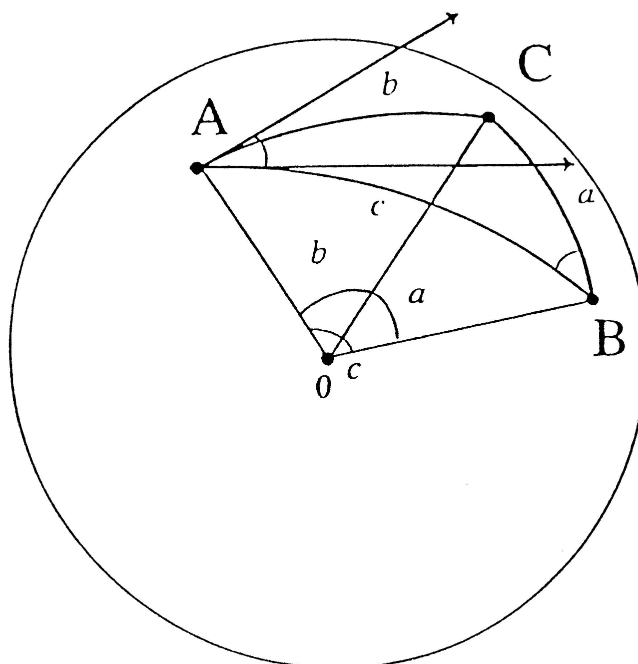


Figure 5. Euler's notations.

when he recapitulates, he interrupts momentarily the course of the text and introduces new notations (see figure 5):

"The previous denominations will be reduced to the present ones in this way:

Previous denominations a, x, s, y, α, ϕ
Present denominations c, b, a, A, B, C."[38]

Regarding the introduction of these new notations, here, right in the middle of the paper, he makes no comment whatsoever. Rather he rewrites with them the dozens of formulas obtained so far and proceeds to show how any problem that can arise in spherical trigonometry can be solved by one of these formulas.

When he begins to deal with the problems, his presentation adopts a new form. It is divided into sections that successively tell how, when one is given any triplet of elements out of the three sides and the three angles of a triangle drawn on a sphere, one can obtain the three others. Moreover, within such sections, he sometimes inserts algebraic computations that transform a formula which could be used to do so into another formula with which the actual computations become more convenient. Hence we have, for example, first a section concerning the case when the three sides are given and the three angles are sought. It is immediately followed by a section presenting how to deal with the case when the

three angles are given and the three sides sought, and so on. But here we are confronted with a singular phenomenon, which brings us back to the problems discussed in the first part of this paper. When comparing section 1 with section 2, and section 3 with section 4, one discovers that the texts of the two pairs of sections correspond to one another formula by formula, step by step. More precisely, if one takes one of the formulas in one of these sections, and transforms capital letters into small letters and conversely, moreover if one inverts certain signs $+$ (resp. $-$) into $-$ (resp. $+$), one gets the corresponding formula in the other section. In correlation with this, where a sentence in one section says "angle", the corresponding sentence in the other section says "side", and conversely —this, however, in a looser way.

This applies to formulas given to obtain a certain element of the triangle, but it also applies to formulas that mark the successive steps in the transformations producing formulas more convenient for numerical computations. In other words: this applies to formulas as well as to algebraic transformations.[39] Therefore:

- The second part of Euler's memoir is given a precise structure, where whole sections correspond to each other by one and the same transformation applied to the text of their formulas and to the text of their sentences.
- This structure is made possible thanks to the notations Euler introduced, as we saw it, in the middle of his paper. Hence there is a correlation between the notations chosen and the structure of the texts built with them.
- The structure of the text shows that angles and sides play symmetrical parts in the body of spherical trigonometry. For us, readers of the twentieth century, this is mathematically meaningful as being a manifestation of a more general phenomenon called duality. We can bring in later mathematics to read the structure.
- On this structure, Euler is silent like Li Ye. The only thing he gives us to read is the structure of the text, only stressing here and there "comme cy-dessus." But, to qualify him as being "silent" refers to a conception of texts according to which only what is said discursively is said. One may as well consider that such is the mode of expression he chose here.

Now, how can we account for this phenomenon? The whole of the *Mémoire* has a meaning which again exceeds the meaning of its sections. This meaning is not expressed discursively and, correlatively, requires a reading of another kind. Here again is a point where the nature of the text matters methodologically to the historian of science.

But, if this is the case, it is because Euler designed a form of text chosen for the expression of certain meanings. And here one finds another perspective from which to consider the relevance of such questions for history of science and for history of text: the design of texts constitutes a crucial aspect of scientific activity and requires to be described as such. Indeed, the two types of questions are inseparable and should be treated together: it is because scientific activity accompanies the elaboration of types of texts that we meet with the problems we have described regarding their reading and interpretation.

This activity involves, in different times and places, elements that might retrospectively seem to be analogous. However, a critical distance sometimes reveals that similar appearances may conceal deep differences in nature: if the *Sea-mirror of the circle*

measurements bears on a drawing, we saw that this drawing has little in common with a drawing to which Euclid's *Elements of geometry* might refer. We seem to have rather the 'fundamental drawing of the right-angled triangle' in a tradition of conceiving such drawings that we can trace back to Liu Hui and Zhao Shuang.[40] Here again the problems of how to read a text, of the elaboration by a tradition of textual elements, and of mathematical issues addressed within this tradition merge.

2.2 *The design of texts*

To confirm our conclusion concerning Euler's design of a type of text, one should check whether this fact is not coincidental: Is it by mere chance, in passing, that Euler elaborates such a text? Several facts speak in favor of the contrary:

- The introduction of the notations and the remarks stressing the similarities between parts of the text show that Euler has a concerted interest in the phenomenon.
- A few years earlier, Euler had presented —probably in Berlin even though it was later to be published in Petersburg— a *Mémoire* on polyhedra (Euler 1750). Again, when dealing with these solids delimited by plane faces, vertices and faces can be treated in the same way. And we now know that this is due to the same phenomenon mentioned earlier: duality. And again, whole parts of Euler's *Mémoire* present faces and vertices as having symmetrical properties, and the proofs given are themselves symmetrical to one another. Euler is aware of this fact, since he regularly introduces a development symmetrical to a previous one with the expression "simili modo, . . . " In this case too, the property of the field that it presents a symmetry is expressed only by the structure of the text. Euler does not seek to account for it from a mathematical point of view. Nor does he, in any case, mention the similarities of these phenomena in spherical trigonometry and in the treatment of polyhedra.

  Hence Euler, confronted roughly at the same time with what we know is the same mathematical phenomenon, expresses it in two cases in the same way: he designs a kind of text the structure of which displays the corresponding symmetry, without further commenting on it. This recurrence makes it all the more difficult for the historian of science to overlook such problems.
- A third fact indicates that Euler's interest in such phenomena is not coincidental. Roughly thirty years later, he comes back again to spherical trigonometry in a *Mémoire* published in 1782 in the *Acta Academiae Scientiarum Imperialis Petropolitanae*. By that time, everything has changed in his treatment of the topic, except that he still wants to give a complete account of spherical trigonometry on the basis of the fewest possible number of principles, that he still uses the same notations as in the *Mémoire* from 1753, and that the symmetry caused by duality is still tacitly central in his presentation of the topic.[41] Moreover, the rule of rewriting which exchanges capital and small letters as well as some signs for computations has become explicit.

But, here, fortunately, if we may say so, Euler makes a mistake which, as in Li Ye's case above, enables us to describe more precisely how he produced this kind of text. In paragraph 22, given as example in figure 6, adding and subtracting 1 to the first and the

Transformatio tertia.

§. 22. Hanc transformationem etiam ex prima forma expedire licet, combinandis his duabus formulis:

$$\text{coſ.}\, a - \text{coſ.}\, b\, \text{coſ.}\, c = \text{ſin.}\, b\, \text{ſin.}\, c\, \text{coſ.}\, A,$$

$$\text{coſ.}\, b - \text{coſ.}\, a\, \text{coſ.}\, c = \text{ſin.}\, a\, \text{ſin.}\, c\, \text{coſ.}\, B;$$

quarum illa per hanc diuiſa praebet,

$$\frac{\text{coſ.}\, a - \text{coſ.}\, b\, \text{coſ.}\, c}{\text{coſ.}\, b - \text{coſ.}\, a\, \text{coſ.}\, c} = \frac{\text{ſin.}\, b\, \text{coſ.}\, A}{\text{ſin.}\, a\, \text{coſ.}\, B} = \frac{\text{ſin.}\, B\, \text{coſ.}\, A}{\text{ſin.}\, A\, \text{coſ.}\, B}.$$

Addatur vtrinque vnitas, fietque

$$(\text{coſ.}\, a + \text{coſ.}\, b)(1 - \text{coſ.}\, c) = \frac{\text{ſin.}\, (A + B)}{\text{ſin.}\, A\, \text{coſ.}\, B},$$

ſubtrahatur vtrinque vnitas, prodibit

$$(\text{coſ.}\, a - \text{coſ.}\, b)(1 + \text{coſ.}\, c) = \frac{\text{ſin.}\, (B - A)}{\text{ſin.}\, A\, \text{coſ.}\, B},$$

quae aequatio per priorem diuiſa dat

$$\frac{\text{coſ.}\, a - \text{coſ.}\, b}{\text{coſ.}\, a + \text{coſ.}\, b}.\, \text{cot.}\, \tfrac{1}{2} c^2 = \frac{\text{ſin.}\, (B - A)}{\text{ſin.}\, (B + A)}.$$

Conſtat autem eſſe

$$\frac{\text{coſ.}\, p - \text{coſ.}\, q}{\text{coſ.}\, p + \text{coſ.}\, q} = \text{tang.}\, \frac{q + p}{2}\, \text{tang.}\, \frac{q - p}{2},$$

vnde colligitur:

$$\text{tang.}\, \frac{b - a}{2}.\, \text{tang.}\, \frac{b + a}{2}.\, \text{cot.}\, \tfrac{1}{2} c^2 = \frac{\text{ſin.}\, (B - A)}{\text{ſin.}\, (B + A)}.$$

Transformatio quarta.

§. 24. Haec ſimili modo deducitur ex his formulis:

$$\text{coſ.}\, A + \text{coſ.}\, B\, \text{coſ.}\, C = \text{ſin.}\, B\, \text{ſin.}\, C\, \text{coſ.}\, a$$

$$\text{coſ.}\, B + \text{coſ.}\, A\, \text{coſ.}\, C = \text{ſin.}\, A\, \text{ſin.}\, C\, \text{coſ.}\, b$$

quarum illa per hanc diuiſa praebet

$$\frac{\text{coſ.}\, A + \text{coſ.}\, B\, \text{coſ.}\, C}{\text{coſ.}\, B + \text{coſ.}\, A\, \text{coſ.}\, C} = \frac{\text{ſin.}\, B\, \text{coſ.}\, a}{\text{ſin.}\, A\, \text{coſ.}\, b} = \frac{\text{ſin.}\, b\, \text{coſ.}\, a}{\text{ſin.}\, a\, \text{coſ.}\, b}.$$

Vnde vnitatem tam addendo quam ſubtrahendo ſequentes nouae deriuantur aequationes:

$$(\text{coſ.}\, A + \text{coſ.}\, B)(1 + \text{coſ.}\, C) = \frac{\text{ſin.}\, (a + b)}{\text{ſin.}\, a\, \text{coſ.}\, b}$$

$$(\text{coſ.}\, A - \text{coſ.}\, B)(1 - \text{coſ.}\, C) = \frac{\text{ſin.}\, (b - a)}{\text{ſin.}\, a\, \text{coſ.}\, b},$$

et diuidendo illam per hanc nanciſcimur:

$$\frac{\text{coſ.}\, A + \text{coſ.}\, B}{\text{coſ.}\, A - \text{coſ.}\, B}\, \text{cot.}\, \tfrac{1}{2} C^2 = \frac{\text{ſin.}\, (a + b)}{\text{ſin.}\, (b - a)}, \text{ ſiue}$$

$$\text{tang.}\, \frac{B - A}{2}.\, \text{tang.}\, \frac{B + A}{2} = \text{cot.}\, \tfrac{1}{2} C^2.\, \frac{\text{ſin.}\, (b - a)}{\text{ſin.}\, (b + a)},$$

Figure 6. Two paragraphs symmetrical to each other in [Euler 1781].

third members of

$$\frac{\cos a - \cos b \cos c}{\cos b - \cos a \cos c} = \frac{\sin b \cos A}{\sin a \cos B} = \frac{\sin B \cos A}{\sin A \cos B}$$

he should obtain:

$$\frac{(\cos a + \cos b)(1 - \cos c)}{\cos b - \cos a \cos c} = \frac{\sin (A + B)}{\sin A \cos B}$$

and

$$\frac{(\cos a - \cos b)(1 + \cos c)}{\cos b - \cos a \cos c} = \frac{\sin (B - A)}{\sin A \cos B}$$

But here Euler forgets the denominators of both first members. Then, he divides this equation by the previous one, hence the two identical forgotten denominators compensate for each other, and thus the result is valid.

The key fact for us here is that a symmetrical mistake can be found in § 24 (see figure 6), symmetrical to this one, step-by-step. Indeed, adding and subtracting 1 to the first and the third members of

$$\frac{\cos A + \cos B \cos C}{\cos B + \cos A \cos C} = \frac{\sin B \cos a}{\sin A \cos b} = \frac{\sin b \cos a}{\sin a \cos b}$$

he should again obtain

$$\frac{(\cos A + \cos B)(1 + \cos C)}{\cos B + \cos A \cos C} = \frac{\sin (a + b)}{\sin a \cos b}$$

and

$$\frac{(\cos A - \cos B)(1 - \cos C)}{\cos B + \cos A \cos C} = \frac{\sin (b - a)}{\sin a \cos b}$$

But here, again, Euler forgets the denominator of both first members. As previously, he divides this equation by the previous one; hence, here as before, the two identical denominators he forgot compensate for each other, and thus the result is valid.[42]

Such a pair of mistakes cannot be explained except by assuming that the latter proof has been obtained by a mere translation of the former one, where each step was transformed in accordance with the rule of rewriting involving an exchange between capital and small letters. This implies that Euler used the notations he introduced, which are so appropriate to deal with this symmetry in spherical trigonometry, to translate parts of the text into other parts. Such is the way in which he gave his text this structure. This phenomenon shows, as in Li Ye's case,

- Euler's awareness of the structure of the text.
- his will to produce it, his will to express something in this way.
- how he produced it: mathematical operations are involved in the making of the text.
- that the text supports certain mathematical operations and is meant to do so.

2.3 *A history developing in the structure of the texts*

The structure of the text manifests mathematical knowledge and mathematical activity. This should suffice to prove that we, as historians, need to discuss what is given to be read there when we account for its content. And Euler's case, like Li Ye's, shows that this description is by no means immediate and requires elaboration. However, if we need to consider such questions, this is not only because Euler "meant" his text to have a "tacit" meaning. In this case, where we possess writings concerning spherical

trigonometry composed by scientists who read Euler's *Mémoires*, other factors enter into consideration:

- The form of Euler's texts was read as such by subsequent mathematicians. They reproduced his notations or produced new notations presenting the same properties regarding duality. And, in correlation with this, they composed texts with similar forms. A kind of text gets stabilized by the existence of a community of readers, synchronically as well as diachronically.
- The form of the text itself was taken up and improved in its function of displaying symmetries that affect certain parts of mathematics in the same way as spherical trigonometry. This shows that forms of texts have a history too.[43] In this case, the process reached a stable state at the beginning of the nineteenth century in the *Annales de mathématiques pures et appliquées*, whose editor, Gergonne, devoted specific attention to this range of phenomena in mathematics. To this end, he published a series of papers dealing with mathematical topics in which such symmetry occurs, and he designed for them a specific form of presentation in double columns (see figure 7), where not only formulas, but also statements corresponded to each other thanks to a systematic rule of translation. He also rewrote the presentations of these domains of mathematics so as to fit them into this form. Moreover, by asking problems in a symmetrical way in his journal (see figure 8), Gergonne incited readers to work on such symmetries within this way of presentation. Such texts can be found even today in certain mathematical publications.
- Because various domains of mathematics were likely to receive such a formal presentation, the idea that this was due to one and the same mathematical phenomenon came out and was explored with texts of this form. Such was the way in which research on duality, as a property of space, began.[44] Hence the kind of knowledge expressed by such forms has its history as well.

Let me recapitulate what, it seems to me, we can conclude from this case:

- In order to deal with certain phenomena in mathematics, or more generally, types of texts may be created that allow writers to express these phenomena, communicate them, and work on them. Hence a form of text is consciously created for a specific kind of research.
- These meanings are received and reworked under this form. In our case, a whole history develops in the structure of the text. And, in correlation with this, we can see the form of the text and what it is meant to deal with evolve. Indeed, as regards duality, after a time when it was worked out through the structures of texts, it became a topic of discursive treatment.[45] This shows the continuity of nature between content expressed by the form of a text and that expressed discursively.
- Without reading the meanings conveyed by these specific forms of texts, without a critical conception of what a text is, it would be difficult to discuss their content as well as the later evolution of ideas thus displayed. To assign the beginning of the history of duality to its discursive treatment would represent a mistake of the same nature as those which we analyzed concerning Li Ye's *Sea-mirror of the circle measurements*.

Voilà ce qui nous détermine à faire de cette sorte de géométrie *en parties doubles*, s'il est permis de s'exprimer ainsi, le sujet d'un écrit spécial dans lequel, après avoir rendu manifeste le fait philosophique dont il s'agit, dans l'exposé même des premières notions, nous nous en appuyerons, soit pour démontrer quelque théorèmes nouveaux, soit pour donner de quelques théorèmes déjà connus des démonstrations nouvelles, qui les rendent à l'avenir tout à fait indépendans des relations métriques desquelles on a été jusqu'ici dans l'usage de les déduire.

§. I.

Notions préliminaires.

1. Deux points, distincts l'un de l'autre, donnés dans l'espace, déterminent une droite indéfinie qui, lorsque ces deux points sont désignés par A et B, peut être elle-même désignée par AB.	1. Deux plans, non parallèles, donnés dans l'espace, déterminent une droite indéfinie qui, lorsque ces deux plans sont désignés par A et B, peut être elle-même désignée par AB.
2. Trois points donnés dans l'espace, ne se confondant pas deux à deux et n'appartenant pas à une même ligne droite, déterminent un plan indéfini qui, lorsque ces trois points sont respectivement désignés par A, B, C, peut être lui-même désigné par ABC.	2. Trois plans, non parallèles deux à deux dans l'espace, et ne passant pas par une même ligne droite, déterminent un point qui, lorsque ces trois plans sont respectivement désignés par A, B, C, peut être lui-même désigné par ABC.
3. Un plan peut aussi être déterminé dans l'espace par une droite et par un point qui ne	3. Un point peut aussi être déterminé dans l'espace par une droite et par un plan dans le-

Figure 7. Gergonne's presentation of geometry in double column ([Gergonne 1826], the top paragraph comes from p. 211, and the double column presentation from p. 212).

CONCLUSION

Even if we limit ourselves here to these two examples, the direction of research they open converges with what many other writers have already indicated: various forms of collaboration between history of science and history of text might prove profitable.

Texts are not a historical, transparent forms conveying meanings whose history we would write. Scientific texts as such appear to have taken various forms within space and time, designed as they were through an interaction with local conditions of text production of all kinds: ways of giving names, literary forms, graphics, and writing technology available. These texts have a history which inscribes them in an activity, in a culture, in an environment of readers.

Elaborating a description of these various forms of texts would provide methodological tools to read them, since they can by no means be read without the mediation of a method. Here, the achievements of a history of text would benefit history of science in that it would deliver a better knowledge of the local contexts of text production within

QUESTIONS PROPOSÉES.

Théorèmes de géométrie.

Deux quadrilatères quelconques étant donnés, il existe un angle tétraèdre auquel ces deux quadrilatères sont l'un et l'autre inscriptibles.

Deux angles tétraèdres quelconques étant donnés, il existe un quadrilatère auquel ces deux angles tétraèdres sont l'un et l'autre circonscriptibles.

Problèmes de geométrie.

On a construit sur les deux faces d'un angle dièdre deux triangles tels que les points que déterminent leurs côtés correspondans sont tous trois sur l'arête de l'angle dièdre, et conséquemment en ligne droite, d'où il résulte que, quelle que soit l'ouverture de l'angle dièdre, toujours les droites que détermineront les sommets correspondans des deux triangles passeront par un même point. On suppose que l'on fait varier cette ouverture, et on demande quelle ligne ce point décrira dans l'espace?

Deux angles trièdres sont tels que les plans que déterminent leurs arêtes correspondantes passent tous trois par la droite que déterminent leurs sommets, et se coupent conséquemment suivant une même droite; d'où il résulte que, quelle que soit la distance de leurs sommets, toujours les droites que détermineront les faces correspondantes des deux angles trièdres seront dans un même plan. On suppose que l'on fait varier cette distance, et l'on demande à quelle surface ce plan sera constamment tangent?

Figure 8. Problems set by Gergonne to his readership ([Gergonne 1826], p. 232).

which scientific texts were designed and elaborated. It would describe an environment against which to read our sources.

But, in addition to providing a better grasp of such a textual context for the production of scientific writing, a history of text might give a better awareness of the various ways in which texts were meant to mean and help us discuss how to account for their content. Indeed, the aim of determining with certainty the content of a document —difficult to push aside when such a branch of intellectual history as history of science is conceived as a history of results— appears, in view of the problems previously described, illusory. We presented texts to which it is difficult to ascribe a content, although the stakes are rather high. In particular, we saw that an opposition between form and content, which underlies many ways of reading, would let innumerable aspects of the content escape, even though what escapes is not simple to characterize. This incites us to add another range of issues to those suggested above and support the call for systematically promoting a history of the readings, of the receptions, of scientific texts, which can be developed by examining, when possible, commentaries on, or reactions to, a given writing. The way readers read tells us something about a text, be they in the same textual tradition or not. And a description of the text, including such structures as those described above, could be helpful in characterizing this reading. Such an additional inquiry to complement the quest of assigning results to a writing would inscribe us as readers amidst the actors we observe.

On the other hand, the history of scientific text could become a systematic concern in history of science as such. Indeed, scientists design their texts at the same time as they design concepts and results. This represents a constitutive part of their activity. And this intimate relation can go as far as the fact that concepts and results can be made, as in the case of duality, of texts. Indeed, if texts were not the subject of a specific design, would they vary in form, would their reading call for methodological tools?

How do scientists design these texts? The study of the production of texts would bring materials that would enable us to deal with the question of understanding how scientists benefit from the cultural and textual contexts within which they work.[46] It would give us a grasp of how scientists construct the symbolic tools with which they perform their activities and communicate their results, which in the end are texts. In all these respects, a history of scientific text could then become a specific domain of the history of text.

REHSEIS-CNRS

NOTES

[1] This paper was written for the workshop "History of science, History of text", during my stay at the Wissenschaftskolleg, in Berlin. I gratefully acknowledge the help of the Wissenschaftskolleg, the Otto und Martha Fischbeck Stiftung and the Einstein Forum. It is a pleasure to thank all my colleagues at the Wissenschaftskolleg during this year for the interest they expressed in this project and for the help they gave me in defining it. I am glad to take this opportunity to thank my colleagues Huang Yilong, who helped me to prepare this text, and Fu Daiwie, who invited me to publish it, in a different shape in *Philosophy and the History of Science*, a Taiwanese journal. I owe many thanks to the publisher and the editors of the journal for allowing me to reproduce it in this volume. I am grateful to Chris Fraser and Jeremy Gray for their revision of the English. Needless to say, I am responsible for all the remaining mistakes. The reader can find in (Chemla 1995a) other arguments to support the thesis defended here.

[2] The recent case of the problems raised by the reading of Diophantos' *Arithmetics* with the tools provided by algebraic geometry has brought new material to bear on this question. See (Rashed 1984) and (Chemla, Morelon, Allard 1987), where the reader can find a bibliography on the topic. As to the role played by later mathematics in enabling us to read an ancient source, this case can be compared to Li Ye's, which I present below, as I have argued in "What could be experimentation in the history of mathematics?", VIIIth conference of the International Union for the History and Philosophy of Science, Gand, August 1986 (to appear).

[3] I have discussed this treatise extensively in my Ph. D. thesis (Chemla 1982) and several papers (Chemla 1985, 1988, 1990, 1993a & b), where an updated bibliography can be found.

[4] On the so-called "procedure of the celestial element," see, for instance, (Chemla 1982). Let us recall here that the polynomials in one indeterminate introduced are written down according to a vertical place-value notation:

1
2 stands for $x^2 + 2x + 3$
3•

whereas

1
2• stands for $x + 2 + \frac{3}{x}$
3

What we transcribe as a point going along with the constant term corresponds to a Chinese character. Polynomials are distinguished from equations by means of this mark:

1
2 stands for $x^2 + 2x + 3 = 0$
3

[5] To designate the segments by the term of *lü* states a property and announces a decision concerning the mathematical situation. The concept of *lü* refers to the fact that when all the magnitudes considered are multiplied by the same amount, this does not change the meaning of the whole. This states a property concerning their relationships to each other, and announces the decision to consider the drawing up to a dilatation, and not in the absolute. Concerning the way names are given to objects in some Chinese mathematical texts and the way to read them in consequence, see (Chemla 1993b).

[6] In order to convey an idea of the text conveniently, I associate letters to the points to which Li Ye gives a Chinese name on his drawing, and numbers between brackets to the triangles which he now introduces. See figure 2: the numbers I associate to right-angled triangles are inscribed in their right angle.

[7] In what follows, I shall designate the base by a, the height by b, and the hypotenuse by c.

[8] This constitutes a change since the time when *The ten classics of mathematics* were collated by Li Chunfeng and by the team he headed in the seventh century (see Qian Baocong 1963). There right-angled triangles receive specific attention, but they are not systematically attached to thirteen quantities (see Schrimpf 1963: 216–42). However, in Song-Yuan times, such is the case, and a specific terminology is chosen for these thirteen quantities, even though there are variations from one author to another.

[9] Hereafter I shall abbreviate the title of this book as *The nine chapters*, and, unless otherwise specified, I shall rely on the edition of Qian Baocong (1963). Note that this classic displays knowledge in the form of problems and general algorithms for their solutions. In this respect, it resembles most of the mathematical writings produced in China until the fourteenth century. In using such textual elements as a compendium, Li Ye resorts to a form of presentation examples of which are rare in classical Chinese mathematical literature.

[10] He refers to it and to its commentary by Liu Hui, written in the third century and handed down together with the classic in all surviving editions, in the preface of his two mathematical works.

[11] Elements relevant for the construction of this drawing are discussed in (Chemla 1994a).

[12] Two exceptions appear here:
—Two triangles (5) and (6) are associated with quantities that are not in the list of those attached to a triangle. I discuss this in (Chemla 1993a).
—The quantities $a + b$ and $b - a$, hence the simplest quantities computed with the base and the height of the largest triangle, are missing in the list of quantities obtained as sums of the lengths of the three sides of a triangle. Again, this is discussed in (Chemla 1993a). Yet, it would suffice to add the line drawn by Liu Hui in his proof to Li Ye's drawing to make such triangles appear. And this is precisely what Li Shanlan will do in the nineteenth century when reading the *Ceyuan haijing*, which indicates that he also considers the two texts to refer to each other (See Chemla 1982: 2.55 *sq.*). Is this an additional hint of the connection between the two texts?

[13] Such a figure seems to have been produced by a research program different from some developed on drawings in ancient Greece. In the Chinese texts starting from *The nine chapters* and Liu Hui's commentary onwards, fundamental patterns for figures seem to be sought for. Compare (Chemla 1994a).

[14] I skip the details, which can be found in (Chemla 1993a), if necessary.

[15] His treatment of problem VIII-15 shows that Li Ye is aware of these facts, see (Chemla 1990).

[16] See Chemla 1993b. As we saw above, the names can be read as conveying information about the things they designate. The design of names and, hence, the symbolic operations that a reader performs with them are to be referred to the uses elaborated within certain communities, and not to be taken as invariant. This point confirms the thesis developed in this paper, but I will not elaborate any further here (see Chemla 1993b).

[17] See (Chemla 1990) and (Chemla 1993a).

[18] See Chemla, Martin & Pigeot (eds.) 1995.

[19] As already emphasized, this holds true for names and for drawings, which indicates that two kinds of cultural contexts have to be taken into account: a mathematical one, where such elements as drawings are elaborated in a specific way, and a larger one within which ways of designing names and making quotations present stable features beyond variation. Numbers, names, and the drawing are three kinds of objects for which we suggest here a specific mode of reading. Note that the outcomes draw a coherent picture.

[20] For a more detailed treatment, see Chemla 1990 & 1993a.

[21] This question is treated in (Chemla 1982: 2.39 *sq.*).

[22] This cannot be due to a mere error by a copyist, as one can find a parallel mistake in problem 17 of chapter IV. We shall discuss such mistakes below.
[23] They are the first two formulas of paragraph 49 if one starts numbering the paragraphs at the beginning of the compendium—a paragraph being the textual unit whose first character is printed higher than the others, in the upper margin.
[24] I translate the two verses with which F. Martin illustrates the phenomenon of parallelism in (Martin 1989: 81 *sq*.).
[25] The entire issue 11 of *Extrême-Orient, Extrême-Occident,* (Jullien (ed.)1989), mentioned above is devoted to the question of parallelism in Chinese texts and thought. The two incorrect formulas found in the solutions of problems that I evoked at the beginning of this paragraph are parallel in the same way as those quoted here.
[26] One can find a presentation of this procedure, which amounts to polynomial computations, in any standard introduction to the history of mathematics in China as well as in (Chemla 1982). In order to study Li Ye's text, I shall reformulate parts of his book below and then, contrary to what we find in his text, I shall make use of Arabic numerals.
[27] As indicated on the drawing, we shall designate these unknowns respectively by A and B.
[28] Such is the name of the unknown, on the basis of which polynomials are computed. As announced above, I keep the place-value notation of the polynomials.
[29] I represent the polynomials as Li Ye does, using letters where he uses numerals. Moreover, I use x instead of a character which marks the term of the first degree. I skip here some details that are irrelevant for my purpose. See Chemla 1982, 1985 or 1993a.
[30] See chapter 10 of (Chemla 1982).
[31] I tried in (Chemla 1982), chapter 10, to make use of some details in the *Sea-mirror of the circle measurements* in order to describe the way in which this computation may have been performed.
[32] See Chemla 1995b.
[33] This is very clear from the classical work *The nine chapters* onwards.
[34] This raises the question of the nature of equations in earlier Chinese texts, which I shall not address here.
[35] The difficulties encountered here seem to me comparable to those raised by the reading of Diophantos' *Arithmetics* with the help of algebraic geometry. The tool reveals properties of the text. We would certainly not assign the knowledge of our tool to Diophantos. Still the cause which produces the properties revealed in the text needs to be determined. A discussion of the operations performed on a text when one brings mathematical tools into play to read it should take into account such evidence.
[36] The author of the commentary introduced by *an* "commentary" —until now not identified (see Chemla 1982: 0.3)— states the close relationships linking both parts of the solution and writes (p. 11–2, chapter 2): "The "sketch" is the root of the "method," the "method" is the functioning of the "sketch"." Li Shanlan, in his preface to the new edition, writes (p. 1) : "The "sketch" ... is the "method" to make the method, the origin of the "method"." Closer to us, Li Yan, when he rewrote Li Ye's "sketches" symbolically, tacitly let the steps in the description Li Ye gave in the "methods" appear (see Chemla 1993a).
[37] For this paragraph I mainly rely on (Chemla 2004), where the reader will find all the details, a more careful treatment, and a bibliography which I do not reproduce here. I am grateful to Serge Pahaut, who called my attention to the history of duality, and we started to work together on the influence of research on spherical trigonometry on the emergence of a specific way of dealing with this notion (see Chemla, Pahaut 1988).
[38] "Les dénominations précédentes se réduiront aux présentes de cette manière:
Dénominations précédentes a, x, s, y, α, φ
Dénominations présentes c, b, a, A, B, C." (Euler 1753: 294)
[39] There are exceptions but, in the loose account given here, I shall not comment on them, see Chemla 2004.
[40] See our paper to appear on the question of drawing in the Chinese mathematical tradition.
[41] See figure 6, where two paragraphs symmetrical to each other are reproduced (they come from p. 82 and p. 84 in the original publication).
[42] In his edition of this text, A. Speiser writes symmetrical footnotes to correct both mistakes.
[43] Here we should make precise that so far, for the sake of simplicity, we have simplified the account one should give of the history of such designs of texts as the one found in Euler's *Mémoires* on spherical trigonometry. Indeed, Euler did not initiate a trend of research: there existed before him a whole tradition

of writings about this topic, presenting some symmetries in their composition (one can think of Viète, for instance). Euler himself inherited this form of text, which he modified and extended, thereby introducing changes in the conception of duality itself, and his readers, in turn, carried on this process.

44 See Chemla 1989 & 1994b.

45 In Cavaillès' terms: "thématisé".

46 When he introduces the presentation in double column, Gergonne refers explicitly to bookkeeping. A textual element is found available in the context and transformed so as to serve some specific mathematical research. In the same way, Li Ye can use the grammar of written Chinese of his time to record his formulas. However, he introduces some restrictions on the common use for the sake of mathematics, thanks to which the statements of formulas are made unambiguous. This turns the language he uses into an artificial one —see my paper in collaboration with Alain Peyraube (to appear). This phenomenon is all the more remarkable in that we do not find it either in *The nine chapters on mathematical procedures* or in the writings of a mathematician slightly posterior to Li Ye, Zhu Shijie.

REFERENCES

Chemla, K. 1982. "Etude du livre *Reflets des mesures du cercle sur la mer* de Li Ye." Ph. Diss. in mathematics, University Paris XIII, 12-10-82.

Chemla, K. 1985. "Equations with General Coefficients in the *Ce Yuan Hai Jing*. "*Cahiers du séminaire de Rennes "Science, Technique, Société*," *Publications de l'Institut de Recherche Mathématique de Rennes, Fascicule II*: Science, Histoire, Société: 23–30.

Chemla, K. 1988. "La pertinence du concept de classification pour l'analyse de textes mathématiques chinois." *Extrême-Orient, Extrême-Occident* 10: 61–87.

Chemla, K. 1989. "The Background to Gergonne's Treatment to Duality: Spherical Trigonometry in the Late eighteenth Century." In *The History of Modern Mathematics*, edited by D. Rowe and J. McCleary, 331–359. San Diego: Academic Press, vol. I.

Chemla, K. 1990. "Du parallélisme entre énoncés mathématiques; analyse d'un formulaire rédigé en Chine au 13° siècle." *Revue d'histoire des sciences* XLIII/1: 57–80.

Chemla, K. 1993a. "Li Ye *Ceyuan hai jing* de jiegou ji qi dui shuxue zhishi de biaoshi." *Shuxueshi wenji* 5: 123–142.

Chemla, K. 1993b. "Cas d'adéquation entre noms et réalités mathématiques. Quelques exemples tirés de textes chinois anciens." *Extrême-Orient, Extrême-Occident* 15: 102–137.

Chemla, K. 1994a. "De la signification mathématique de marqueurs de couleurs dans le commentaire de Liu Hui." In *Mélanges en hommage à Alexis Rygaloff*, edited by Alain Peyraube, Irène Tamba & Alain Lucas. *Cahiers de Linguistique—Asie Orientale* 23: 61–76.

Chemla, K. 1994b. "Le rôle joué par la sphère dans la maturation de l'idée de dualité au début du XIX° siècle. Les articles de Gergonne entre 1811 et 1827." *Actes de la Quatrième Université d'Eté d'histoire des mathématiques*, 57–72. Lille: Irem de Lille.

Chemla, K. 1995a. "Histoire des sciences, histoire du texte." *Enquête* 1: 167–80.

Chemla, K. 1995b. "Algebraic Equations East and West until the Middle Ages." In *East Asian Science: Tradition and Beyond*: Papers from the Seventh International Conference on the History of Science in East Asia, Kyoto, 2–7 August 1993, edited by K. Hashimoto, C. Jami, L. Skar, 83–9. Osaka: Kansai University Press.

Chemla, K. 2004. "Euler's Work in Spherical Trigonometry: Contributions and Applications." In *Opera Omnia*, Third series, volume 10, *Über Magnetismus, Elektrizität, und Wärme*, CXXV—CLXXXVII. Basel: Birkhäuser.

Chemla, K., Martin F. & Pigeot, J. (eds.) 1995. *Le travail de la citation en Chine et au Japon, Extrême-Orient, Extrême-Occident* 17. Saint-Denis: Presses Universitaires de Vincennes.

Chemla, K., Morelon, R. & Allard, A. 1986. "La tradition arabe de Diophante d'Alexandrie." *L'Antiquité Classique* LV: 351–375.

Chemla, K. & Pahaut, S. 1988. "Préhistoires de la dualité: explorations algébriques en trigonométrie sphérique (1753–1825). "In *Sciences à l'époque de la Révolution Française*, edited by R. Rashed, 148–200 + 1 planche. Paris: Librairie Scientifique et Technique Albert Blanchard.

Euler, Leonhard 1750. "Elementa doctrinae solidorum." *Novi commentarii academiae scientiarum Petropolitanae* 4, (1752/3), 1758: 109–40. In *Opera Omnia*, vol. XXVI of the first series, edited by A. Speiser, 71–93. Lausanne, MCMLIII.

Euler, L. 1753. "Principes de la trigonométrie sphérique tirés de la méthode des plus grands et des plus petits." *Mémoires de L'Académie des Sciences de Berlin* 9,(1753), 1755: 223–257. In *Opera Omnia*, vol. XXVII of the first series, edited by A. Speiser, 277–308. Lausanne, MCMLIV.

Euler, L. 1781. "Trigonometria sphaerica universa ex primis principiis breviter et dilucide derivata." *Acta Academiae Scientiarum Imperialis Petropolitanae* 3, I (1779), 1782: 72–86. In *Opera Omnia*, vol. XXVI of the first series, edited by A. Speiser, 224–236. Lausanne, MCMLIII.

Gergonne, J.D. 1826. "Considérations philosophiques sur les élémens de la science de l'étendue." *Annales de Mathématiques Pures et Appliquées* XVI, (1826): 209–231.

Jullien, François (ed.) 1989. *Parallélisme et appariement des choses, Extrême-Orient, Extrême-Occident*, 11. Saint-Denis: Presses Universitaires de Vincennes.

Li Ye 1248. *Ceyuan haijing* (Sea-mirror of the circle measurements), quoted according to the edition of the Tongwenguan, 1876.

Martin, F. 1989. "Les vers couplés de la poésie chinoise classique," *Extrême-Orient, Extrême-Occident* 11: 81–98.

Mei Rongzhao 1966. "Li Ye ji qi shuxue zhuzuo." In *Song Yuan shuxueshi lunwenji*, edited by Qian Baocong et al., 104–148. Beijing: Kexue chubanshe.

Qian Baocong 1963. *Suanjing shishu* (*The Ten Classics in Mathematics*), 2 volumes. Beijing: Zhonghua Shuju.

Rashed, Roshdi 1984. *Diophante. Les Arithmétiques* (Volume III: Book IV; Volume IV: Books V, VI, VII). Paris: Les Belles Lettres.

Schrimpf, Robert 1963. "La collection mathématique "Souan King Che Chou," Contribution à l'histoire des mathématiques chinoises des origines au VIIe siècle de notre ère." Ph. D. diss., Université de Rennes.

DAVID R. OLSON

KNOWLEDGE AND ITS ARTIFACTS

ABSTRACT

Knowledge consists of a set of beliefs, that is, mental states, held as true by members of a culture. These beliefs are "represented" in the permanent artifacts of that culture as well as in the non-archived discourse surrounding those artifacts. My question is the effect that the "archiving" of knowledge in the form of public documents and artifacts and the subsequent "reading" of those artifacts, has on the form that knowledge takes and on the minds of those that use them. I will suggest that the form of representation and the ways in which it is used affect what is represented and what, then, is taken to be knowledge. I will illustrate this argument by reference to writing and reading texts such as essays, diagrams, charts and mathematical formulae in medieval and modern times. I will attempt to show that the changes in these quite different forms of representation in fact are parallel to each other and may be traced back to changing practices of writing and reading. I conclude with some general comments on the relation between knowledge and its artifacts.

In her monumental work on the printing press as an agent of change, Eisenstein claimed that the Alexandrian achievements which came to an end with the collapse of the Roman Empire in the fourth century AD, were not surpassed until the invention of printing which allowed one to put "'the world on paper' for all armchair travelers to see" (Eisenstein 1979:503).

The world on paper is an apt metaphor for analyzing one of the most far-reaching implications of literacy, for, by creating texts in which statements, formula and images serve a representational function, one comes to deal not with the world but with the world as depicted or described. This point is readily seen for maps and formulae for which linguistic descriptions seem inadequate; it is less readily seen for written texts which appear to simply transcribe speech. Yet the same relations, it may be argued, hold there as well.

The notion of a paper world was not accepted enthusiastically even by those who most directly contributed to its creation. A common refrain amongst Renaissance writers, Galileo included, was the importance of turning away from books to study the things in themselves. Eisenstein reverses the claim suggesting that it was the accumulation of information *in* books, maps and diagrams that made possible the rapid growth of knowledge that we associate with the Early Modern, that is seventeenth century, science. This accumulation is what Popper has called World Three, the world of "objective knowledge," namely, the theories, models and other artifacts we use to think with (Popper 1972).

K. Chemla (ed.), History of Science, History of Text, 231–245.

Eisenstein is undoubtedly correct regarding the important role that printing played in the establishment of an accumulative archival tradition. That accumulative archival tradition, storing knowledge produced by many minds in a common representational format, was preceded by a new understanding of texts and a new way of reading and writing them, namely, of seeing texts as representations. Indeed, it may be argued that this new way of seeing was as important as the accumulation itself. That is, whereas Eisenstein emphasized the importance of printing for the assembly of knowledge, I wish to emphasize the importance of these new representational systems on the structure of thought itself. Texts not only accumulated knowledge, they allowed one to think about that knowledge in a new way.

As the concept of "representation" plays many and varied roles, it is important to note that, for my purposes, a representation is not just any symbol that evokes a meaning. I can indicate my special use of the term by contrasting symbols that merely evoke an object or idea from those which are taken as standing in for some object or idea. In regard to writing, as long as knowledge was thought of as in the mind, whether the mind of God or man, and expressed in speech, the usefulness of writing was limited; writing could only be seen as reminder, a mnemonic, and not as a representation. To create a representation is not merely to record speeches or to construct mnemonics; it is to construct visible artifacts with a degree of autonomy from their author and with normative standards for controlling how they will be interpreted.

This transformation from mnemonics to representations began in the twelfth century but became conspicuous and dominant in the seventeenth century. Clanchy, in his historical account of the increasingly prominent role that writing played in the English justice system in the twelfth century, points out that at first such documents were augmented by iconic signs (Clanchy 1993). Thus a deed of land may not only spell out the agreement, but also append a physical object such as a stone or a knife or other object that had some intrinsic relation to the deeded property. By the thirteenth century the document alone could represent the deeding of property. Texts, we may say, have made the shift from mnemonics to representations, from memory to written record.

For written texts to bear the burden of official documents, that is, to carry the weight of meanings they came to represent required the formation of a new attitude to signs. Foucault pointed out that by the seventeenth century language and other sign systems had come to be seen in a new way (Foucault 1970). Signs were no longer seen as natural to their object but as conventions: not copies or mimesis (Morrison 1982) but as representations in a medium (Gombrich 1960.) It is that transformation in the role and significance of writing that must be explained.

1. READING AND MEANING

It may be argued (Olson 1994) that this new orientation to signs was a product of the invention of a new way of reading. That shift has been set out primarily in the work of Smalley (Smalley 1941) and more recently of Morrison (Morrison 1990.) It is somewhat difficult to imagine that the ways of reading, interpreting and understanding a text could have changed historically in any fundamental way. We tend to think that we just read

and understand. Not so. The conceptual changes that ushered in modernity, that is, those changes that occurred between the Middle Ages and the Renaissance were, in part, a matter of learning to read in a new way. The shift from oral to silent reading may have played a part in this shift (Saenger 1982, 1997), but most importantly, it involved a new assumption about meaning. The shift was a matter from moving away from a form of reading which Morrison has described as "seeking epiphanies between the lines" to reading what was on the lines (Morrison 1990). This involved the recognition of sentence meaning, that is, the information explicitly represented in the text by means of its linguistic form and its textual context. It was the invention of what we would now call "strict reading." When the strategy was applied to writing, the product was "strict writing," that is, the attempt to make the texts themselves the embodiment of meaning, or, as one may say, of putting the meaning into the text.

In the Middle Ages, texts were seen as a boundless resource from which one could take an inexhaustible supply of meanings. The assumption thrived in part because the texts in question were religious, primarily the Scripture. Other writing was seen as pagan and hence of little significance. The arts of rhetoric, various devices for inventive readings for devotion and edification, provided means for rich reading—readings of great diversity but of questionable validity. Such reading, however, began to raise a general alarm by the twelfth century. The problem was not that readers had difficulty in finding meaning in texts; the problem was that readers found too much meaning. New techniques for reading and for justifying interpretations had to be found if any of them were to be taken as authoritative, a major concern of the Church.

A few other characteristics of such pre-modern reading may be mentioned. Reading was not distinguished from memorizing, the committing of important texts to memory for subsequent pondering and reflection. Reading was more of an aesthetic activity than an intellectual one. Hence the best preparation for imbibing or drinking in the sacred word was self-purification and mortification rather than theological study (Green 1994). The purpose of reading was to find a truth, a revelation, the light behind the words, the spirit behind the letter. Reminded by St. Paul that "the letter killeth but the spirit giveth life" (II Cor. 3:5), readers indulged in luxuriant methods of exegesis—the phrase is Karl Morrison's (Morrison 1990: 247)—and a profusion of readings that would put any postmodernist or deconstructionist to shame.

The written texts played a somewhat secondary role to oral speech and oral memory in this period. The written text was not itself thought about; rather the written text was used to permit memorization and to check memory. Difficult as it is for us to imagine, it appears that most thinkers in the Middle Ages and in the Medieval period did their thinking and composing orally and publicly as speeches and sermons, rather than in writing. Students could record what had been said but the act of composition was oral. Carruthers, in her interesting book on memory (Carruthers 1990), points out that St. Thomas Aquinas composed his magisterial *Summa Theologica* orally, pacing around a large room dictating to a bank of secretaries, each of whom took responsibility for transcribing a part. St. Thomas, gazing skywards spoke as if he were reading from a book in the heavens that only he himself could see.

It was this way of reading and writing that gave way and set the stage for the early modern period. Perhaps alarmed by the richness and diversity of interpretation, church

fathers beginning with Hugh of St. Victor in the twelfth century and his student Andrew, began to search for ways to weed out the more of the luxuriant interpretations which were being generated. A new way of reading was in the making.

Their contribution was to advance the understanding of the "historical meaning" of a text, an achievement made by borrowing both knowledge and technique from his Jewish colleagues. The "Jewish way" of reading the Old Testament, as Andrew referred to it, was to render according to the actual words of the text thereby providing new insights into its meaning. Andrew admired this Jewish way even if in the eyes of Andrew and his twelfth century contemporaries, Rabbinical scholars employed "serpentine wile" to evade the "manifest witness to Christ" which, to Andrew, was conspicuous in the Scripture (Smalley 1941: 139–142.) Smalley adds: "No Western commentator before [Andrew] had set out to give a purely literal interpretation of the Old Testament, though many had attempted a purely spiritual one." (Smalley 1941: 140).

This new form of reading is immediately recognizable to us. It required close analysis of the verbal form, its context, as well as of the author, his intended audience and his choice of expression. It was this kind of reading which Luther came to see as providing the one true meaning of scripture, its historical meaning, open for all to see. As is well known, Luther came to see the traditional doctrines of the Church as being accretions and digressions whereas his own beliefs, he believed, were based on the "clear teaching of the divine word" (Stearns 1947: 107).

A satisfactory procedure for deciding what exactly was "given" or meant by a text allowed the clear formation of the question as to the source of all those other meanings which readers traditionally had seen in a text. That source came increasingly to be seen as the product of the imagination. This is, in my view, how reading contributed to the growth of subjectivity, the increasing consciousness of the mind that developed in this period (Olson 1994, Chapter 11).

The new attitude to signs produced not only a new way of reading—according to the literal meanings—, but also a new way of writing, writing as the creation of "representations." This new way of reading old texts was responsible for the development of a way of writing new texts, a new variety of literary form or genre. In this new way of writing, expressions were intended to be taken literally as meaning neither more nor less than what they said. The result was a 'neutral' and 'objective' scientific form of discourse with a "mathematical plainness of style" as the Royal Society of London (Sprat [1667] 1966) was to phrase it. Unlike Medieval discourse in which "a speaker said one thing so that another would be understood" (Morrison 1990: 54) texts were written to honor the new principles of reading, namely, the management of voice, the management of intention, the management of linguistic meaning, and the establishment of a new court of appeal for judging the correctness of an interpretation, namely, the common reader.

This new way of reading, reading designed to specify or determine the one, true, historical meaning, had a second implication. It allowed, indeed encouraged, one to read the "Book of nature" according to the same formula, in both cases on the basis of visible properties open for all to see. Reading a text according to its literal meaning, the meaning "grounded openly in the text," was sufficiently radical that reading the book of Scripture yielded new heresies, one species of which succeeded as the Reformation, and that reading the book of Nature according to the same principles produced Early Modern

science. Writing a text according to this formula produces texts with just those properties that strike modern theorists as exemplifying an analytical form of discourse, and the new understanding of signs as *representations*, that is, as knowledge objects upon which new operations may be performed in the production of new knowledge.

We must set aside temporarily our postmodern suspicions that their goal was unachievable. What they invented, it turned out, was not a royal road to the ultimate truth of things but a new mode of discourse. What we are now more likely to acknowledge but which they did not was that even a simple description of an observed fact is not merely a true representation but an assertion made by a speaker, an insight elevated to systematic theory in the work of J. L. Austin (Austin 1962). The speaker and the speaker's attitude was not eliminated from the discourse but merely hidden or "occulted" as Reiss puts it (Reiss 1982). As we would now acknowledge, there is no representation without intention and interpretation. I return to this point at the end of the chapter.

Just what has to be done to change mnemonic signs into representations? In my view the critical achievement is the creation of what I have called "autonomous" texts, that is, texts which purport to say no more and no less than they mean. The model is that of mathematical expressions in which the form exhausts the meaning; no appeal is made to extralinguistic intentions of the writer or imagination of the reader in determining what they mean. Recall that it is the goal rather than the achievement that is at stake here. Harris has called such texts "authorless" or "unsponsored" in that the author no longer speaks for himself or herself, nor does he or she speak for God as the evangelists did (Harris 1989). Rather, the author's attempt was merely to report, to reveal to the reader what was there for them to see with their own eyes. As mentioned this rule was to apply equally to reading the book of Scripture and to reading "the Book of nature." The achievement was a kind of text with a new set of properties: a clear distinction between observation and interpretation, an elaborated metalanguage for indicating how expressions were to be "taken," a new genre of writing, and, as mentioned, a new, text-based conception of meaning.

We can see just how far some seventeenth century artists and writers succeeded in their attempt to honor these rules for representing the world on paper by examining the evolution of representations in a variety of domains: representational paintings of Dutch art, the representation of the world in maps, representation of physical motion in mathematical notations, the representation of botanical species in herbals, and the representation of imaginative events in fiction all of which I discussed somewhat briefly in *The world on paper*. Here I will consider only drawings and maps, and scientific texts. The cases are chosen to illustrate the impact upon the structure of knowledge and consequently the ways of thinking as one began to examine the world by means of explicit attention to the ways of representing it.

2. REPRESENTING THE WORLD IN PICTURES: DUTCH ART OF THE SEVENTEENTH CENTURY

In her fascinating book *The Art of Describing* (Alpers 1983) Alpers attempts to show the common intellectual basis of Dutch art, geographical maps and microscopic anatomical drawings that co-occurred in the seventeenth century. The common ground, she suggests

was the art of "description." Description is usually thought of as a verbal art, a branch of rhetoric, the power of words to evoke people, places, and events. But for seventeenth century Dutch artists, the notion of describing referred to the ways in which images could parallel the use of words in giving a true description. Medieval icons did not simply or truly describe; they were objects of veneration. For Dutch artists of the seventeenth century the descriptive function of images was to be distinguished from the rhetorical one (Alpers 1983: 136). Literal description is the genre which best exemplifies what, above, I have called *representation*.

Alpers provides convincing evidence that Northern, primarily Dutch, art of the seventeenth century can be understood as a sustained attack on the interpretive tradition in art, the tradition that sees significance or meaning in all that is depicted. The Dutch concern with "mere" description made it difficult for viewers, critics and artists alike to see any point to Dutch art of the seventeenth century. Dutch art, Alpers points out, is noteworthy for its peculiar devotion to the life-like portrayal of objects such as radishes, dead swans, and herring as well as simple domestic scenes such as a cook pouring milk into a pot or map-like landscapes, uninhabited by people. An interpretive stance, which seeks meaning or significance, is inappropriate for Dutch art of this period.

Indeed, contemporary viewers looking for "meaning" were puzzled by such art. Alpers cites Fromentin, a nineteenth century commentator as asking "What motive had a Dutch painter in painting a picture?" And then providing the appropriate answer to that question: "None." (Alpers 1983: xviii). And Joshua Reynolds, the first president of the British Royal Academy after his trip to examine Dutch art said "The account which has now been given of the Dutch pictures is, I confess, more barren of entertainment, than I expected . . . their merit often consists in the truth of representation alone" (Alpers 1983: xviii).

The Dutch art of the seventeenth century, Alpers argues, exchanged narrative depth for surface description. The pictures were constructed not as mnemonics for remembered classical themes but as literal representations of things visible in the world. Typical of the commentators of the day was Samuel van Hoogstraten who in 1678 urged that drawing should be "imitating things after life even as they appear (Alpers 1983: 38). The importance of such careful visual inspection was not merely to see the world better but to disentangle what was there to be seen, from the interpretations read into it. Hoogstraten, a critic of the day, criticized the Italian Renaissance tradition of which Michaelangelo and Raphael were leading exponents, for emphasizing beauty over truth in art, and "chides those who read *meanings* into the clouds of the sky" (Alpers 1983: 77). Hoogstraten urged painters to use their eyes to see clouds as clouds and not as symbols of the heavens!

Alpers traces the Dutch descriptive tradition to the personal contacts between Dutch artists and the leaders of the British empirical or scientific tradition as represented by Bacon, Hooke and Boyle. They exemplified in their art what these scientists had urged in their science. They attempted to do in their art just what Bacon had claimed that scientists should do:

> all depends upon keeping the eye steadily fixed upon the facts of nature and so receiving their images simply as they are. For God forbid that we should give out a dream of our own imagination for a pattern of the world (*The great instauration,* (Bacon [1620] 1965: 323)

Secondly, Alpers supports her claim that Dutch pictures called for close looking and not for "interpreting" by pointing out the accuracy of the pictures. A map which serves as the background for a Vermeer painting was so carefully drawn that the original of which it is a painting has recently been located in Paris (Alpers 1983: 120).

Thirdly, Alpers points out the close relation between picture and map making. A whole landscape tradition was developed by the Dutch in which factual map-making held common cause with picture-making. Even the Dutch word "landschap" was used to refer to both what the surveyor measured and the artist painted. Northern map-makers and artists conceived of a picture as a "surface to inscribe the world" rather than for the portrayal of significant human action. They provide "disinterested" observation, what Alpers calls a "mapped landscape."

The close relation between maps and landscapes is also indicated by the fact that the point of view from which the artist viewed the landscape was often similar to that assumed in a map—the view from nowhere. Alpers points out that for the Dutch of the period, there was no strict distinction between maps and art, between knowledge and decoration, for "pictures challenged texts as a central way of understanding the world" (Alpers 1983: 126).

A final illustration of the Dutch artist's choice of the "true testimony" of the eye over what he took to be subjective and misleading interpretations, comes from the print made by Pieter Saendredam in 1628 of the cross-section of an old apple tree. He made the drawing to repudiate the widely held belief that the dark wood forming the pith of the



Figure 1. Etching by Saendredam of the 'false picture'.

apple tree contained the miraculous images of Catholic clergy. In Protestant Holland, recently at war with Catholic Spain, there were important social reasons for contesting the belief. Saendredam's method is important. He identifies which tree he has cut down, draws the core with great accuracy, contrasting it with the false picture, dates the drawing and publishes an etching of it to repudiate the false picture. His strategy is to "separate the object seen from those beliefs or interpretations to which it had given rise" (Alpers 1983: 81). This, of course, complies with Bacon's charge to distinguish the "dreams of the imagination" from the "patterns in the world." It was those patterns in the world which were to be captured in true representations.

3. REPRESENTING THE WORLD IN MAPS

Again, let us consider two further ways in which the world on paper played a decisive role in thinking about and consequently exploring the real world. The first example of using graphic models as devices for thinking about the world comes from the role that maps and charts played in the voyages of exploration. In 1665 the Royal Society of London published a volume of "Directions for seamen, bound for far voyages." Such "voyages of discovery" were by then common and provided the basis for a new world picture. The world has had it share of itinerants, wanderers and travelers but explorers were new to the fifteenth century. The purpose of the Royal Society was to help explorers relate what they found to a new organized vision. That vision in fact generated the voyages of discovery in that it was a theoretical conception of the world as represented by maps and globes. Two important examples of the application of that vision were the "discovery" of America by the Europeans, and the search for the Southern Continent.

The 'discovery' of America has recently been turned into the question of who found whom. But the analysis here has less to do with who was found than to do with who was looking. Further, it is sometimes assumed that Columbus was a voyager of the old school, simply sailing a few miles farther than his countrymen, inadvertently making a discovery and recording it. This is clearly not the case. Columbus was employing a theoretical model of the world on paper in making calculations about routes, directions and distances (Boas 1962.) Columbus knew that, being a sphere, the earth could be represented as 360 degrees. Further, he concluded on the basis of reading Ptolemy's second century *Geographia* (then recently retranslated back into Latin from Arabic) that each degree corresponded to some fifty miles. He also estimated, on the basis of information provided by Marco Polo's land-based excursion to Cathay (China), that the extent of the old, known world was between 225 and 255 degrees. Since the world sphere is only 360 degrees, that left only some 100 degrees, at most 5000 miles, between China and Spain, traveling westward. He also knew that Cipangu (Japan) was some 1500 miles east of China so he could infer that sailing due west for 3500 miles, some thirty-five days, he would arrive in Japan. All these were inferences from a paper world. Indeed when he arrived in Cuba, a voyage that indeed had taken some 35 days, he took it to be Indo-China and only a few days of sail from India. In fact, 120 degrees of the circle, some one-third of the earth including the Americas and the Pacific, had yet to be admitted into the new world picture.

An equally impressive illustration of the conceptual importance of the paper world was the search for the Southern Continent. By the mid-sixteenth century the Pacific Ocean had become not only a trade route but the subject of a second grand illusion sponsored by the

paper world (Skelton 1958.) The concept of a vast, inhabited continent, reaching from the South Pole to the Tropics and being bounded by the Atlantic, Pacific and Indian Oceans seemed obvious. Something must fill the void on the charts and maintain the equilibrium with the northern continents if the planet was not to spin off its axis! James Cook's 1786 charge of taking Royal Society astronomers to Tahiti included secret instructions to search for the continent which "there is reason to imagine ... may be found to the Southward" (Skelton 1958: 233.) Needless, to say, repeated voyages failed to turn up such a continent. Cook's voyages were the first clear scientific voyages of discovery, not merely mapping the world—transcribing it—but exploring it. They were not simply a matter of putting the world on paper so much as exploring the world *from the map's point of view.* The map is the model or theory of which the voyages are the empirical tests. Progressively, "the known" came to be seen as that represented on paper.

Ong has anticipated the conclusion:

> Only after print and the extensive experience with maps that print implemented would human beings, when they thought about the cosmos or universe or 'world,' think primarily of something laid out before their eyes, as in a modern printed atlas, a vast surface or assemblage of surfaces ... ready to be 'explored.' The ancient oral world knew few 'explorers,' though it did know many itinerants, travelers, voyagers, adventurers, and pilgrims. (Ong 1982: 73)

4. REPRESENTATION OF NATURE: GALILEO'S MATHEMATIZATION OF MOTION

Although geographers had taken the critical step of representing the physical world by means of an abstract geometry—seeing the world in terms of a geometrical sphere with its known mathematical properties—, the more dramatic achievement was to see non-spatial properties of nature, motion in particular, in terms of such geometrical representations. Galileo is justly celebrated for just this achievement (Haugeland 1987). We may recall Galileo's celebrated claim that the book of nature was written in mathematics. True he occasionally rolled balls down inclined planes but he formulated his laws using geometrical methods. His method was to take logical propositions as postulates, the truth of which were to be established "when we find that the inferences from it correspond to and agree perfectly with experiment" (Galileo [1638] 1954: 172). The view is perfectly modern. The theory has a logical form, the implications of the theory are tested by experiment. Further, the model allows one to infer new knowledge; geometrical representation enables integration without calculus.

Consider his Theorem 1, Proposition 1 on the properties of uniform motion:

> If a moving particle, carried uniformly at a constant speed, traverses two distances the time-intervals required are to each other in the ratio of these distances.

The theory is a physical theory but he proceeds to prove it by geometrical methods:

> Let a particle move uniformly with constant speed through two distances AB, BC, and let the time required to traverse AB be represented by DE; the time required to traverse BC, by EF; then I say that the distance AB is to the distance BC as the time DE is to the time EF. (Galileo [1638] 1954: 155)

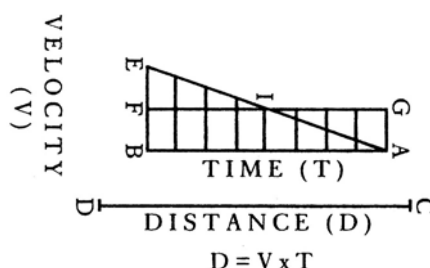


Figure 2. Drawing after Galileo.

Here distance is readily represented by the length of a line; but time is now represented spatially by a similar line. The relations between distance and time are then represented by constant ratios: AB/BC = DE/EF.

Naturally accelerated motion required a somewhat more complex set of geometrical representations (see Figure 2.)

Time is represented by a line in space, distance by an aggregate area and the proof of the relation between uniform and accelerated motion reduces to the proof that the area of a triangle with twice the base and equal height to that of a parallelogram has the same area. Geometry with known laws, rules and properties serves as a model for representing the properties of motion. Nature is seen in terms of this mathematical model, represented on paper.

Note too that geometry is not merely a metaphor for acceleration but a representation, or model, of acceleration. The difference is not simply terminological. The representation specifies in a precise way how each of the features in the representation map on to the properties of the objects in question and how the rules for interpretation of the representation—geometrical deductions—represent possible states of those objects. The representation is to be interpreted as literally true of the represented world; operation on and deductions from the model are taken as true of the represented world. It is worth noting that while Euclid's geometry is used by Galileo, the difference is that Euclid's geometry applied to ideal objects or at best to the spatial properties of objects; Galileo, on the other hand, uses a spatial representation to represent non-spatial properties such as time and velocity and consequently to work out the relations among those properties for any given set of values. Galileo's model allows one to make new inferences and thus to think about velocity in a new way.

5. REPRESENTATIONS AS EMBODIMENTS OF KNOWLEDGE

What is common to the construction of these representations, this world on paper, whether in drawings, maps, descriptions or proofs, was the attempt to create documents which represented the world independently of the particular author who created them, what I have called "autonomous" text. These are texts which are not only clues to the retrieval of known information but which could, in addition, allow one to think about the world in

terms of that representation. Knowledge came to be seen as embodied in text; one could consult the text to determine what was known and to infer new knowledge.

How did texts do this? They claimed to be transparent to the world they represented, in Boyle's words, to inform the reader of what he himself would see if they had been present. This is the new standard of objectivity and it is a standard that continues to be taken as the ideal for modern scientific discourse (Geisler 1994).

This involved a new way of using and understanding language as well as the creation of a new form of discourse. The firm distinction between the descriptive and the interpretive was a product of the new way of reading texts and nature, according to their detectable, surface properties. Texts, including drawings and charts, were designed to neither mean, allude to nor hint at anything; they simply purport to describe and they aspire to be transparent to the reality they describe. They state what is available for anyone to see; they assert, insist, imply nothing. Even links in the discourse are not seen as conclusions or inferences made by a writer. They are seen as simply "following" from what went before. They excise or hide the presence of the writer.

It may be argued that they achieve this by a new management of illocutionary force (Austin 1962.) The writer or artist moves from being an orator with a persuasive message to being a mere reporter who simply tells what we would see *if we were there ourselves*. Even Galileo's demonstrative proofs regarding motion were presented as logical inferences that even the innocent interlocutor, Simplico, could follow. To serve this representational function, language itself had to be reformed so that it could present the object to the reader and by naming, represent it.

This optimistic view of the objectivity of representation is no longer so appealing to this generation of readers. Even the most neutral reports of facts have an illocutionary force as well as a propositional content. Even simple descriptions which purport to point to "the things themselves" are, at base, assertions which express not only a content but also the speaker's "attitude" to that content. The factual description "Grass is green" or "Two plus two is four" are, Austin has argued, abridged versions of "I assert that grass is green" and "I claim that two plus two is four." Statements express both a content and the speaker's attitude to that content, the latter of which indicates to the listener how that content is to be taken (Austin 1962). Consequently, the presumably simple verbal art of describing is one in which that authorial stance is not eliminated but rather hidden.

Pictures are to be analyzed in a similar way. The achievement of the Dutch masters was, as Alpers pointed out, to represent the visible world free from the beliefs and assumptions traditionally read into such representations; they could represent without "meaning" or symbolizing anything. Yet, as is now widely acknowledged, pictures do not, contra the Dutch masters, simply depict things as they are. Even if the intentions of the artist seem to be neutral, they represent a stance of the describer (Schama 1987) within a "vocabulary of forms" (Gombrich 1950). Visual descriptions, then, are disguised assertions. But they are assertions with a new warrant as to their truthfulness. The warrant for the truthfulness of the depiction is the assumption that if the viewer were present at the time the picture was made, the viewer would have seen just what the artist depicted. Just as Boyle turned his readers into "virtual witnesses" of his experiments (Shapin 1984), so to did the Dutch artists of the seventeenth century.

Twentieth century readers are more likely to acknowledge that all observation is, as we say, theory driven, and we need to revise our interpretation of the seventeenth century achievement accordingly. We realize that a "sincere eye and a faithful hand" are not enough to see all that it there; it requires, in addition, a prepared mind. It was the great task of our present generation to discover, report, and so far as possible bring into consciousness the attitudes implicit in these simple objective reports.

But the seventeenth century representations we have surveyed, whether in Dutch art, in cartography, indeed, the very attempt at verbal representation, yielded a new understanding of the world. The paper world, therefore, did not simply provide a means for accumulating and storing what was learned. Rather it was a matter of inventing the conceptual means for coordinating the bits of geographical, biological, mechanical, and other forms of knowledge acquired from many sources into an adequate and common frame of reference. This common frame of reference became the theoretical model into which local knowledge was inserted and reorganized. This is the sense, I believe, in which western science of that period acquired the distinctive property of being theoretical science.

But their attempts at objectivity had a second set of implications. As the distinction between what was "in the world" to be seen and what was sometimes "seen" as being in the world became sharpened, it not only produced more objective accounts of the world but provided a ground for ascribing other aspects of knowledge to the mind. It provided a new understanding of subjectivity that was to become the concern of Descartes and the British empiricist philosophers (Olson 1994, Chapter 11).

6. KNOWLEDGE AND ITS ARTIFACTS

But now a further, unanticipated problem arises: What precisely is the relation between human knowledge and the artifacts created to represent that knowledge? Is it possible, as Popper has argued, to create knowledge objects, objects that embody knowledge in a way separable from the knower (Popper 1970). In one sense this is not problematic for it is clearly the case that knowledge is preserved across generations, lost and later recovered and so on. But in a deeper sense, that defended by Popper, knowledge is inherent in the objects created whether in a text, a model or a computer program. This is the sense that seems to me to be wrong.

Because artifacts, primarily texts, have come to occupy such a central place in both the form and the growth of knowledge it is tempting to see these representations as "embodiments" of knowledge. It is indeed tempting to regard texts as themselves as making up the accumulated knowledge of the culture only parts of which are mastered by any individual reader. Furthermore, I am particularly sensitive on this point because in some of my own writing on meaning I attempted to show that the growth of this particular textual tradition involved the attempt to create texts that "mean neither more nor less than what they say," such that they can stand in the place of the original author's intention (Olson 1977). In such a text, so the argument goes, it is of no help to ask the author what he or she intended; the text stands as an autonomous representation of meaning. The attempt to put the meaning into the text, that is to make the text an autonomous representation of meaning, continues to be, I believe, a laudatory, if unachievable goal. However, if taken

as an achievement rather than as a goal, the idea that texts embody knowledge can be seriously misleading. Textbooks are often treated as if they were the cultural embodiment of "the known" and a student's obligation then is taken as the assimilation of the known. Textbooks are written in such a way as to remove the marks of the author's personal intentions and are treated as simple compilations of the given.

This, I now believe, is one of the fateful illusions of modernism, the idea that knowledge can be embodied in a text or a computer program or other artifact. Texts are more accurately seen as artifacts, notational devices for representation and thought. Knowledge remains the possession of the knower not of the artifact.

The assumption that texts may embody the known derives from the Scriptural traditions of the great "religions of the book," those religions in which the authority is seen as residing in the text as much as in its putative author. Further, the assumption serves as the goal of writers who attempt to contribute autonomous text to the archival store (Geisler 1994). Yet, as mentioned, it is misleading to think that the goal is in fact achievable, that the text can embody meaning rather than represent it. How, then, do artifacts represent?

One way to proceed is to acknowledge that knowledge is psychological; knowledge is the set of justified beliefs possessed by human minds, not the properties of artifacts such as texts. An author attempts to represent those beliefs in a common code or convention in such a way that a reader will reconstruct a belief which is sufficiently similar to that of the writer so that similar inferences can be drawn and that fruitful agreement and disagreement is possible.

Speech is a suitable medium for the representation of most beliefs. But writing becomes important for "spelling out" beliefs and theories of some complexity in such a way that they can be criticized and analyzed in detail and, if appropriate, revised. Authors revise texts in anticipation of such criticism. But more importantly, the text becomes an object for other's readings, criticisms, interpretations and extrapolations and thus provides the occasion for the creation of new texts that attempt to circumvent the problems of the older ones. In this way a scientific textual tradition is created with the genuine prospects of advancing knowledge. The artifacts, the texts, improve in the process in that they represent the beliefs of the writers and readers more precisely. But the growth of knowledge is not in those texts themselves but in the new understandings they allow in readers and writers.

This is not to say that suitably created texts cannot make routine some of the functions of thought. An adequate notational system in mathematics makes many problems solvable by algorithmic, essentially mindless, procedures. The notation, like the maps of discovery discussed above, provides a new tool to think with, a tool for producing and sharing new knowledge rather than embodying that knowledge.

A culture's stock of knowledge is represented in its archival resources. But it is a mistake, I believe, to hold that those archives embody the knowledge of the culture. The knowledge is psychological, embodied in the people who create and use them. The growth of the archival resource is the creation of representations which allow new readers to both grasp what others have believed or thought and to formulate their own thoughts in relation to them. Scientific texts require something more; unless those constructed by the reader bear rather precisely on those intended by the author, the growth of knowledge

would be impossible. And that is why the construction of those texts is so important and why the texts themselves are so venerated.

To summarize. First, I have argued that writing about nature depends more upon the habits of writing and reading of the author, what one may call the textual tradition, than on the facts of nature themselves. Second, changes in these modes of reading and writing once worked out in one domain, namely Scripture, transferred rather directly into quite a different domain, Nature. As a result, it is quite defensible to talk of distinctive Medieval, Early Modern, and contemporary ways of reading and writing texts. Conceptions of both Scripture and Nature have changed accordingly.

Ontario Institute for Studies in Education, University of Toronto

REFERENCES

Alpers, S. 1983. *The art of describing: Dutch art in the seventeenth century*. Chicago: University of Chicago Press.
Austin, J. L. 1962. *How to do things with words*. Cambridge, MA: Harvard University Press.
Boas, M. 1962. *The scientific Renaissance* 1450–1630. New York: Harper.
Bacon, F. [1620] 1965. *The great instauration*. In *Francis Bacon: A selection of his works*, edited by S. Warhaft, 298–324. Toronto: Macmillan. (Original work published 1620).
Carruthers, M. J. 1990. *The book of memory: A study of memory in medieval culture*. Cambridge: Cambridge University Press.
Clanchy, M. T. 1993. *From memory to written record: England, 1066–1307*, 2nd ed. Oxford: Blackwell.
Eisenstein, E. 1979. *The printing press as an agent of change*. Cambridge: Cambridge University Press.
Foucault, M. 1970. *The order of things*. London: Tavistock.
Galileo, G. [1638] [1914] 1954. *Dialogues concerning two new sciences*. Translated by H. Grew & A. de Salvio (1914 ed.). New York: Dover. (Original work published 1638).
Geisler, C. 1994. *Academic literacy and the nature of expertise: Reading, writing and knowing in academic philosophy*. Hillsdale, NJ: Erlbaum.
Gombrich, E. H. 1950. *The story of art*. Oxford: Phaidon.
Gombrich, E. H. 1960. *Art and illusion: A study in the psychology of pictorial representation*. New York: Bollingen/Pantheon Books.
Green, D. H. 1994. *Medieval listening and reading: The primary reception of German literature 800–1300*. Cambridge: Cambridge University Press.
Harris, R. 1989. "How does writing restructure thought?" *Language and Communication* 9: 99–106.
Haugeland, J. 1987. *Artificial intelligence: The very idea*. Cambridge: Cambridge University Press.
Morrison, K. F. 1982. *The mimetic tradition of reform in the West*. Princeton, NJ: Princeton University Press.
Morrison, K. F. 1990. *History as a visual art in the twelfth-century Renaissance*. Princeton, NJ: Princeton University Press.
Olson, D. R. 1977. "From utterance to text: The bias of language in speech and writing." *Harvard Educational Review* 47 (3): 257–281.
Olson, D. R. 1994. *The world on paper: The conceptual and cognitive implications of writing and reading*. Cambridge: Cambridge University Press.
Ong, W. 1982. *Orality and literacy: The technologizing of the word*. London: Methuen.
Popper, K. 1972. *Objective knowledge: An evolutionary approach*. Oxford: Clarendon.
Ptolemy. [2nd Century] 1966. *Geographia.* Edited by S. Munster. Basel, 1540, with an introduction by R. A. Skelton. Amsterdam: Theatrum Orbis Terrarum.
Reiss, T. J. 1982. *The discourse of modernism*. Ithaca, NY: Cornell University Press.
Saenger, P. 1982. "Silent reading: Its impact on late medieval script and society." *Viator* 13: 367–414.
Saenger, P. 1997. *Space between words: The origins of silent reading.* Stanford, CA: Stanford University Press.

Schama, S. 1987. *The embarrassment of riches: An interpretation of Dutch culture in the golden age*. New York: Alfred A. Knopf.

Shapin, S. 1984. "Pump and circumstance: Robert Boyle's literary technology." *Social Studies of Science* 14: 481–520.

Skelton, R. A. 1958. *Explorer's maps: Chapters in the cartographic record of geographical discovery*. London: Routledge & Kegan Paul.

Smalley, B. 1941. *The study of the Bible in the Middle Ages*. Oxford: Clarendon.

Sprat, T. [1667] 1966. *History of the Royal Society of London for the improving of natural knowledge*. Edited by J. I. Cope & H. W. Jones. St. Louis: Washington University Press. (Original work published 1667).

Stearns, R. P. 1947. *Pageant of Europe; Sources and selections from the Renaissance to the present day*. New York: Harcourt, Brace.

SUBJECT INDEX

NAME INDEX

Boston Studies in the Philosophy of Science

Editor: Robert S. Cohen, *Boston University*

1. M.W. Wartofsky (ed.): *Proceedings of the Boston Colloquium for the Philosophy of Science, 1961/1962.* [Synthese Library 6] 1963 ISBN 90-277-0021-4
2. R.S. Cohen and M.W. Wartofsky (eds.): *Proceedings of the Boston Colloquium for the Philosophy of Science, 1962/1964.* In Honor of P. Frank. [Synthese Library 10] 1965 ISBN 90-277-9004-0
3. R.S. Cohen and M.W. Wartofsky (eds.): *Proceedings of the Boston Colloquium for the Philosophy of Science, 1964/1966.* In Memory of Norwood Russell Hanson. [Synthese Library 14] 1967 ISBN 90-277-0013-3
4. R.S. Cohen and M.W. Wartofsky (eds.): *Proceedings of the Boston Colloquium for the Philosophy of Science, 1966/1968.* [Synthese Library 18] 1969 ISBN 90-277-0014-1
5. R.S. Cohen and M.W. Wartofsky (eds.): *Proceedings of the Boston Colloquium for the Philosophy of Science, 1966/1968.* [Synthese Library 19] 1969 ISBN 90-277-0015-X
6. R.S. Cohen and R.J. Seeger (eds.): *Ernst Mach, Physicist and Philosopher.* [Synthese Library 27] 1970 ISBN 90-277-0016-8
7. M. Čapek: *Bergson and Modern Physics.* A Reinterpretation and Re-evaluation. [Synthese Library 37] 1971 ISBN 90-277-0186-5
8. R.C. Buck and R.S. Cohen (eds.): *PSA 1970.* Proceedings of the 2nd Biennial Meeting of the Philosophy and Science Association (Boston, Fall 1970). In Memory of Rudolf Carnap. [Synthese Library 39] 1971 ISBN 90-277-0187-3; Pb 90-277-0309-4
9. A.A. Zinov'ev: *Foundations of the Logical Theory of Scientific Knowledge (Complex Logic).* Translated from Russian. Revised and enlarged English Edition, with an Appendix by G.A. Smirnov, E.A. Sidorenko, A.M. Fedina and L.A. Bobrova. [Synthese Library 46] 1973 ISBN 90-277-0193-8; Pb 90-277-0324-8
10. L. Tondl: *Scientific Procedures.* A Contribution Concerning the Methodological Problems of Scientific Concepts and Scientific Explanation.Translated from Czech. [Synthese Library 47] 1973 ISBN 90-277-0147-4; Pb 90-277-0323-X
11. R.J. Seeger and R.S. Cohen (eds.): *Philosophical Foundations of Science.* Proceedings of Section L, 1969, American Association for the Advancement of Science. [Synthese Library 58] 1974 ISBN 90-277-0390-6; Pb 90-277-0376-0
12. A. Grünbaum: *Philosophical Problems of Space and Times.* 2nd enlarged ed. [Synthese Library 55] 1973 ISBN 90-277-0357-4; Pb 90-277-0358-2
13. R.S. Cohen and M.W. Wartofsky (eds.): *Logical and Epistemological Studies in Contemporary Physics.* Proceedings of the Boston Colloquium for the Philosophy of Science, 1969/72, Part I. [Synthese Library 59] 1974 ISBN 90-277-0391-4; Pb 90-277-0377-9
14. R.S. Cohen and M.W. Wartofsky (eds.): *Methodological and Historical Essays in the Natural and Social Sciences.* Proceedings of the Boston Colloquium for the Philosophy of Science, 1969/72, Part II. [Synthese Library 60] 1974 ISBN 90-277-0392-2; Pb 90-277-0378-7
15. R.S. Cohen, J.J. Stachel and M.W. Wartofsky (eds.): *For Dirk Struik.* Scientific, Historical and Political Essays in Honor of Dirk J. Struik. [Synthese Library 61] 1974 ISBN 90-277-0393-0; Pb 90-277-0379-5

16. N. Geschwind: *Selected Papers on Language and the Brains.* [Synthese Library 68] 1974 ISBN 90-277-0262-4; Pb 90-277-0263-2
17. B.G. Kuznetsov: *Reason and Being.* Translated from Russian. Edited by C.R. Fawcett and R.S. Cohen. 1987 ISBN 90-277-2181-5
18. P. Mittelstaedt: *Philosophical Problems of Modern Physics.* Translated from the revised 4th German edition by W. Riemer and edited by R.S. Cohen. [Synthese Library 95] 1976 ISBN 90-277-0285-3; Pb 90-277-0506-2
19. H. Mehlberg: *Time, Causality, and the Quantum Theory.* Studies in the Philosophy of Science. Vol. I: *Essay on the Causal Theory of Time.* Vol. II: *Time in a Quantized Universe.* Translated from French. Edited by R.S. Cohen. 1980
    Vol. I: ISBN 90-277-0721-9; Pb 90-277-1074-0
    Vol. II: ISBN 90-277-1075-9; Pb 90-277-1076-7
20. K.F. Schaffner and R.S. Cohen (eds.): *PSA 1972.* Proceedings of the 3rd Biennial Meeting of the Philosophy of Science Association (Lansing, Michigan, Fall 1972). [Synthese Library 64] 1974 ISBN 90-277-0408-2; Pb 90-277-0409-0
21. R.S. Cohen and J.J. Stachel (eds.): *Selected Papers of Léon Rosenfeld.* [Synthese Library 100] 1979 ISBN 90-277-0651-4; Pb 90-277-0652-2
22. M. Čapek (ed.): *The Concepts of Space and Time.* Their Structure and Their Development. [Synthese Library 74] 1976 ISBN 90-277-0355-8; Pb 90-277-0375-2
23. M. Grene: *The Understanding of Nature.* Essays in the Philosophy of Biology. [Synthese Library 66] 1974 ISBN 90-277-0462-7; Pb 90-277-0463-5
24. D. Ihde: *Technics and Praxis.* A Philosophy of Technology. [Synthese Library 130] 1979 ISBN 90-277-0953-X; Pb 90-277-0954-8
25. J. Hintikka and U. Remes: *The Method of Analysis.* Its Geometrical Origin and Its General Significance. [Synthese Library 75] 1974 ISBN 90-277-0532-1; Pb 90-277-0543-7
26. J.E. Murdoch and E.D. Sylla (eds.): *The Cultural Context of Medieval Learning.* Proceedings of the First International Colloquium on Philosophy, Science, and Theology in the Middle Ages, 1973. [Synthese Library 76] 1975 ISBN 90-277-0560-7; Pb 90-277-0587-9
27. M. Grene and E. Mendelsohn (eds.): *Topics in the Philosophy of Biology.* [Synthese Library 84] 1976 ISBN 90-277-0595-X; Pb 90-277-0596-8
28. J. Agassi: *Science in Flux.* [Synthese Library 80] 1975 ISBN 90-277-0584-4; Pb 90-277-0612-3
29. J.J. Wiatr (ed.): *Polish Essays in the Methodology of the Social Sciences.* [Synthese Library 131] 1979 ISBN 90-277-0723-5; Pb 90-277-0956-4
30. P. Janich: *Protophysics of Time.* Constructive Foundation and History of Time Measurement. Translated from German. 1985 ISBN 90-277-0724-3
31. R.S. Cohen and M.W. Wartofsky (eds.): *Language, Logic, and Method.* 1983 ISBN 90-277-0725-1
32. R.S. Cohen, C.A. Hooker, A.C. Michalos and J.W. van Evra (eds.): *PSA 1974.* Proceedings of the 4th Biennial Meeting of the Philosophy of Science Association. [Synthese Library 101] 1976 ISBN 90-277-0647-6; Pb 90-277-0648-4
33. G. Holton and W.A. Blanpied (eds.): *Science and Its Public.* The Changing Relationship. [Synthese Library 96] 1976 ISBN 90-277-0657-3; Pb 90-277-0658-1
34. M.D. Grmek, R.S. Cohen and G. Cimino (eds.): *On Scientific Discovery.* The 1977 Erice Lectures. 1981 ISBN 90-277-1122-4; Pb 90-277-1123-2

35. S. Amsterdamski: *Between Experience and Metaphysics.* Philosophical Problems of the Evolution of Science. Translated from Polish. [Synthese Library 77] 1975
ISBN 90-277-0568-2; Pb 90-277-0580-1
36. M. Marković and G. Petrović (eds.): *Praxis.* Yugoslav Essays in the Philosophy and Methodology of the Social Sciences. [Synthese Library 134] 1979
ISBN 90-277-0727-8; Pb 90-277-0968-8
37. H. von Helmholtz: *Epistemological Writings.* The Paul Hertz / Moritz Schlick Centenary Edition of 1921. Translated from German by M.F. Lowe. Edited with an Introduction and Bibliography by R.S. Cohen and Y. Elkana. [Synthese Library 79] 1977
ISBN 90-277-0290-X; Pb 90-277-0582-8
38. R.M. Martin: *Pragmatics, Truth and Language.* 1979
ISBN 90-277-0992-0; Pb 90-277-0993-9
39. R.S. Cohen, P.K. Feyerabend and M.W. Wartofsky (eds.): *Essays in Memory of Imre Lakatos.* [Synthese Library 99] 1976 ISBN 90-277-0654-9; Pb 90-277-0655-7
40. Not published.
41. Not published.
42. H.R. Maturana and F.J. Varela: *Autopoiesis and Cognition.* The Realization of the Living. With a Preface to "Autopoiesis' by S. Beer. 1980
ISBN 90-277-1015-5; Pb 90-277-1016-3
43. A. Kasher (ed.): *Language in Focus: Foundations, Methods and Systems.* Essays in Memory of Yehoshua Bar-Hillel. [Synthese Library 89] 1976
ISBN 90-277-0644-1; Pb 90-277-0645-X
44. T.D. Thao: *Investigations into the Origin of Language and Consciousness.* 1984
ISBN 90-277-0827-4
45. F.G.-I. Nagasaka (ed.): *Japanese Studies in the Philosophy of Science.* 1997
ISBN 0-7923-4781-1
46. P.L. Kapitza: *Experiment, Theory, Practice.* Articles and Addresses. Edited by R.S. Cohen. 1980 ISBN 90-277-1061-9; Pb 90-277-1062-7
47. M.L. Dalla Chiara (ed.): *Italian Studies in the Philosophy of Science.* 1981
ISBN 90-277-0735-9; Pb 90-277-1073-2
48. M.W. Wartofsky: *Models.* Representation and the Scientific Understanding. [Synthese Library 129] 1979 ISBN 90-277-0736-7; Pb 90-277-0947-5
49. T.D. Thao: *Phenomenology and Dialectical Materialism.* Edited by R.S. Cohen. 1986
ISBN 90-277-0737-5
50. Y. Fried and J. Agassi: *Paranoia.* A Study in Diagnosis. [Synthese Library 102] 1976
ISBN 90-277-0704-9; Pb 90-277-0705-7
51. K.H. Wolff: *Surrender and Cath.* Experience and Inquiry Today. [Synthese Library 105] 1976 ISBN 90-277-0758-8; Pb 90-277-0765-0
52. K. Kosík: *Dialectics of the Concrete.* A Study on Problems of Man and World. 1976
ISBN 90-277-0761-8; Pb 90-277-0764-2
53. N. Goodman: *The Structure of Appearance.* [Synthese Library 107] 1977
ISBN 90-277-0773-1; Pb 90-277-0774-X
54. H.A. Simon: *Models of Discovery* and Other Topics in the Methods of Science. [Synthese Library 114] 1977 ISBN 90-277-0812-6; Pb 90-277-0858-4
55. M. Lazerowitz: *The Language of Philosophy.* Freud and Wittgenstein. [Synthese Library 117] 1977 ISBN 90-277-0826-6; Pb 90-277-0862-2

56. T. Nickles (ed.): *Scientific Discovery, Logic, and Rationality.* 1980
ISBN 90-277-1069-4; Pb 90-277-1070-8
57. J. Margolis: *Persons and Mind.* The Prospects of Nonreductive Materialism. [Synthese Library 121] 1978 ISBN 90-277-0854-1; Pb 90-277-0863-0
58. G. Radnitzky and G. Andersson (eds.): *Progress and Rationality in Science.* [Synthese Library 125] 1978 ISBN 90-277-0921-1; Pb 90-277-0922-X
59. G. Radnitzky and G. Andersson (eds.): *The Structure and Development of Science.* [Synthese Library 136] 1979 ISBN 90-277-0994-7; Pb 90-277-0995-5
60. T. Nickles (ed.): *Scientific Discovery.* Case Studies. 1980
ISBN 90-277-1092-9; Pb 90-277-1093-7
61. M.A. Finocchiaro: *Galileo and the Art of Reasoning.* Rhetorical Foundation of Logic and Scientific Method. 1980 ISBN 90-277-1094-5; Pb 90-277-1095-3
62. W.A. Wallace: *Prelude to Galileo.* Essays on Medieval and 16th-Century Sources of Galileo's Thought. 1981 ISBN 90-277-1215-8; Pb 90-277-1216-6
63. F. Rapp: *Analytical Philosophy of Technology.* Translated from German. 1981
ISBN 90-277-1221-2; Pb 90-277-1222-0
64. R.S. Cohen and M.W. Wartofsky (eds.): *Hegel and the Sciences.* 1984
ISBN 90-277-0726-X
65. J. Agassi: *Science and Society.* Studies in the Sociology of Science. 1981
ISBN 90-277-1244-1; Pb 90-277-1245-X
66. L. Tondl: *Problems of Semantics.* A Contribution to the Analysis of the Language of Science. Translated from Czech. 1981 ISBN 90-277-0148-2; Pb 90-277-0316-7
67. J. Agassi and R.S. Cohen (eds.): *Scientific Philosophy Today.* Essays in Honor of Mario Bunge. 1982 ISBN 90-277-1262-X; Pb 90-277-1263-8
68. W. Krajewski (ed.): *Polish Essays in the Philosophy of the Natural Sciences.* Translated from Polish and edited by R.S. Cohen and C.R. Fawcett. 1982
ISBN 90-277-1286-7; Pb 90-277-1287-5
69. J.H. Fetzer: *Scientific Knowledge.* Causation, Explanation and Corroboration. 1981
ISBN 90-277-1335-9; Pb 90-277-1336-7
70. S. Grossberg: *Studies of Mind and Brain.* Neural Principles of Learning, Perception, Development, Cognition, and Motor Control. 1982
ISBN 90-277-1359-6; Pb 90-277-1360-X
71. R.S. Cohen and M.W. Wartofsky (eds.): *Epistemology, Methodology, and the Social Sciences.* 1983. ISBN 90-277-1454-1
72. K. Berka: *Measurement.* Its Concepts, Theories and Problems. Translated from Czech. 1983 ISBN 90-277-1416-9
73. G.L. Pandit: *The Structure and Growth of Scientific Knowledge.* A Study in the Methodology of Epistemic Appraisal. 1983 ISBN 90-277-1434-7
74. A.A. Zinov'ev: *Logical Physics.* Translated from Russian. Edited by R.S. Cohen. 1983 [*see also* Volume 9] ISBN 90-277-0734-0
75. G-G. Granger: *Formal Thought and the Sciences of Man.* Translated from French. With and Introduction by A. Rosenberg. 1983 ISBN 90-277-1524-6
76. R.S. Cohen and L. Laudan (eds.): *Physics, Philosophy and Psychoanalysis.* Essays in Honor of Adolf Grünbaum. 1983 ISBN 90-277-1533-5
77. G. Böhme, W. van den Daele, R. Hohlfeld, W. Krohn and W. Schäfer: *Finalization in Science.* The Social Orientation of Scientific Progress. Translated from German. Edited by W. Schäfer. 1983 ISBN 90-277-1549-1

78. D. Shapere: *Reason and the Search for Knowledge.* Investigations in the Philosophy of Science. 1984 ISBN 90-277-1551-3; Pb 90-277-1641-2
79. G. Andersson (ed.): *Rationality in Science and Politics.* Translated from German. 1984 ISBN 90-277-1575-0; Pb 90-277-1953-5
80. P.T. Durbin and F. Rapp (eds.): *Philosophy and Technology.* [*Also* Philosophy and Technology Series, Vol. 1] 1983 ISBN 90-277-1576-9
81. M. Marković: *Dialectical Theory of Meaning.* Translated from Serbo-Croat. 1984 ISBN 90-277-1596-3
82. R.S. Cohen and M.W. Wartofsky (eds.): *Physical Sciences and History of Physics.* 1984. ISBN 90-277-1615-3
83. É. Meyerson: *The Relativistic Deduction.* Epistemological Implications of the Theory of Relativity. Translated from French. With a Review by Albert Einstein and an Introduction by Milić Čapek. 1985 ISBN 90-277-1699-4
84. R.S. Cohen and M.W. Wartofsky (eds.): *Methodology, Metaphysics and the History of Science.* In Memory of Benjamin Nelson. 1984 ISBN 90-277-1711-7
85. G. Tamás: *The Logic of Categories.* Translated from Hungarian. Edited by R.S. Cohen. 1986 ISBN 90-277-1742-7
86. S.L. de C. Fernandes: *Foundations of Objective Knowledge.* The Relations of Popper's Theory of Knowledge to That of Kant. 1985 ISBN 90-277-1809-1
87. R.S. Cohen and T. Schnelle (eds.): *Cognition and Fact.* Materials on Ludwik Fleck. 1986 ISBN 90-277-1902-0
88. G. Freudenthal: *Atom and Individual in the Age of Newton.* On the Genesis of the Mechanistic World View. Translated from German. 1986 ISBN 90-277-1905-5
89. A. Donagan, A.N. Perovich Jr and M.V. Wedin (eds.): *Human Nature and Natural Knowledge.* Essays presented to Marjorie Grene on the Occasion of Her 75th Birthday. 1986 ISBN 90-277-1974-8
90. C. Mitcham and A. Hunning (eds.): *Philosophy and Technology II.* Information Technology and Computers in Theory and Practice. [*Also* Philosophy and Technology Series, Vol. 2] 1986 ISBN 90-277-1975-6
91. M. Grene and D. Nails (eds.): *Spinoza and the Sciences.* 1986 ISBN 90-277-1976-4
92. S.P. Turner: *The Search for a Methodology of Social Science.* Durkheim, Weber, and the 19th-Century Problem of Cause, Probability, and Action. 1986 ISBN 90-277-2067-3
93. I.C. Jarvie: *Thinking about Society.* Theory and Practice. 1986 ISBN 90-277-2068-1
94. E. Ullmann-Margalit (ed.): *The Kaleidoscope of Science.* The Israel Colloquium: Studies in History, Philosophy, and Sociology of Science, Vol. 1. 1986 ISBN 90-277-2158-0; Pb 90-277-2159-9
95. E. Ullmann-Margalit (ed.): *The Prism of Science.* The Israel Colloquium: Studies in History, Philosophy, and Sociology of Science, Vol. 2. 1986 ISBN 90-277-2160-2; Pb 90-277-2161-0
96. G. Márkus: *Language and Production.* A Critique of the Paradigms. Translated from French. 1986 ISBN 90-277-2169-6
97. F. Amrine, F.J. Zucker and H. Wheeler (eds.): *Goethe and the Sciences: A Reappraisal.* 1987 ISBN 90-277-2265-X; Pb 90-277-2400-8
98. J.C. Pitt and M. Pera (eds.): *Rational Changes in Science.* Essays on Scientific Reasoning. Translated from Italian. 1987 ISBN 90-277-2417-2
99. O. Costa de Beauregard: *Time, the Physical Magnitude.* 1987 ISBN 90-277-2444-X

100. A. Shimony and D. Nails (eds.): *Naturalistic Epistemology.* A Symposium of Two Decades. 1987 ISBN 90-277-2337-0
101. N. Rotenstreich: *Time and Meaning in History.* 1987 ISBN 90-277-2467-9
102. D.B. Zilberman: *The Birth of Meaning in Hindu Thought.* Edited by R.S. Cohen. 1988 ISBN 90-277-2497-0
103. T.F. Glick (ed.): *The Comparative Reception of Relativity.* 1987 ISBN 90-277-2498-9
104. Z. Harris, M. Gottfried, T. Ryckman, P. Mattick Jr, A. Daladier, T.N. Harris and S. Harris: *The Form of Information in Science.* Analysis of an Immunology Sublanguage. With a Preface by Hilary Putnam. 1989 ISBN 90-277-2516-0
105. F. Burwick (ed.): *Approaches to Organic Form.* Permutations in Science and Culture. 1987 ISBN 90-277-2541-1
106. M. Almási: *The Philosophy of Appearances.* Translated from Hungarian. 1989 ISBN 90-277-2150-5
107. S. Hook, W.L. O'Neill and R. O'Toole (eds.): *Philosophy, History and Social Action.* Essays in Honor of Lewis Feuer. With an Autobiographical Essay by L. Feuer. 1988 ISBN 90-277-2644-2
108. I. Hronszky, M. Fehér and B. Dajka: *Scientific Knowledge Socialized.* Selected Proceedings of the 5th Joint International Conference on the History and Philosophy of Science organized by the IUHPS (Veszprém, Hungary, 1984). 1988 ISBN 90-277-2284-6
109. P. Tillers and E.D. Green (eds.): *Probability and Inference in the Law of Evidence.* The Uses and Limits of Bayesianism. 1988 ISBN 90-277-2689-2
110. E. Ullmann-Margalit (ed.): *Science in Reflection.* The Israel Colloquium: Studies in History, Philosophy, and Sociology of Science, Vol. 3. 1988 ISBN 90-277-2712-0; Pb 90-277-2713-9
111. K. Gavroglu, Y. Goudaroulis and P. Nicolacopoulos (eds.): *Imre Lakatos and Theories of Scientific Change.* 1989 ISBN 90-277-2766-X
112. B. Glassner and J.D. Moreno (eds.): *The Qualitative-Quantitative Distinction in the Social Sciences.* 1989 ISBN 90-277-2829-1
113. K. Arens: *Structures of Knowing.* Psychologies of the 19th Century. 1989 ISBN 0-7923-0009-2
114. A. Janik: *Style, Politics and the Future of Philosophy.* 1989 ISBN 0-7923-0056-4
115. F. Amrine (ed.): *Literature and Science as Modes of Expression.* With an Introduction by S. Weininger. 1989 ISBN 0-7923-0133-1
116. J.R. Brown and J. Mittelstrass (eds.): *An Intimate Relation.* Studies in the History and Philosophy of Science. Presented to Robert E. Butts on His 60th Birthday. 1989 ISBN 0-7923-0169-2
117. F. D'Agostino and I.C. Jarvie (eds.): *Freedom and Rationality.* Essays in Honor of John Watkins. 1989 ISBN 0-7923-0264-8
118. D. Zolo: *Reflexive Epistemology.* The Philosophical Legacy of Otto Neurath. 1989 ISBN 0-7923-0320-2
119. M. Kearn, B.S. Philips and R.S. Cohen (eds.): *Georg Simmel and Contemporary Sociology.* 1989 ISBN 0-7923-0407-1
120. T.H. Levere andW.R. Shea (eds.): *Nature, Experiment and the Science.* Essays on Galileo and the Nature of Science. In Honour of Stillman Drake. 1989 ISBN 0-7923-0420-9
121. P. Nicolacopoulos (ed.): *Greek Studies in the Philosophy and History of Science.* 1990 ISBN 0-7923-0717-8

122. R. Cooke and D. Costantini (eds.): *Statistics in Science.* The Foundations of Statistical Methods in Biology, Physics and Economics. 1990 ISBN 0-7923-0797-6
123. P. Duhem: *The Origins of Statics.* Translated from French by G.F. Leneaux, V.N. Vagliente and G.H. Wagner. With an Introduction by S.L. Jaki. 1991 ISBN 0-7923-0898-0
124. H. Kamerlingh Onnes: *Through Measurement to Knowledge.* The Selected Papers, 1853-1926. Edited and with an Introduction by K. Gavroglu and Y. Goudaroulis. 1991 ISBN 0-7923-0825-5
125. M. Čapek: *The New Aspects of Time: Its Continuity and Novelties.* Selected Papers in the Philosophy of Science. 1991 ISBN 0-7923-0911-1
126. S. Unguru (ed.): *Physics, Cosmology and Astronomy, 1300-1700.* Tension and Accommodation. 1991 ISBN 0-7923-1022-5
127. Z. Bechler: *Newton's Physics on the Conceptual Structure of the Scientific Revolution.* 1991 ISBN 0-7923-1054-3
128. É. Meyerson: *Explanation in the Sciences.* Translated from French by M-A. Siple and D.A. Siple. 1991 ISBN 0-7923-1129-9
129. A.I. Tauber (ed.): *Organism and the Origins of Self.* 1991 ISBN 0-7923-1185-X
130. F.J. Varela and J-P. Dupuy (eds.): *Understanding Origins.* Contemporary Views on the Origin of Life, Mind and Society. 1992 ISBN 0-7923-1251-1
131. G.L. Pandit: *Methodological Variance.* Essays in Epistemological Ontology and the Methodology of Science. 1991 ISBN 0-7923-1263-5
132. G. Munévar (ed.): *Beyond Reason.* Essays on the Philosophy of Paul Feyerabend. 1991 ISBN 0-7923-1272-4
133. T.E. Uebel (ed.): *Rediscovering the Forgotten Vienna Circle.* Austrian Studies on Otto Neurath and the Vienna Circle. Partly translated from German. 1991 ISBN 0-7923-1276-7
134. W.R. Woodward and R.S. Cohen (eds.): *World Views and Scientific Discipline Formation.* Science Studies in the [former] German Democratic Republic. Partly translated from German by W.R. Woodward. 1991 ISBN 0-7923-1286-4
135. P. Zambelli: *The Speculum Astronomiae and Its Enigma.* Astrology, Theology and Science in Albertus Magnus and His Contemporaries. 1992 ISBN 0-7923-1380-1
136. P. Petitjean, C. Jami and A.M. Moulin (eds.): *Science and Empires.* Historical Studies about Scientific Development and European Expansion. ISBN 0-7923-1518-9
137. W.A. Wallace: *Galileo's Logic of Discovery and Proof.* The Background, Content, and Use of His Appropriated Treatises on Aristotle's *Posterior Analytics.* 1992 ISBN 0-7923-1577-4
138. W.A.Wallace: *Galileo's Logical Treatises.* A Translation, with Notes and Commentary, of His Appropriated Latin Questions on Aristotle's *Posterior Analytics.* 1992 ISBN 0-7923-1578-2
     Set (137 + 138) ISBN 0-7923-1579-0
139. M.J. Nye, J.L. Richards and R.H. Stuewer (eds.): *The Invention of Physical Science.* Intersections of Mathematics, Theology and Natural Philosophy since the Seventeenth Century. Essays in Honor of Erwin N. Hiebert. 1992 ISBN 0-7923-1753-X
140. G. Corsi, M.L. dalla Chiara and G.C. Ghirardi (eds.): *Bridging the Gap: Philosophy, Mathematics and Physics.* Lectures on the Foundations of Science. 1992 ISBN 0-7923-1761-0
141. C.-H. Lin and D. Fu (eds.): *Philosophy and Conceptual History of Science in Taiwan.* 1992 ISBN 0-7923-1766-1

142. S. Sarkar (ed.): *The Founders of Evolutionary Genetics.* A Centenary Reappraisal. 1992 ISBN 0-7923-1777-7
143. J. Blackmore (ed.): *Ernst Mach – A Deeper Look.* Documents and New Perspectives. 1992 ISBN 0-7923-1853-6
144. P. Kroes and M. Bakker (eds.): *Technological Development and Science in the Industrial Age.* New Perspectives on the Science-Technology Relationship. 1992 ISBN 0-7923-1898-6
145. S. Amsterdamski: *Between History and Method.* Disputes about the Rationality of Science. 1992 ISBN 0-7923-1941-9
146. E. Ullmann-Margalit (ed.): *The Scientific Enterprise.* The Bar-Hillel Colloquium: Studies in History, Philosophy, and Sociology of Science, Volume 4. 1992 ISBN 0-7923-1992-3
147. L. Embree (ed.): *Metaarchaeology*. Reflections by Archaeologists and Philosophers. 1992 ISBN 0-7923-2023-9
148. S. French and H. Kamminga (eds.): *Correspondence, Invariance and Heuristics.* Essays in Honour of Heinz Post. 1993 ISBN 0-7923-2085-9
149. M. Bunzl: *The Context of Explanation*. 1993 ISBN 0-7923-2153-7
150. I.B. Cohen (ed.): *The Natural Sciences and the Social Sciences.* Some Critical and Historical Perspectives. 1994 ISBN 0-7923-2223-1
151. K. Gavroglu, Y. Christianidis and E. Nicolaidis (eds.): *Trends in the Historiography of Science.* 1994 ISBN 0-7923-2255-X
152. S. Poggi and M. Bossi (eds.): *Romanticism in Science.* Science in Europe, 1790–1840. 1994 ISBN 0-7923-2336-X
153. J. Faye and H.J. Folse (eds.): *Niels Bohr and Contemporary Philosophy.* 1994 ISBN 0-7923-2378-5
154. C.C. Gould and R.S. Cohen (eds.): *Artifacts, Representations, and Social Practice.* Essays for Marx W. Wartofsky. 1994 ISBN 0-7923-2481-1
155. R.E. Butts: *Historical Pragmatics.* Philosophical Essays. 1993 ISBN 0-7923-2498-6
156. R. Rashed: *The Development of Arabic Mathematics: Between Arithmetic and Algebra.* Translated from French by A.F.W. Armstrong. 1994 ISBN 0-7923-2565-6
157. I. Szumilewicz-Lachman (ed.): *Zygmunt Zawirski: His Life and Work.* With Selected Writings on Time, Logic and the Methodology of Science. Translations by Feliks Lachman. Ed. by R.S. Cohen, with the assistance of B. Bergo. 1994 ISBN 0-7923-2566-4
158. S.N. Haq: *Names, Natures and Things.* The Alchemist J̄abir ibn Ḣayyān and His *Kitāb al-Ahjār* (Book of Stones). 1994 ISBN 0-7923-2587-7
159. P. Plaass: *Kant's Theory of Natural Science*. Translation, Analytic Introduction and Commentary by Alfred E. and Maria G. Miller. 1994 ISBN 0-7923-2750-0
160. J. Misiek (ed.): *The Problem of Rationality in Science and its Philosophy.* On Popper vs. Polanyi. The Polish Conferences 1988–89. 1995 ISBN 0-7923-2925-2
161. I.C. Jarvie and N. Laor (eds.): *Critical Rationalism, Metaphysics and Science*. Essays for Joseph Agassi, Volume I. 1995 ISBN 0-7923-2960-0
162. I.C. Jarvie and N. Laor (eds.): *Critical Rationalism, the Social Sciences and the Humanities*. Essays for Joseph Agassi, Volume II. 1995 ISBN 0-7923-2961-9
Set (161–162) ISBN 0-7923-2962-7
163. K. Gavroglu, J. Stachel and M.W. Wartofsky (eds.): *Physics, Philosophy, and the Scientific Community.* Essays in the Philosophy and History of the Natural Sciences and Mathematics. In Honor of Robert S. Cohen. 1995 ISBN 0-7923-2988-0

164. K. Gavroglu, J. Stachel and M.W. Wartofsky (eds.): *Science, Politics and Social Practice.* Essays on Marxism and Science, Philosophy of Culture and the Social Sciences. In Honor of Robert S. Cohen. 1995 ISBN 0-7923-2989-9
165. K. Gavroglu, J. Stachel and M.W. Wartofsky (eds.): *Science, Mind and Art*. Essays on Science and the Humanistic Understanding in Art, Epistemology, Religion and Ethics. Essays in Honor of Robert S. Cohen. 1995 ISBN 0-7923-2990-2
Set (163–165) ISBN 0-7923-2991-0
166. K.H. Wolff: *Transformation in the Writing.* A Case of Surrender-and-Catch. 1995
ISBN 0-7923-3178-8
167. A.J. Kox and D.M. Siegel (eds.): *No Truth Except in the Details.* Essays in Honor of Martin J. Klein. 1995 ISBN 0-7923-3195-8
168. J. Blackmore: *Ludwig Boltzmann, His Later Life and Philosophy, 1900–1906.* Book One: A Documentary History. 1995 ISBN 0-7923-3231-8
169. R.S. Cohen, R. Hilpinen and R. Qiu (eds.): *Realism and Anti-Realism in the Philosophy of Science.* Beijing International Conference, 1992. 1996 ISBN 0-7923-3233-4
170. I. Kuçuradi and R.S. Cohen (eds.): *The Concept of Knowledge.* The Ankara Seminar. 1995 ISBN 0-7923-3241-5
171. M.A. Grodin (ed.): *Meta Medical Ethics*: The Philosophical Foundations of Bioethics. 1995 ISBN 0-7923-3344-6
172. S. Ramirez and R.S. Cohen (eds.): *Mexican Studies in the History and Philosophy of Science.* 1995 ISBN 0-7923-3462-0
173. C. Dilworth: *The Metaphysics of Science*. An Account of Modern Science in Terms of Principles, Laws and Theories. 1995 ISBN 0-7923-3693-3
174. J. Blackmore: *Ludwig Boltzmann, His Later Life and Philosophy, 1900–1906* Book Two: The Philosopher. 1995 ISBN 0-7923-3464-7
175. P. Damerow: *Abstraction and Representation.* Essays on the Cultural Evolution of Thinking. 1996 ISBN 0-7923-3816-2
176. M.S. Macrakis: *Scarcity's Ways: The Origins of Capital.* A Critical Essay on Thermodynamics, Statistical Mechanics and Economics. 1997 ISBN 0-7923-4760-9
177. M. Marion and R.S. Cohen (eds.): *Québec Studies in the Philosophy of Science.* Part I: Logic, Mathematics, Physics and History of Science. Essays in Honor of Hugues Leblanc. 1995 ISBN 0-7923-3559-7
178. M. Marion and R.S. Cohen (eds.): *Québec Studies in the Philosophy of Science.* Part II: Biology, Psychology, Cognitive Science and Economics. Essays in Honor of Hugues Leblanc. 1996 ISBN 0-7923-3560-0
Set (177–178) ISBN 0-7923-3561-9
179. Fan Dainian and R.S. Cohen (eds.): *Chinese Studies in the History and Philosophy of Science and Technology.* 1996 ISBN 0-7923-3463-9
180. P. Forman and J.M. Sánchez-Ron (eds.): *National Military Establishments and the Advancement of Science and Technology.* Studies in 20th Century History. 1996
ISBN 0-7923-3541-4
181. E.J. Post: *Quantum Reprogramming*. Ensembles and Single Systems: A Two-Tier Approach to Quantum Mechanics. 1995 ISBN 0-7923-3565-1
182. A.I. Tauber (ed.): *The Elusive Synthesis: Aesthetics and Science.* 1996
ISBN 0-7923-3904-5
183. S. Sarkar (ed.): *The Philosophy and History of Molecular Biology: New Perspectives.* 1996 ISBN 0-7923-3947-9

184. J.T. Cushing, A. Fine and S. Goldstein (eds.): *Bohmian Mechanics and Quantum Theory: An Appraisal.* 1996 ISBN 0-7923-4028-0
185. K. Michalski: *Logic and Time.* An Essay on Husserl's Theory of Meaning. 1996 ISBN 0-7923-4082-5
186. G. Munévar (ed.): *Spanish Studies in the Philosophy of Science.* 1996 ISBN 0-7923-4147-3
187. G. Schubring (ed.): *Hermann Günther Graßmann (1809–1877): Visionary Mathematician, Scientist and Neohumanist Scholar.* Papers from a Sesquicentennial Conference. 1996 ISBN 0-7923-4261-5
188. M. Bitbol: *Schrödinger's Philosophy of Quantum Mechanics.* 1996 ISBN 0-7923-4266-6
189. J. Faye, U. Scheffler and M. Urchs (eds.): *Perspectives on Time.* 1997 ISBN 0-7923-4330-1
190. K. Lehrer and J.C. Marek (eds.): *Austrian Philosophy Past and Present*. Essays in Honor of Rudolf Haller. 1996 ISBN 0-7923-4347-6
191. J.L. Lagrange: *Analytical Mechanics*. Translated and edited by Auguste Boissonade and Victor N. Vagliente. Translated from the *Mécanique Analytique, novelle édition* of 1811. 1997 ISBN 0-7923-4349-2
192. D. Ginev and R.S. Cohen (eds.): *Issues and Images in the Philosophy of Science.* Scientific and Philosophical Essays in Honour of Azarya Polikarov. 1997 ISBN 0-7923-4444-8
193. R.S. Cohen, M. Horne and J. Stachel (eds.): *Experimental Metaphysics.* Quantum Mechanical Studies for Abner Shimony, Volume One. 1997 ISBN 0-7923-4452-9
194. R.S. Cohen, M. Horne and J. Stachel (eds.): *Potentiality, Entanglement and Passion-at-a-Distance*. Quantum Mechanical Studies for Abner Shimony, Volume Two. 1997 ISBN 0-7923-4453-7; Set 0-7923-4454-5
195. R.S. Cohen and A.I. Tauber (eds.): *Philosophies of Nature: The Human Dimension.* 1997 ISBN 0-7923-4579-7
196. M. Otte and M. Panza (eds.): *Analysis and Synthesis in Mathematics.* History and Philosophy. 1997 ISBN 0-7923-4570-3
197. A. Denkel: *The Natural Background of Meaning*. 1999 ISBN 0-7923-5331-5
198. D. Baird, R.I.G. Hughes and A. Nordmann (eds.): *Heinrich Hertz: Classical Physicist, Modern Philosopher.* 1999 ISBN 0-7923-4653-X
199. A. Franklin: *Can That be Right?* Essays on Experiment, Evidence, and Science. 1999 ISBN 0-7923-5464-8
200. D. Raven, W. Krohn and R.S. Cohen (eds.): *The Social Origins of Modern Science.* 2000 ISBN 0-7923-6457-0
201. Reserved
202. Reserved
203. B. Babich and R.S. Cohen (eds.): *Nietzsche, Theories of Knowledge, and Critical Theory.* Nietzsche and the Sciences I. 1999 ISBN 0-7923-5742-6
204. B. Babich and R.S. Cohen (eds.): *Nietzsche, Epistemology, and Philosophy of Science.* Nietzsche and the Science II. 1999 ISBN 0-7923-5743-4
205. R. Hooykaas: *Fact, Faith and Fiction in the Development of Science*. The Gifford Lectures given in the University of St Andrews 1976. 1999 ISBN 0-7923-5774-4
206. M. Fehér, O. Kiss and L. Ropolyi (eds.): *Hermeneutics and Science.* 1999 ISBN 0-7923-5798-1

207. R.M. MacLeod (ed.): *Science and the Pacific War.* Science and Survival in the Pacific, 1939–1945. 1999 ISBN 0-7923-5851-1
208. I. Hanzel: *The Concept of Scientific Law in the Philosophy of Science and Epistemology.* A Study of Theoretical Reason. 1999 ISBN 0-7923-5852-X
209. G. Helm; R.J. Deltete (ed./transl.): *The Historical Development of Energetics.* 1999 ISBN 0-7923-5874-0
210. A. Orenstein and P. Kotatko (eds.): *Knowledge, Language and Logic.* Questions for Quine. 1999 ISBN 0-7923-5986-0
211. R.S. Cohen and H. Levine (eds.): *Maimonides and the Sciences.* 2000 ISBN 0-7923-6053-2
212. H. Gourko, D.I. Williamson and A.I. Tauber (eds.): *The Evolutionary Biology Papers of Elie Metchnikoff.* 2000 ISBN 0-7923-6067-2
213. S. D'Agostino: *A History of the Ideas of Theoretical Physics.* Essays on the Nineteenth and Twentieth Century Physics. 2000 ISBN 0-7923-6094-X
214. S. Lelas: *Science and Modernity.* Toward An Integral Theory of Science. 2000 ISBN 0-7923-6303-5
215. E. Agazzi and M. Pauri (eds.): *The Reality of the Unobservable.* Observability, Unobservability and Their Impact on the Issue of Scientific Realism. 2000 ISBN 0-7923-6311-6
216. P. Hoyningen-Huene and H. Sankey (eds.): *Incommensurability and Related Matters*. 2001 ISBN 0-7923-6989-0
217. A. Nieto-Galan: *Colouring Textiles*. A History of Natural Dyestuffs in Industrial Europe. 2001 ISBN 0-7923-7022-8
218. J. Blackmore, R. Itagaki and S. Tanaka (eds.): *Ernst Mach's Vienna 1895–1930*. Or Phenomenalism as Philosophy of Science. 2001 ISBN 0-7923-7122-4
219. R. Vihalemm (ed.): *Estonian Studies in the History and Philosophy of Science*. 2001 ISBN 0-7923-7189-5
220. W. Lefèvre (ed.): *Between Leibniz, Newton, and Kant.* Philosophy and Science in the Eighteenth Century. 2001 ISBN 0-7923-7198-4
221. T.F. Glick, M. Á. Puig-Samper and R. Ruiz (eds.): *The Reception of Darwinism in the Iberian World.* Spain, Spanish America and Brazil. 2001 ISBN 1-4020-0082-0
222. U. Klein (ed.): *Tools and Modes of Representation in the Laboratory Sciences*. 2001 ISBN 1-4020-0100-2
223. P. Duhem: *Mixture and Chemical Combination.* And Related Essays. Edited and translated, with an introduction, by Paul Needham. 2002 ISBN 1-4020-0232-7
224. J.C. Boudri: *What was Mechanical about Mechanics*. The Concept of Force Betweem Metaphysics and Mechanics from Newton to Lagrange. 2002 ISBN 1-4020-0233-5
225. B.E. Babich (ed.): *Hermeneutic Philosophy of Science, Van Gogh's Eyes, and God*. Essays in Honor of Patrick A. Heelan, S.J. 2002 ISBN 1-4020-0234-3
226. D. Davies Villemaire: *E.A. Burtt, Historian and Philosopher*. A Study of the Author of The Metaphysical Foundations of Modern Physical Science. 2002 ISBN 1-4020-0428-1
227. L.J. Cohen: *Knowledge and Language*. Selected Essays of L. Jonathan Cohen. Edited and with an introduction by James Logue. 2002 ISBN 1-4020-0474-5
228. G.E. Allen and R.M. MacLeod (eds.): *Science, History and Social Activism: A Tribute to Everett Mendelsohn*. 2002 ISBN 1-4020-0495-0
229. O. Gal: *Meanest Foundations and Nobler Superstructures*. Hooke, Newton and the "Compounding of the Celestiall Motions of the Planetts". 2002 ISBN 1-4020-0732-9

230. R. Nola: *Rescuing Reason*. A Critique of Anti-Rationalist Views of Science and Knowledge. 2003 Hb: ISBN 1-4020-1042-7; Pb ISBN 1-4020-1043-5
231. J. Agassi: *Science and Culture*. 2003 ISBN 1-4020-1156-3
232. M.C. Galavotti (ed.): *Observation and Experiment in the Natural and Social Science.* 2003 ISBN 1-4020-1251-9
233. A. Simões, A. Carneiro and M.P. Diogo (eds.): *Travels of Learning*. A Geography of Science in Europe. 2003 ISBN 1-4020-1259-4
234. A. Ashtekar, R. Cohen, D. Howard, J. Renn, S. Sarkar and A. Shimony (eds.): *Revisiting the Foundations of Relativistic Physics*. Festschrift in Honor of John Stachel. 2003 ISBN 1-4020-1284-5
235. R.P. Farell: *Feyerabend and Scientific Values*. Tightrope-Walking Rationality. 2003 ISBN 1-4020-1350-7
236. D. Ginev (ed.): *Bulgarian Studies in the Philosophy of Science*. 2003 ISBN 1-4020-1496-1
237. C. Sasaki: *Descartes Mathematical Thought*. 2003 ISBN 1-4020-1746-4
238. K. Chemla (ed.): *History of Science, History of Text*. 2004 ISBN 1-4020-2320-0
239. C.R. Palmerino and J.M.M.H. Thijssen (eds.): *The Reception of the Galilean Science of Motion in Seventeenth-Century Europe*. 2004 ISBN 1-4020-2454-1
240. J. Christianidis (ed.): *Classics in the History of Greek Mathematics*. 2004 ISBN 1-4020-0081-2

Also of interest:
R.S. Cohen and M.W. Wartofsky (eds.): *A Portrait of Twenty-Five Years Boston Colloquia for the Philosophy of Science, 1960-1985*. 1985 ISBN Pb 90-277-1971-3

Previous volumes are still available.
